教育部「5G行動寬頻人才培育跨校教學聯盟計畫」
5G行動網路協定與核網技術聯盟中心

課程: 5G系統層模擬技術

第八週：實體層萃取、鏈結層效應與Effective SINR

國立中正大學
National Chung Cheng University

Outline

- 8.1 Link-to-System Interface
- 8.2 Effective SINR
  - Effective SINR Mapping(ESM)
  - Capacity ESM(CESM)
  - Exponential ESM(EESM)
  - Mutual Information ESM(MIESM)
- 8.3 EESM推導

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8.1 Link-to-System Interface

- Block error rate(BLER)並未被包含在系統層模擬中，而是在實體層(Link-level)模擬，因此SLS在模擬實際傳輸狀況前，需要向LLS取得BLER。
- SLS需要將數個資訊交給LLS，讓LLS去模擬出BLER，包含：Effective SINR、Modulation Scheme、Coding Rate、Block Size…等。

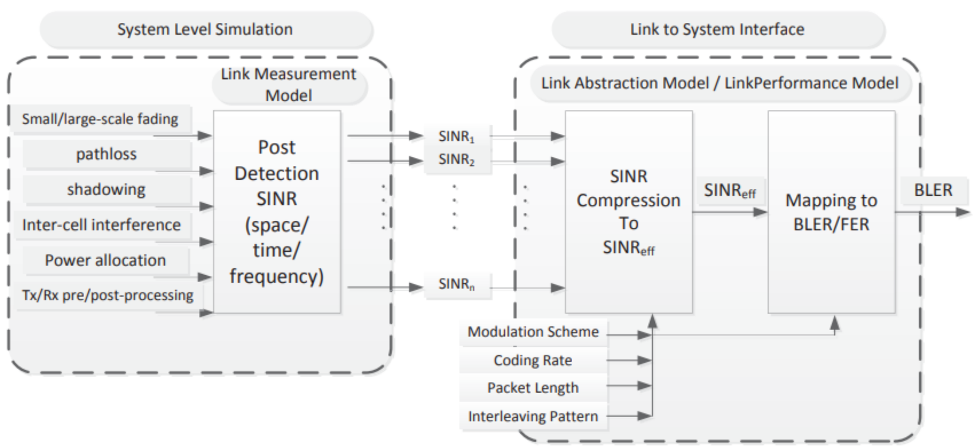

Fig. 2. Link-to-System interface mapping.

Ref:[1] Z. Hanzaz and H. D. Schotten, "Analysis of effective SINR mapping models for MIMO OFDM in LTE system"

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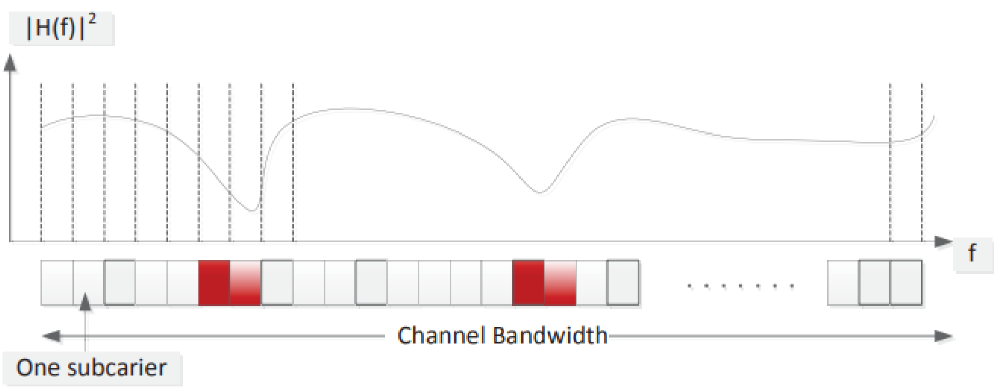
8.2 Effective SINR

- OFDM技術的運用，使得每個transport block中會有數個subcarriers，每個subcarrier所在的頻率皆不同
- Multipath fading會導致在不同subcarrier所感受到的干擾不同(frequency selective fading)，因此，每個subcarrier的SINR會不同
  - Different channel quality for different subcarrier



Ref:[1]　Fig. 1.　Effect of multipath fading on one transmitted block.

8.2 Effective SINR

- 由於，依據數個SINR值來決定一個transport block整體的BLER是十分的困難
  - 需要一個機制使數個SINR整合成一個SINR(Effective SINR)

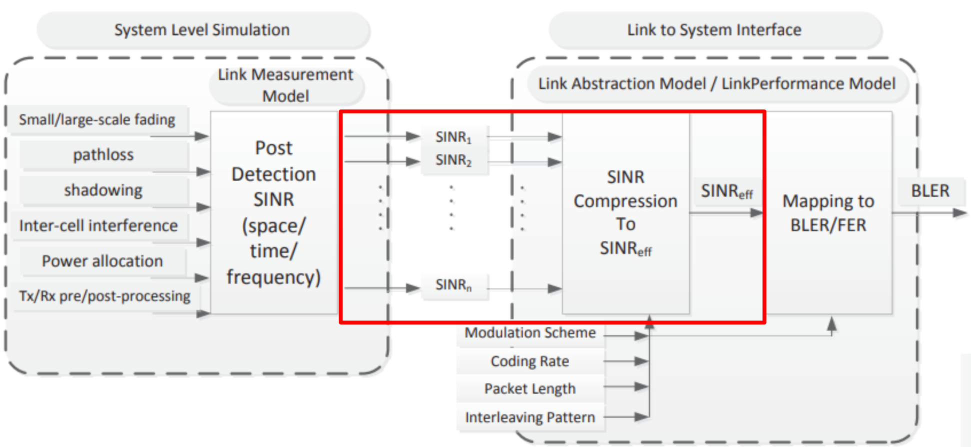


Fig. 2. Link-to-System interface mapping.

Ref:[1]

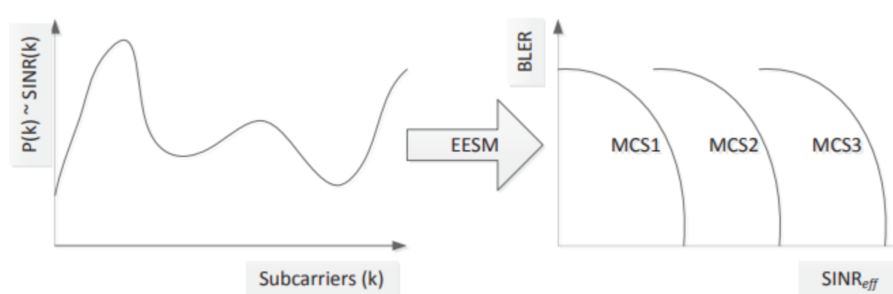


Fig. 3. Mapping in EESM model.

Effective SINR Mapping(ESM)

- 以下將簡單列出數個ESM作法
  - Capacity ESM(CESM)
  - Exponential ESM(EESM)
  - Mutual Information ESM(MIESM)

CESM

- [1]This approach is based on the channel capacity measure and it is called the Capacity Effective SINR Mapping (CESM). The main formula to calculate the information measure based channel capacity is as follows:

$$I_{\gamma} = \log(1+\gamma)$$

- The effective SINR value as follows:

先算出每個subcarrier 的capacity

$$SINR_{eff} = \beta * (2^{\left(\frac{1}{N}\sum_{k=1}^{N}\log_2\left(1+\frac{SINR_k}{\beta}\right)\right)} - 1)$$

得到平均的capacity後，回推為對應的SINR，即為effective SINR

Exponential ESM

- [1]The EESM model, as its name refers, computes the information measure based on exponential function φ(SINR) = exp (−SINRs) .
- The final derivation of the formula is as follows:

$$\mathrm{SINR}_{eff} = -\beta \ln\left(\frac{1}{N}\sum_{k=1}^{N}\exp\left(-\frac{SINR_k}{\beta}\right)\right)$$

Exponential ESM

- Assume the 2-state channel is characterized by an SNR vector **γ**=[γ1 γ2], where the two states γ1 and γ2 occur with probability p_1 and p_2 respectively
  - What's the expected BPSK (UB) BER Pb?
  - $P_{b1} = e^{-\gamma 1}$, $P_{b2} = e^{-\gamma 2}$
  - $e^{-\gamma eff} = E[Pb] = p_1 * e^{-\gamma 1} + p_2 * e^{-\gamma 2}$
  - $\gamma_{eff} = -\ln(p_1 * e^{-\gamma 1} + p_2 * e^{-\gamma 2})$
    - Assume $p_1=p_2=0.5$, γ1 = 0dB (1), γ2 = 6dB (4)
    - $P_{b1} = 0.3679$, $P_{b2} = 0.0183$
    - E[Pb] = 0.1931, γ_{eff} = 2.16dB (1.645)

詳細推導在8.3 EESM推導

- The generalized exponential ESM

$$\gamma_{eff} = -\beta \ln\left(\frac{1}{N} \sum_{i=1}^{N} e^{-\frac{\gamma_i}{\beta}} \right)$$

Reference: 3GPP R1-031303

MIESM

- The information measure is made based on the mutual information function *φ*(*SINR*) = *I*(*SINRk*).
- The general formula as follows:

$$\mathrm{SINR}_{eff} = \beta . I^{-1}\left(\frac{1}{N}\sum_{k=1}^{N} I\left(\frac{SINR_k}{\beta}\right)\right)$$

$$\mathrm{I}_{mp}(x) = m_p - E_y\left\{\frac{1}{2^{mp}}\sum_{i=1}^{mp}\sum_{b=0}^{1}\sum_{z\in X_b^i}\log\frac{\sum_{\hat{x}\in X}\exp(A)}{\sum_{\tilde{x}\in X_b^i}\exp(A)}\right\}$$

$$\text{where } A = -|Y - \sqrt{x/\beta}(\tilde{x} - z)|^2$$

Ref:[1]

- where, I_{mp} is the bits per symbol for the selected modulation scheme, *X* is the set of symbols, X_b^i is the set of symbols for which bit i equals b. *Y* is zero mean unit variance complex Gaussian variable.

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Gaussian (Normal) Distribution

$$\mathcal{N}(\mu, \sigma^2)$$

- The N**ormal or Gaussian distribution**, is an important family of *continuous* probability distributions
- The mean ("average", μ) and variance (standard deviation squared, σ^2) are the defining parameters
- The **standard normal distribution** is the normal distribution with zero mean ($\mu=0$) and unity variance ($\sigma^2=1$)
- Many measurements, from psychological to thermal noise can be approximated by the Gaussian distribution.

Gaussian RV

- A Gaussian RV, with mean value m, variance σ^2, is denoted as $N(m, \sigma^2)$. Probability density function(pdf) is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}, x \in R$$

- Also known as normal distribution.
- N(0,1): standard normal distribution, pdf and CDF are given respectively as:

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad F_X(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$$

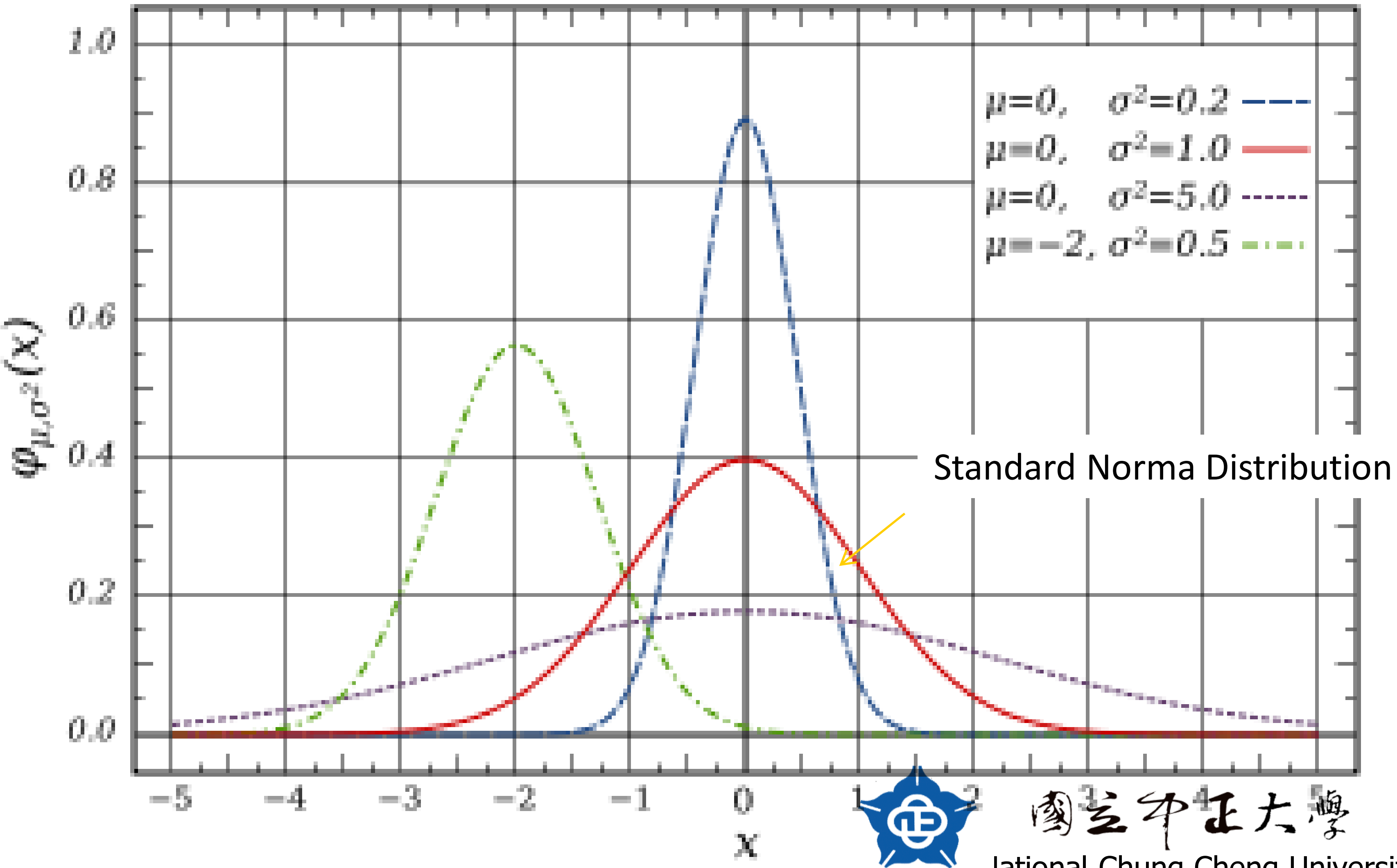
- Define function Q(x) as the tail integration of normal Gaussian:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{x^2}{2}} dx$$

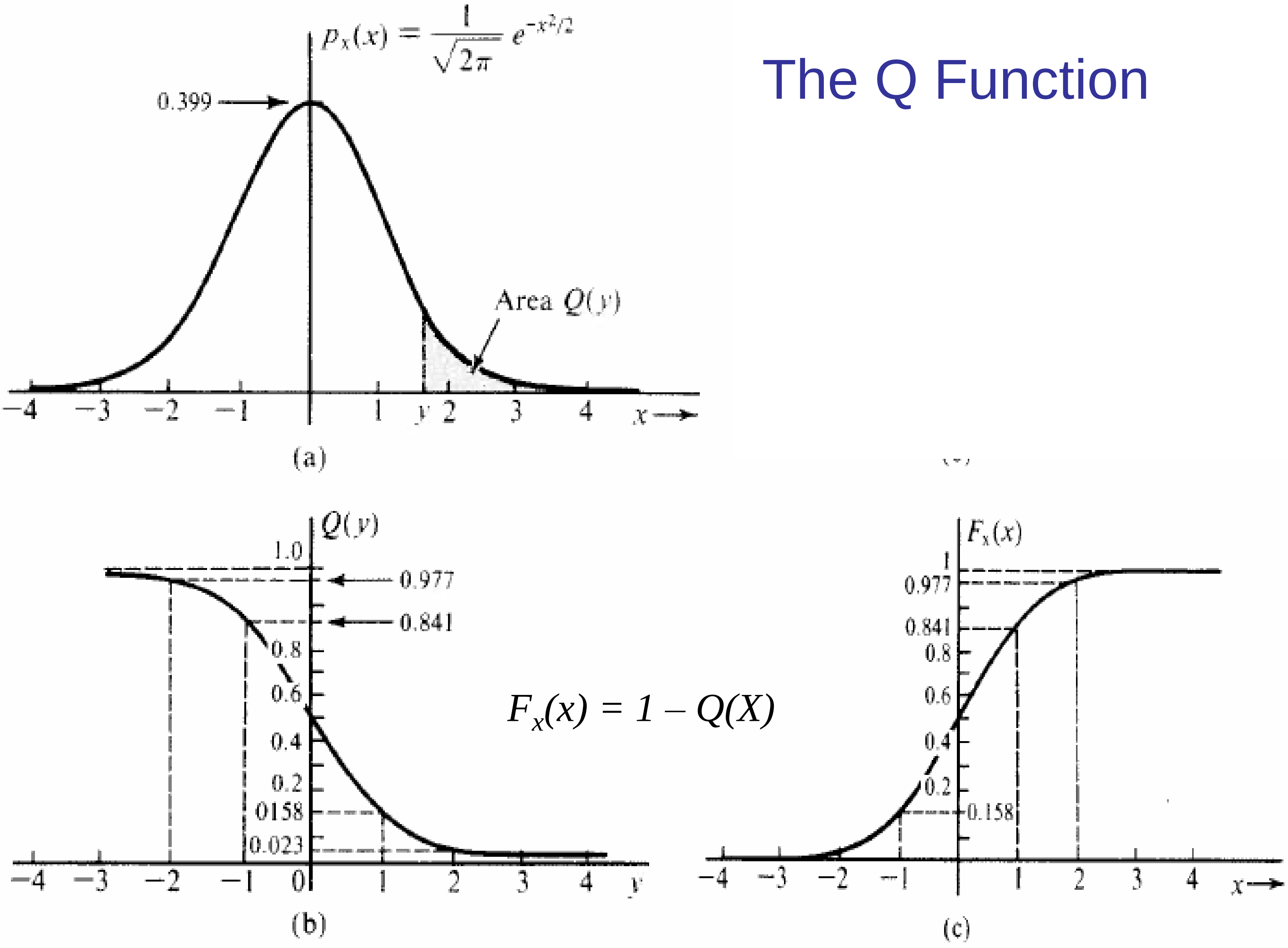
- From definition of Q-function, we can see that

$$F_X(x) = 1 - Q(x) = Q(-x)$$

PDF of Gaussian Distribution



The Q Function



$$F_x(x) = 1 - Q(X)$$

Additive white Gaussian noise and error

- Receive signal: $\mathbf{y}(t) = \mathbf{H}(t)\mathbf{x}(t) + \mathbf{n}(t)$
  - a random **n(**t**)** is called white process if it has a flat power spectral density (PSD)
  - The white process, from thermal noise, can be approximated as a Gaussian process
  - The PSD is given by

    h=6.6e-34 Joules sec: Planck's constant
    k=1.38e-23 Joules/Kelvin: Boltzmann's constant
    T: temperature in Kelvin degree

$$S_n(f) = \frac{\hbar f}{2\left(e^{\hbar f/kT} - 1\right)} \quad \text{watts/Hz}$$

$$\approx \frac{kT}{2} \text{watts/Hz} = \frac{N_0}{2} \text{watts/Hz}$$

  - N_0 [dBm/Hz]
    = 10Log_{10}(1.38e-23 x 290[16.85ºC] x 1e3[mW/W])
    = -174 [dBm/Hz]
- BER is the ratio of erroneous bits to correct bits
- BER is an important measure of digital communication link
- BER depends on the signal and noise power (Signal to Noise Ratio)

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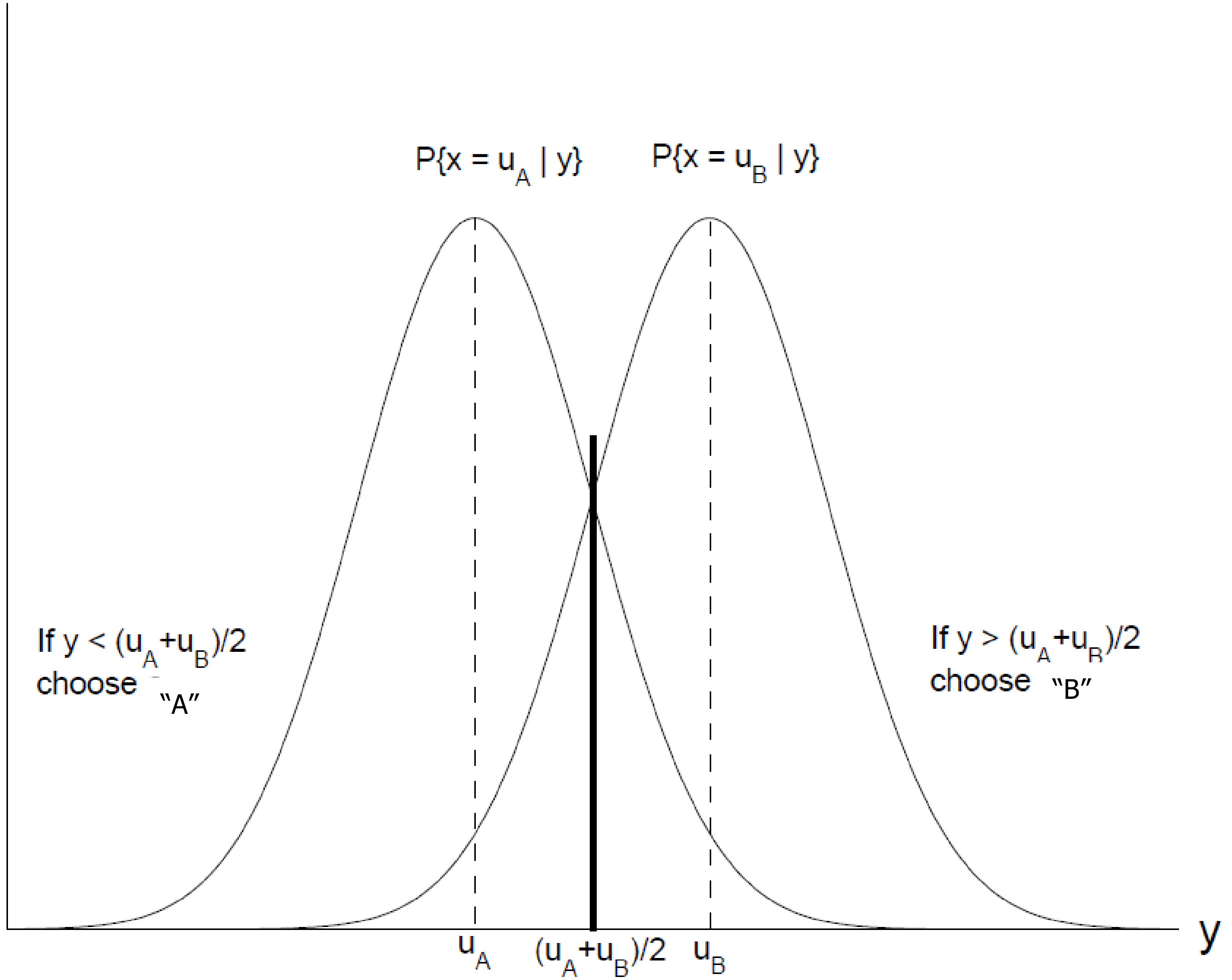
Maximum likelihood selection: the nearest neighbor

- $\mathbf{y} = \mathbf{x} + \mathbf{n}$, $\mathbf{n} \sim N(0, N_0/2)$



Error probability

$$P_b = P_r\left\{ y < \frac{u_A + u_B}{2} \middle| \text{ x=u}_\text{B} \right\} = P_r\left\{ y > \frac{u_A + u_B}{2} \middle| \text{ x=u}_\text{A} \right\}$$

$$= P_r\left\{ v > \frac{|u_B - u_A|}{2} \right\} = Q\left(\frac{|u_B - u_A|/2}{\sqrt{\frac{N_0}{2}}} \right) = Q\left(\frac{|u_B - u_A|}{2\sqrt{\frac{N_0}{2}}} \right)$$

令$v = y - u_A$ if $x = u_B$
或$v = y - u_B$ if $x = u_A$

$$\text{if } u_B = \sqrt{E_b}, u_A = -\sqrt{E_b} \rightarrow P_b = Q\left(\sqrt{\frac{2E_b}{N_0}} \right)$$

Q($\sqrt{2SNR}$) 就是
BPSK的bit error rate

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Q function upper Chernoff bound

- Generic Chernoff bound for a random variable X
  - For every t > 0

$$\Pr\{X \ge a\} = \Pr\{e^{tX} \ge e^{ta}\} \le E[e^{tX-ta}] = e^{-ta}E[e^{tX}]$$

- Assume X ~ N(0,1)

$$E[e^{tX}] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{tu} e^{-\frac{u^2}{2}} du = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{(u-t)^2}{2}+\frac{t^2}{2}} du = e^{\frac{t^2}{2}}$$

- Let t = x in generic Chernoff bound inequality

$$Q(x) = P\{X > x\} \le e^{-tx}E[e^{tX}] = e^{-x^2}e^{\frac{x^2}{2}} = e^{-\frac{x^2}{2}}$$

$$Q(x) \le e^{-x^2/2}$$

BPSK BER $P_b = Q(\sqrt{2SNR}) \le e^{-SNR}$

BPSK symbols on multiple subcarriers

- Bit length *L* block error rate of BPSK without coding/scrambling
  - BLEP = $1 - (1 - P_b)^L$
- How about different SNR values in each sub-carriers of OFDM ?
  - For each subcarrier → costly input for link level simulation
  - If only 2 states → (5 + 15) / 2 ?? (5dB + 15dB) /2 ??
- Basic principles of an Effective SINR Mapping (ESM)
  - From an instantaneous channel state, such as the instantaneous SINR for each sub-carrier in case of OFDM, to a corresponding block-error probability (BLEP)
  - Map the instantaneous channel state, e.g. the set of sub-carrier SNRs $\{\gamma_k\}$ in case of OFDM, into an instantaneous effective SNR γ_{eff} (a scalar value)
  - The effective SNR is then used to find an estimate of the block-error probability from basic AWGN link-level performance
  - $\text{BLEP}(\{\gamma_k\}) \approx \text{BLEP}(\gamma_{eff})$

Exponential ESM

- Assume the 2-state channel is characterized by an SNR vector **γ**=[γ1 γ2], where the two states γ1 and γ2 occur with probability p_1 and p_2 respectively
  - What's the expected BPSK (UB) BER Pb?
  - $P_{b1} = e^{-\gamma 1}$, $P_{b2} = e^{-\gamma 2}$
  - $e^{-\gamma eff} = E[Pb] = p_1 * e^{-\gamma 1} + p_2 * e^{-\gamma 2}$
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$$\gamma_{eff} = -\beta \ln\left(\frac{1}{N} \sum_{i=1}^{N} e^{-\frac{\gamma_i}{\beta}} \right)$$

Reference: 3GPP R1-031303

Representation of Signals

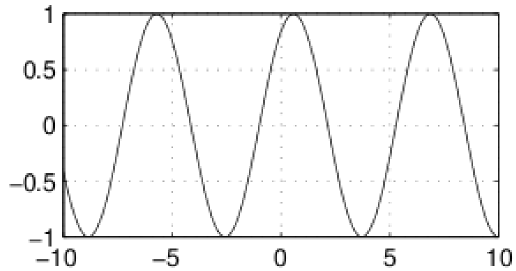
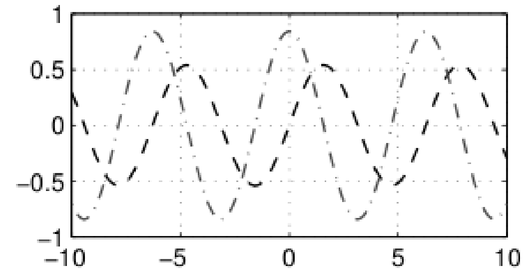
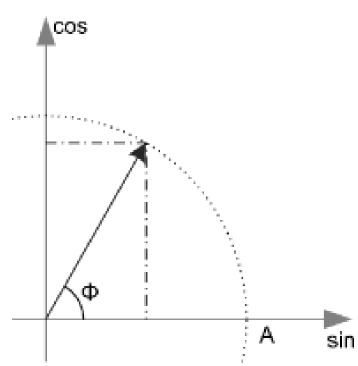
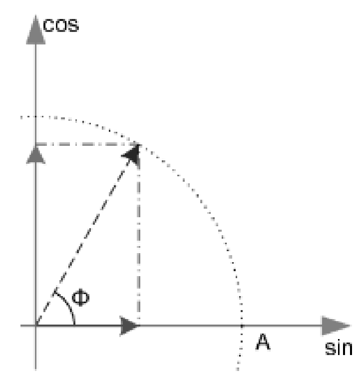
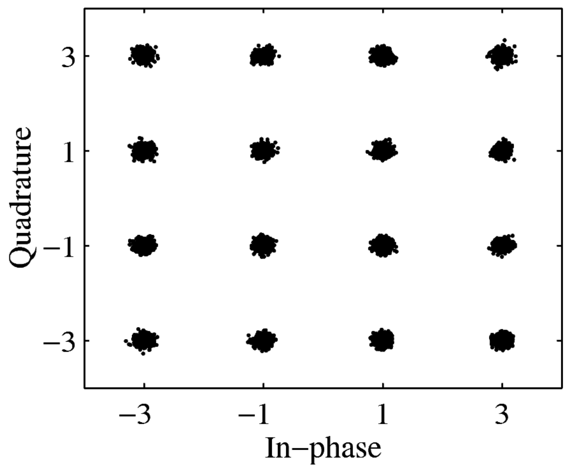
	Envelope/Phase $\mathbf{A}, \phi$	In-phase/Quadrature $(\mathbf{A}\sin\phi), (\mathbf{A}\cos\phi)$
Time	$\mathbf{A}\sin(\omega t + \phi)$	$(\mathbf{A}\mathbf{cos}\phi)\sin(\omega t)$ $+(\mathbf{A}\mathbf{sin}\phi)\cos(\omega t)$
Waveform		
Vector		

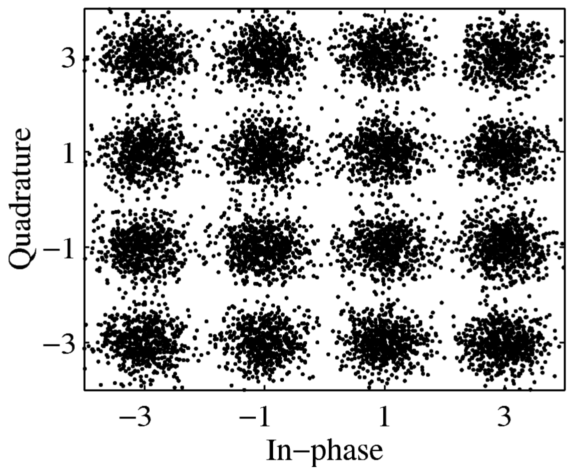
Table 3.1: Signal representations of $A\sin(\omega t + \phi)$

High order modulation

- Nearest neighbor detection



AWGN N(0,0.01)



AWGN N(0,0.25)

EESM Derivation of high order modulation

- Basic idea is to find an equivalent SIR in the AWGN channel that results in the same BLER, using the Union-Chernoff bound to relate the error probability to the corresponding SIR in a channel/subchannel with an approximately constant channel response
- An adjustment factor (b) is necessary for QPSK and higher-order modulation schemes
- Less accurate in case of M-QAM, for M > 4
- The corresponding BLER performance can be found through table look-up from the pre-computed AWGN BLER performance for each modulation and coding combination.

Reference: 3GPP R1-031303

Approximation of error probability

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BPSK		$P_b = Q\left(\sqrt{2\gamma_b}\right)$
QPSK	$P_s \approx 2Q\left(\sqrt{\gamma_s}\right)$	$P_b \approx Q\left(\sqrt{2\gamma_b}\right)$
MPSK	$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)\right)$	$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{2\gamma_b \log_2 M}\sin\left(\frac{\pi}{M}\right)\right)$
M-QAM	$P_s \approx 4Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\overline{\gamma}_b \log_2 M}{M-1}}\right)$

Table 1: Approximate symbol and bit error probabilities for coherent modulation

Ref: [2]http://www.unilim.fr/pages_perso/vahid/notes/ber_awgn.pdf

參考資料

[1] Z. Hanzaz and H. D. Schotten, ‘‘Analysis of effective SINR mapping models for MIMO OFDM in LTE system,’’ in Proc. 9th Int. Wireless Commun. Mobile Comput. Conf. (IWCMC), Sardinia, Italy, Jul. 2013, pp. 1509–1515.

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