教育部「5G行動寬頻人才培育跨校教學聯盟計畫」
5G行動網路協定與核網技術聯盟中心

課程: 5G系統層模擬技術

第五週：Channel Model (1/2)

Large-Scale Fading & Small-Scale Fading

國立中正大學
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Outline

5.1 Large-Scale Fading

- Introduction to Radio Wave Propagation
- The Three Basic Propagation Mechanisms
- Free Space Propagation Model
- Free Space Path Loss
- Path Loss Model for General Ground-Wave Propagations
- Log-Normal Shadowing
- Large-Scale Fading
- Percentage of Coverage Area
- Empirical Models for Outdoor Path Loss
- Path Loss Model for General Ground-Wave Propagations – Hata's Model

5.2 Small-Scale Fading

- Introduction
- Doppler Shift
- Impulse Response Model of a Multipath Channel
- Channel Correlation Functions and Spectrum
- Parameters of Mobile Multipath Channels
- Types of Small-Scale Fading
- Received Envelope and Phase Distribution

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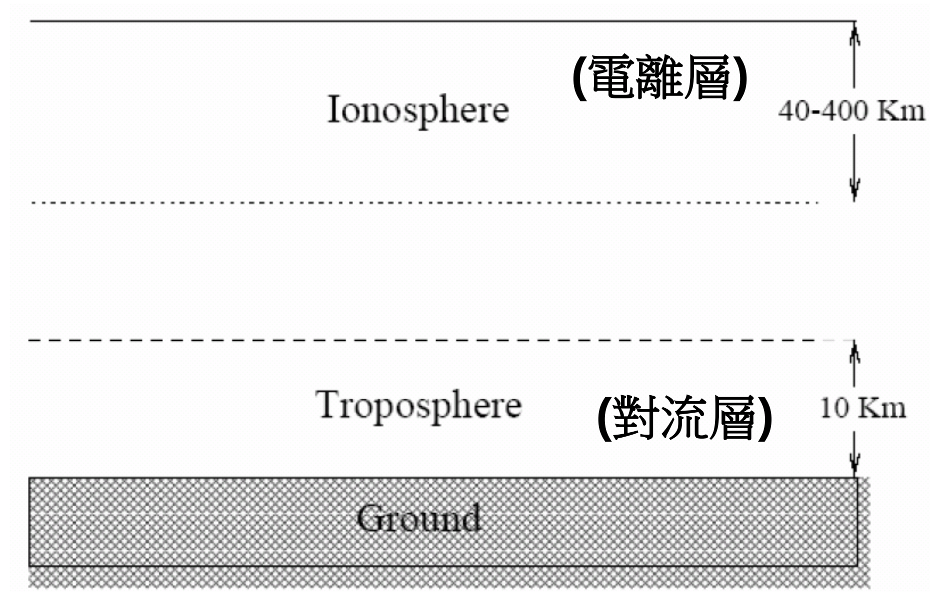
Introduction to Radio Wave Propagation(1/8)

- The term "radio communication" refers to the transfer of information using electromagnetic (EM) waves over the atmosphere, rather using wired-line medium.
- The radio medium is susceptible to interference (from competing users), noise, blockage and multi-path.
  - These impediments result in attenuation, delay and even complete distortion of the transmitted signal.
- Thus, in order to be able to design good wireless systems, it becomes imperative to have a good understanding of radio wave propagation.

Introduction to Radio Wave Propagation(2/8)

- Depending on the frequency of transmission, there are three main propagation means:
  - Ground wave
  - Troposphere wave
  - Ionosphere wave (or Sky wave)



Introduction to Radio Wave Propagation(3/8)

- **Ground-Wave Propagation**
  - **The ground wave travels in contact with the earth's surface by scattering off buildings, vegetation, hills, mountains, and other irregularities in the earth's surface.**
    - **Nevertheless, unless there is a LOS (Line of Sight) path between TX and RX, the ground wave propagation provides the dominant local signal at RX.**
  - **The ground-wave propagation effects all frequencies.**
  - **The signal dies off rapidly as the distance from the transmitter increases.**

Introduction to Radio Wave Propagation(4/8)

- **Troposphere-Wave Propagation**
  - **For the portion of EM radiation close to the earth's surface, troposphere wave results from the radio waves being reflected from the troposphere.**
  - **It is relatively inconsequential below 30 MHz.**
  - **This troposphere refraction can make VHF (30~300 MHz) communication possible over distances far greater than those by the ordinary ground waves.**
    - **➔For VHF and UHF propagation used for cellular and PCS communications, the troposphere wave becomes an annoyance that causes interference.**

Introduction to Radio Wave Propagation(5/8)

- **Sky-Wave (Ionosphere-Wave) Propagation**
  - **The ionosphere is where ions and electrons exist in sufficient quantities to reflect the radio waves.**
  - **The sky wave arises from radio waves that leave the antenna at angles somewhat above the horizontal.**
  - **Thus, the radio waves reaching the ionosphere can be sufficiently reflected by the ionosphere to the ground again at distances ranging from 0 to about 4,000 km from the transmitter through successive reflections.**
  - **This separation becomes more pronounced with increasing frequencies.**

Introduction to Radio Wave Propagation(6/8)

- The ground waves generally propagate according to three mechanisms:
  - Reflection (反射)
  - Diffraction (繞射)
  - Scattering (散射)
- The transmitted radio signal is not only susceptible to noise and interference (in additive manner), but also distorted by the propagation channel (in multiplicative manner).

Introduction to Radio Wave Propagation(7/8)

- "Fading" is used to describe the variations in received power with time caused by changes of the propagation channel.
  - Large-scale fading
  - Small-scale fading

Introduction to Radio Wave Propagation(8/8)

- **Large-scale fading**
  - **The variation in received signal power over relatively large distances due to path loss and shadowing**
    - **"Path loss" is the reduction in power (attenuation) of an electromagnetic wave as it propagates through space.**
    - **"Shadowing" is the attenuation of an electromagnetic radio signal by obstacles.**
- **Small-scale fading**
  - **The variation in received signal power occurs over very short distances (on the order of the signal wavelength)**
    - **Light speed $C = 3 \times 10^8$ m/s**

$$\Rightarrow \boxed{\overline{PL}(d)=\overline{PL}(d_0)-10n\log(d_0)+10n\log(d)}$$

$\frac{P_r(d)}{P_t}$ (dB)

$=1/\ \overline{PL}(d)$ (dB)

for path loss alone

$\overline{PL}(d)$

$=10\log\frac{P_t}{\overline{P}_r(d)}$

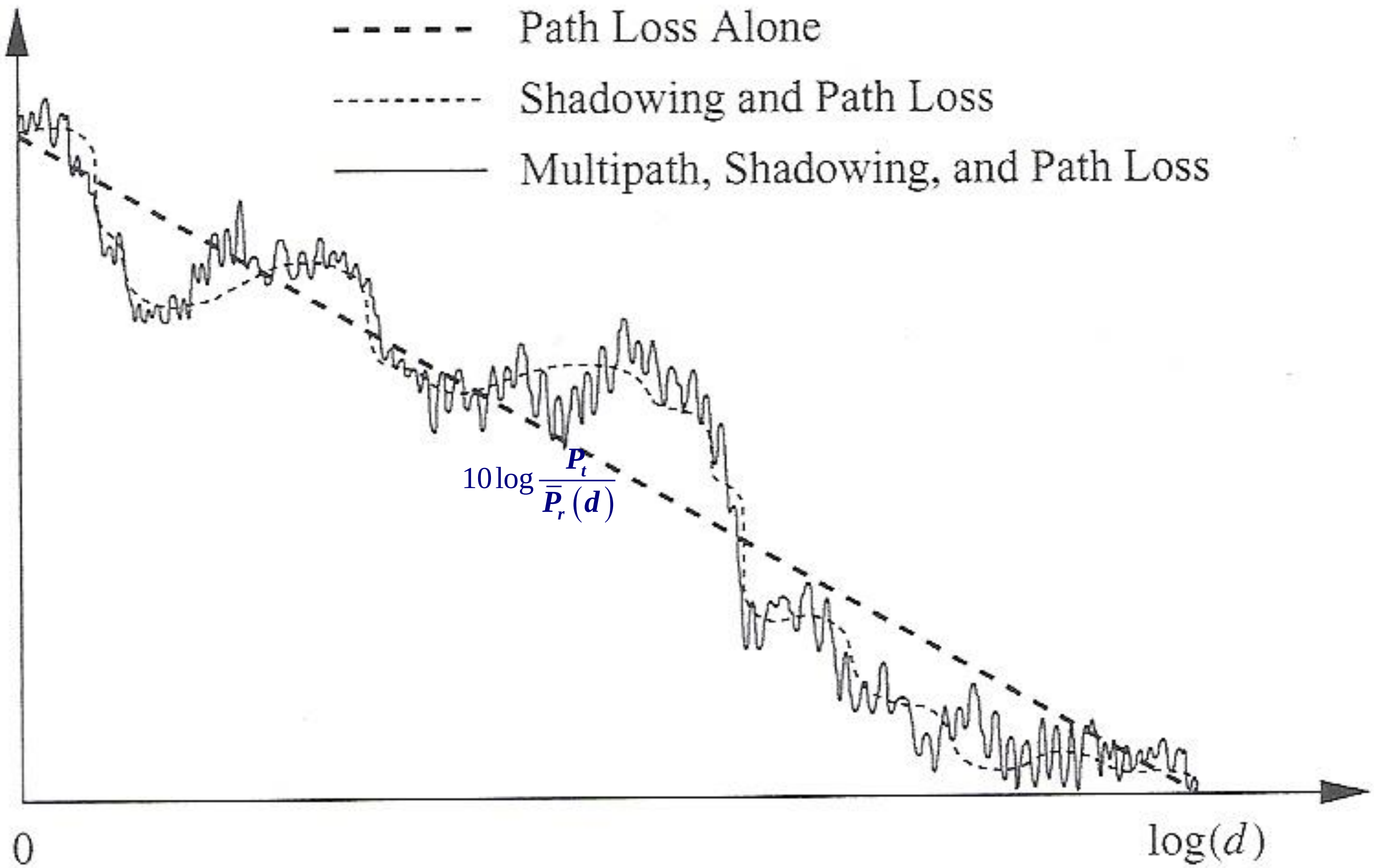


Figure 2.1: Path loss, shadowing, and multipath versus distance.

***d*: distance**

[3, p.28]

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The Three Basic Propagation Mechanisms(1/3)

1. **Reflection**
   - **Reflection occurs when a propagating electromagnetic (EM) wave impinges upon an object with dimensions that are very large compared to the wavelength of the propagating wave.**
     - **If the EM wave is incident on a dielectric, part of the energy is transmitted into the dielectric object, and part of the energy is reflected back.**
     - **If the EM wave is incident on a conductor, then all incident energy is reflected back.**

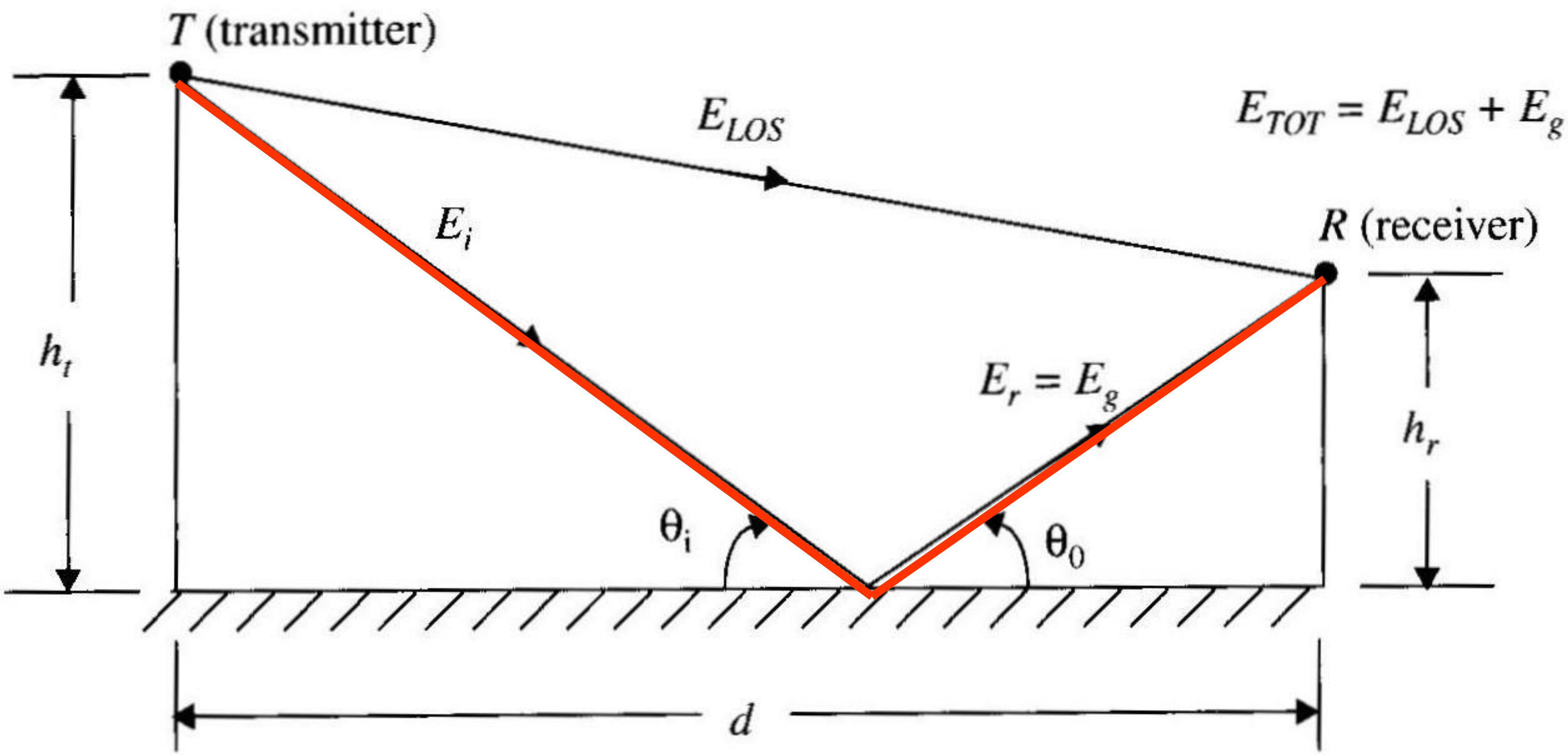

Figure 4.7 Two-ray ground reflection model.

[1, p.121]

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The Three Basic Propagation Mechanisms(2/3)

2. **Diffraction**
   - **Diffraction arises when the radio path between the TX and RX is obstructed by a surface that has sharp edges.**
   - **The transmitted signal would "bend around" the sharp edges to the receiver.**
     - **That is, the EM wave propagates behind the obstacle even when an LOS path does not exist.**
   - **Diffraction allows radio signals to propagate around the curved surface of the earth.**

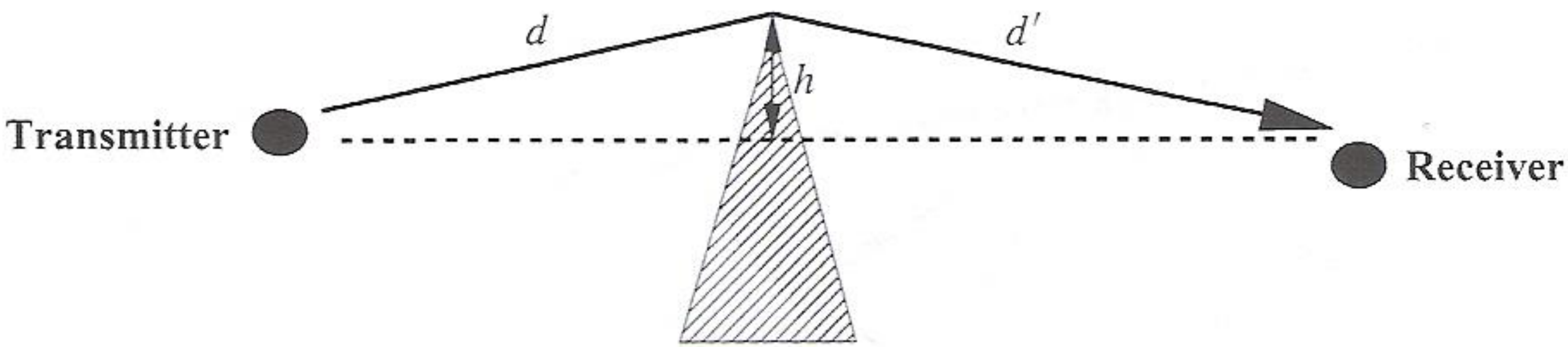
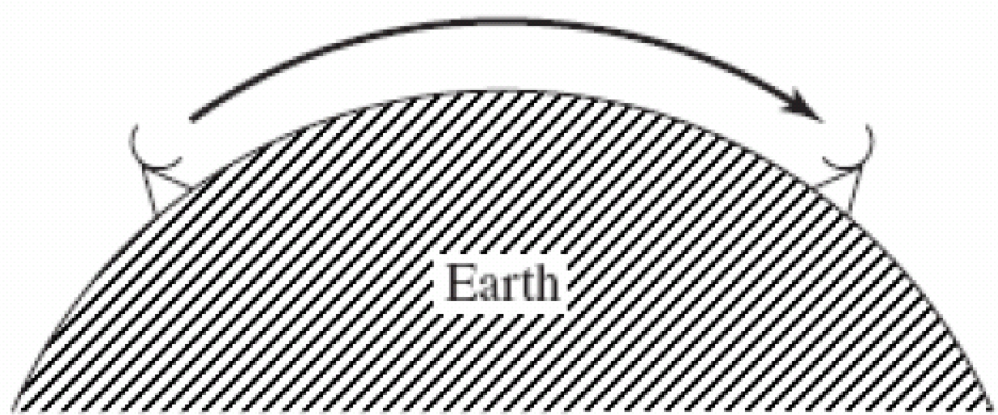


Figure 2.7: Knife-edge diffraction.

[3,p.39]



The Three Basic Propagation Mechanisms(3/3)

3. **Scattering**
   - **Scattering occurs when the propagation waves are incident upon an object whose dimensions are on the order of a wavelength or small compared to a wavelength.**
   - **It causes the energy to be redirected in many directions.**
   - **In practice, foliage, street signs, and lamp posts induce scattering in a mobile communication system.**

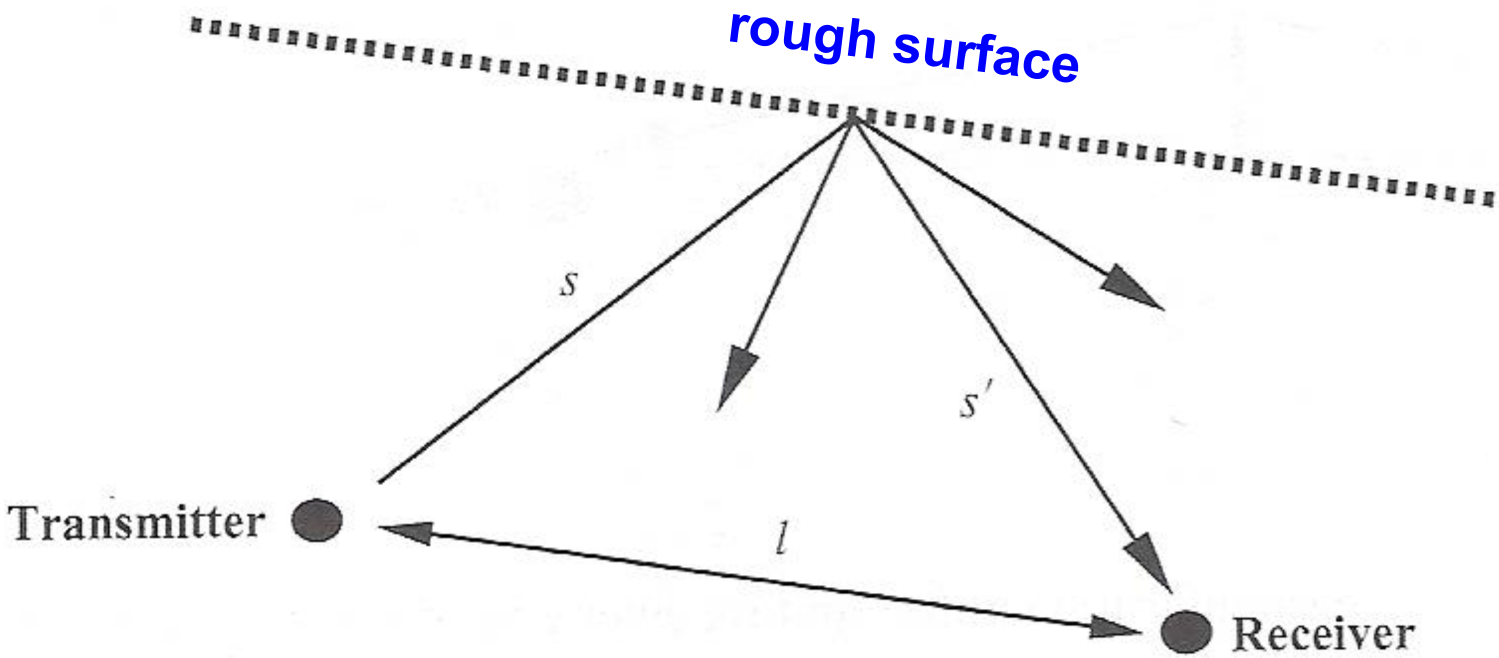


Figure 2.8: Scattering.

[3,p.40]

Free Space Propagation Model(1/3)

- **To obtain insight into large-scale fading, the first natural step is to consider propagation in free space.**
  - **Propagation in free space ➔ i.e., a clear, unobstructed line-of-sight (LOS) path between TX and RX.**
  - **Satellite communication systems typically undergo free-space propagation.**
- **We then refine our understanding to propagation close to the earth's surface.**
  - **Sky-wave is not the dominant mode of propagation for some part of the EM spectrum.**

Free Space Propagation Model(2/3)

- The free-space received power at distance d, $P_r(d)$

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} \propto \frac{1}{d^2}$$

– P_t : transmitted power

– G_t (G_r): transmitter (receiver) antenna gain

– λ : wavelength in meter

– d : radio path length from TX to RX

➔ $P_r(d)$ decays with distance at a rate of 20 dB/decade. (A gain drop of 20 dB for each 10-fold increase in distance.)

$$10\log\left\{P_r(d)/P_r(10d)\right\} = 10\log\left\{10^{-2}\right\} = -20\ \mathbf{dB}$$

Free Space Propagation Model(3/3)

- **Notes**
  - The gain of an antenna $(G_t \text{ or } G_r)$ is related to its effective aperture, A_e

$$\boxed{G = \frac{4\pi A_e}{\lambda^2}}$$

$\left(A_e = \dfrac{\lambda^2}{\Omega_A} \right.$ is related to the physical **size of the antenna.**

$\left. \Omega_A \text{ is the beam area (or beam solid angle).} \right)$

  - The wavelength is related to the carrier frequency, f

$$\boxed{\lambda = \frac{c}{f}}$$

(The speed of light $c = 3 \times 10^8$ meters/s)

➔ **It follows that** $P_r(d) = \dfrac{P_t G_t G_r \lambda^2}{(4\pi d)^2} \propto \dfrac{1}{d^2}, \dfrac{1}{f^2}, \lambda^2.$

- The free space model is only valid for values of d which are in the **far-field region**.
- The far-field region is the region, where the **radiation pattern** is essentially **independent of distance from the source**.

Free Space Path Loss(1/3)

- Path Loss, $PL(d)$
  - Path loss is the power difference (measured in dB) between the effective transmitted power and the received power.
    - It may or may not include the effect of the antenna gains.

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}$$

$$PL(d) = 10\log P_t - 10\log P_r(d) = 10\log \frac{P_t}{P_r(d)}$$

$$= \begin{cases} 10\log\left[\dfrac{(4\pi)^2 d^2}{G_t G_r \lambda^2}\right] = 10\log\left[\dfrac{(4\pi)^2 d^2 f^2}{G_t G_r c^2}\right] \text{ dB } (\text{including the antenna gains}) \\ 10\log\left[\dfrac{(4\pi)^2 d^2}{\lambda^2}\right] = 10\log\left[\dfrac{(4\pi)^2 d^2 f^2}{c^2}\right] \text{ dB } (\text{excluding the antenna gains}) \end{cases}$$

Ideal or isotropic antenna (i.e., unit antenna gain)

Free Space Path Loss(2/3)

- **Notes**
  - **The propagation path loss depends not only on the distance and wavelength, but also on the antenna gains (radiation patterns).**
  - **This makes the theoretical prediction of path loss difficult.**
  - **An easy solution to these difficulties is to measure the average received signal power, $\bar{P}_r(d_0)$, at a close-in distance d_0 in advance.**

Free Space Path Loss(3/3)

- **Alternative expression of free-space received power**
  - **Using $\overline{P}_r(d_0)$ as the received power reference d_0, it follows that the alternative expression of average free-space received power at distance d is given by**

$$\overline{P}_r(d) = \overline{P}_r(d_0)\left(\frac{d_0}{d}\right)^2,\ d \ge d_0$$

$$\Leftarrow \overline{P}_r(d) : \overline{P}_r(d_0) = \frac{1}{d^2} : \frac{1}{d_0^2}$$

- **A simplest free-space path loss model**

$$\overline{PL}(d) = 10\log P_t - 10\log \overline{P}_r(d) = 10\log\frac{P_t}{\overline{P}_r(d)} = 10\log\left[\frac{P_t}{\overline{P}_r(d_0)(d_0/d)^2}\right]$$

$$= 10\log\frac{P_t}{\overline{P}_r(d_0)} + 10\log\left(\frac{d}{d_0}\right)^2$$

$$= \overline{PL}(d_0) + 20\log\left(\frac{d}{d_0}\right),\ \text{where } \overline{PL}(d_0) = 10\log\frac{P_t}{\overline{P}_r(d_0)}.$$

Path Loss Model for General Ground-Wave Propagations

- Free-space path loss model does not reflect the local terrain characteristics such as buildings and hills.
- Both theoretical and measurement-based propagation models indicate that

  the average path loss for general surrounding environments with arbitrary T-R separation is given by

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n, \quad d \geq d_0 \Rightarrow \boxed{\overline{PL}(d) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)} \text{ in dB}$$

  - n is the path loss exponent which strongly depends on the cell size and local terrain characteristics.
  - When plotted on a log-log scale, the modeled path loss is a straight line with a slope equal to $10n$ dB per decade. 投影片 12

$$\Rightarrow \boxed{\overline{PL}(d) = \overline{PL}(d_0) - 10n\log(d_0) + 10n\log(d)}$$

Log-Normal Shadowing(1/2)

- **The log-normal distribution is the probability distribution of any random variable (RV) whose logarithm is normally distributed.**
  - **If *X* is log-normally distributed, then log(*X*) is normally distributed.**
- **"Shadowing" effect**
  - **A signal transmitted through a radio channel will typically experience attenuation due to blockage from obstacles.**
  - **The attenuation will be varied slowly due to the changes in reflecting surfaces and scattering objects.**
- **Zero-mean log-normal distribution (normal in dB) has been empirically confirmed to accurately model the shadowing effect.**

  ➔ log-normal shadowing

Log-Normal Shadowing(2/2)

- **Let the log-normal random variable X_σ (in dB) with standard deviation σ (also in dB) describe the random shadowing effects.**
  - **The parameter σ typically ranges from 5 to 12 dB, with 8 dB being a typical value.**
  - **Log-normal shadowing has been observed to be nearly independent of the radio path length d.**

Large-Scale Fading

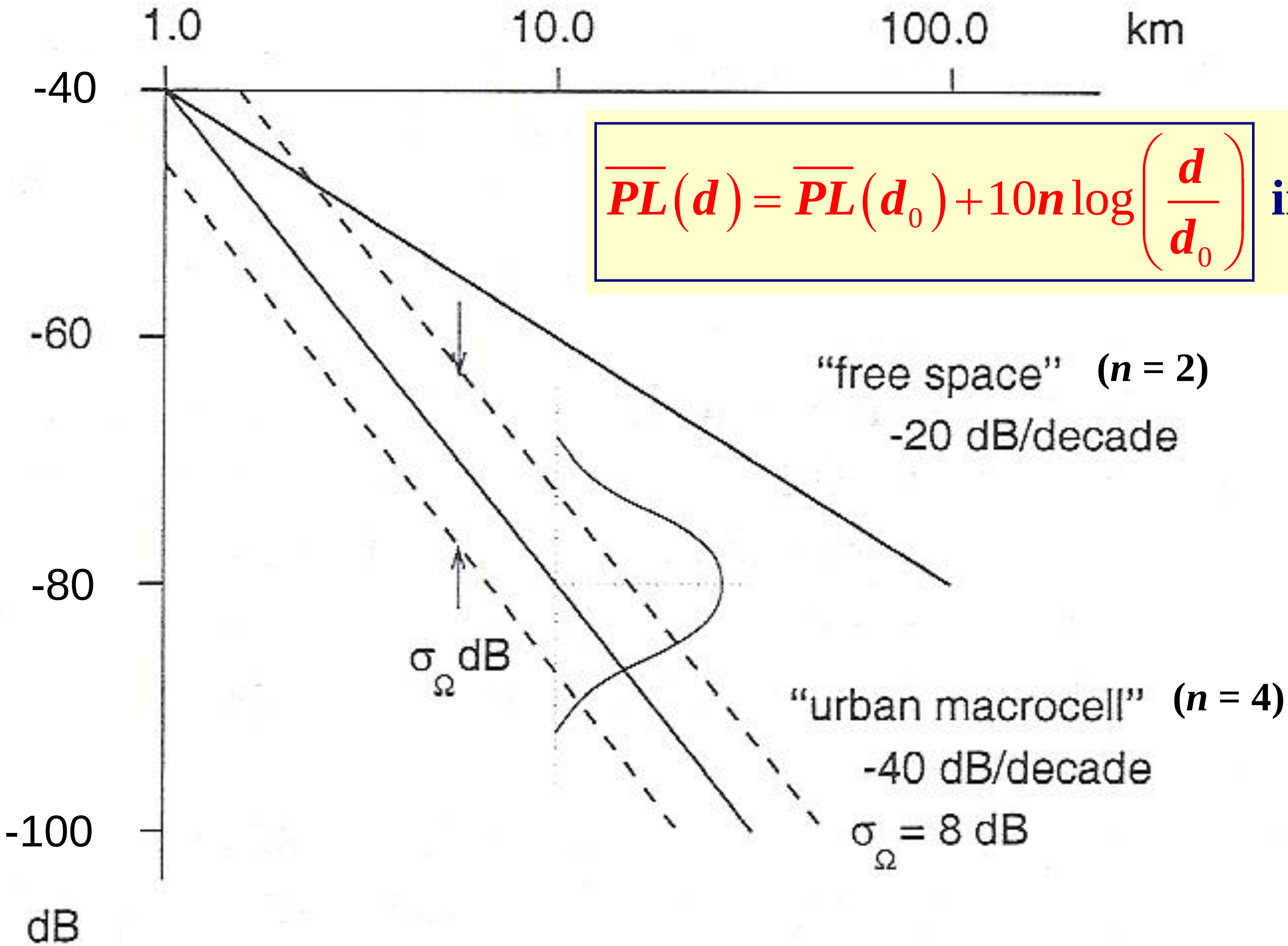
- Combining path loss and shadowing results in large-scale fading.

$$\Omega_\sigma(d) = \overline{PL}(d) + X_\sigma, \text{where } \boxed{\overline{PL}(d) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)}$$

$$= \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma \ \text{ in dB}$$

- $\Omega_\sigma(d)$ in dB is a random variable of normal distribution with mean $\overline{PL}(d)$

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) \text{ in dB}$$



[2,p.22]

Percentage of Coverage Area(1/4)

- The "cell coverage area" in a cellular system is the expected percentage of locations within a cell where the received power at these locations is above a given minimum.
  - Due to random effects of shadowing, some locations within a coverage area will be below a particular desired received signal threshold.
- Let $U(\gamma)$ be the cell coverage area within a cell with cell radius R.
  - γ is the minimum received signal threshold (in dB).

Percentage of Coverage Area(2/4)

- The random received signal power (in dB) at a distance d

$$P_{r\,\mathrm{dB}}(d) = \boxed{\bar{P}_{r\mathrm{dB}}(d) + X_\sigma},$$ where $\bar{P}_{r\mathrm{dB}}(d) = 10\log \bar{P}_r(d)$ due to path loss

$P_{r\,\mathrm{dB}}(d)$ has a mean of $\bar{P}_{r\mathrm{dB}}(d)$.

X_σ is the RV of log-normal shadowing.

- Note on $\bar{P}_{r\mathrm{dB}}(d)$

$$\overline{PL}(d) = 10\log\frac{P_t}{\bar{P}_r(d)} = \underbrace{10\log P_t}_{\equiv P_{t\mathrm{dB}}} - \underbrace{10\log \bar{P}_r(d)}_{\equiv \bar{P}_{r\mathrm{dB}}(d)}$$

$$\Rightarrow \bar{P}_{r\mathrm{dB}}(d) = P_{t\mathrm{dB}} - \underbrace{\overline{PL}(d)}_{=\overline{PL}(d_0)+10n\log\left(\frac{d}{d_0}\right)}$$

$$\Rightarrow \boxed{\bar{P}_{r\mathrm{dB}}(d) = P_{t\mathrm{dB}} - \left(\overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)\right)}$$

Percentage of Coverage Area(3/4)

- **Q-function,** $$Q(z) \equiv \int_z^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-y^2/2\right\} dy$$
  - **A Gaussian random variable X with mean of m and variance of σ^2**
  - **Then the probability that X exceeds x_0, is evaluated as**

$$\Pr\left(X \geq x_0\right) = \int_{x_0}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\left(x-m\right)^2 / \left(2\sigma^2\right)\right\} dx$$

$$= \int_{\frac{x_0-m}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-y^2/2\right\} dy, \quad \textbf{letting } y \equiv \frac{x-m}{\sigma}$$

$$= \Pr\left(y \geq \frac{x_0-m}{\sigma}\right)$$

$$= Q\left(\frac{x_0-m}{\sigma}\right)$$

Percentage of Coverage Area(4/4)

- **The Cell Coverage Area**

$$U(\gamma) \equiv \frac{1}{\pi R^2}\int \Pr\left[P_{r\,\mathrm{dB}}(r) > \gamma\right] dA = \boxed{\frac{1}{\pi R^2}\int_0^{2\pi}\int_0^{R} \Pr\left[P_{r\,\mathrm{dB}}(r) > \gamma\right] r\,dr\,d\theta}$$

$$-\quad \Pr\left[P_{r\,\mathrm{dB}}(r) > \gamma\right] = Q\left(\frac{\gamma - \bar{P}_{r\mathrm{dB}}(r)}{\sigma}\right)$$

$$= Q\left[\left(\gamma - \left[P_{t\mathrm{dB}} - \left(\overline{PL}(d_0) + 10n\log\left(\frac{r}{d_0}\right)\right)\right]\right)\Big/\sigma\right]$$

- **Outage Probability under Path Loss and Shadowing**

$$\Pr\left[P_{r\mathrm{dB}}(r) \le \gamma\right] = 1 - \Pr\left[P_{r\mathrm{dB}}(r) > \gamma\right]$$

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Empirical Models for Outdoor Path Loss

- **The propagation models that we have seen thus far attempt to predict path loss for ground-wave transmission close to the Earth's surface (in an average manner).**
- **However, radio communication often takes place over irregular terrain.**
- **The *terrain profile* of a particular area needs to be taken into account for obtaining better estimates of path loss.**
- **A number of propagation models are available to predict path loss over irregular terrain.**
  - **These models are *empirical* i.e., based on a systematic interpretation of measurement data (usually curve fitting) obtained in the service area.**
- **Here, we only present the Hata's model. The interested reader is referred to the text book for additional reading.**

Path Loss Model for General Ground-Wave Propagations– Hata's Model(1/3)

- Hata's model is based on measurements in Tokyo, Japan.
- It is intended for use in Japanese landscapes (urban or suburban) and performs poorly for American suburban terrain.
- Hata's Path Loss Model

$$\underset{\text{(in dB)}}{L_p} = \begin{cases} A + B\log_{10}(d) & \textit{for urban area} \\ A + B\log_{10}(d) - C & \textit{for suburban area} \\ A + B\log_{10}(d) - D & \textit{for open area} \end{cases}$$

Path Loss Model for General Ground-Wave Propagations – Hata's Model(1/3)

- **Hata's Path Loss Model (Cont.)**

Carrier Frequency : $150 \text{ MHz} \leq f_c \leq 1500 \text{ MHz}$
Base Station Height : $30\text{m} \leq h_b \leq 200\text{m}$
Mobile Station Height: $1\text{m} \leq h_m \leq 10\text{m}$
T-R distance : $1\text{km} \leq d \leq 20\text{km}$

$$A = 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55 \log_{10}(h_b)$$
$$C = 5.4 + 2[\log_{10}(f_c/28)]^2$$
$$D = 40.94 + 4.78 [\log_{10}(f_c)]^2 - 18.33 \log_{10}(f_c)$$

Path Loss Model for General Ground-Wave Propagations – Hata's Model(1/3)

- **Hata's Path Loss Model (Cont.)**

A correction factor for effective mobile antenna height

(for a small to medium sized city)

$$a(h_m) = \begin{cases} [1.1 \log_{10}(f_c) - 0.7]h_m - 1.56 \log_{10}(f_c) - 0.8 \\ 8.28 [\log_{10}(1.54 h_m)]^2 - 1.1 \\ 3.2 [\log_{10}(11.75 h_m)]^2 - 4.97 \end{cases}$$

(for a large city and $f_c \le 300$MHz)

(for a large city and $f_c \ge 300$MHz)

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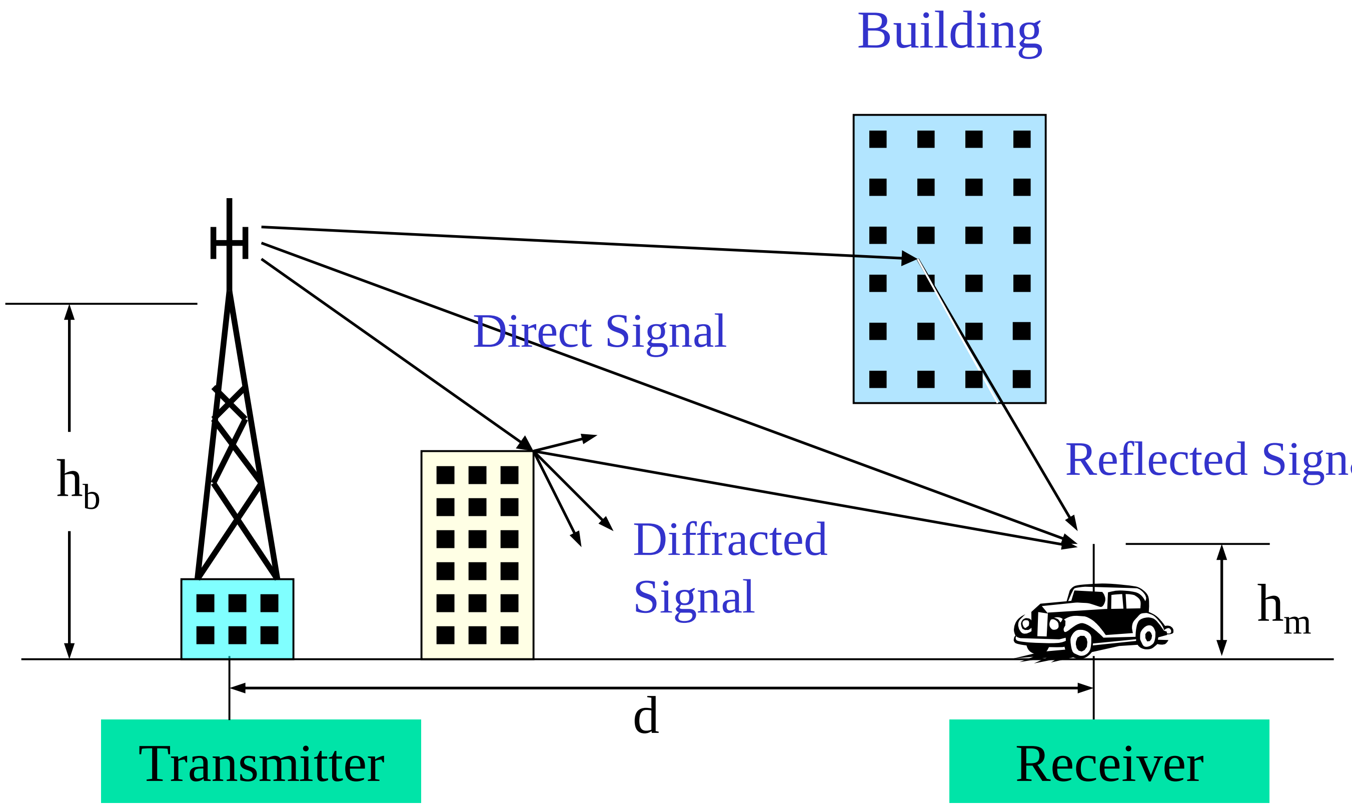
Introduction(1/4)

- **Multipath**
  - **Due to obstructions in the path from the base station to the mobile unit,**
    - **The receiver will receive different copies of the same signal from various paths with different time delays.**
    - **➔Such a phenomenon is called multipath which results in inter-symbol interference (ISI).**
  - **Multipath usually includes:**
    - **Line-of-sight (LOS): the direct connection between the TX and the RX without obstructions.**
    - **Non-line-of-sight (NLOS): the path arrivals resulting from reflections from obstacles.**
  - **The resultant signal from the combination of multipath waves at RX can vary widely in amplitude and phase.**

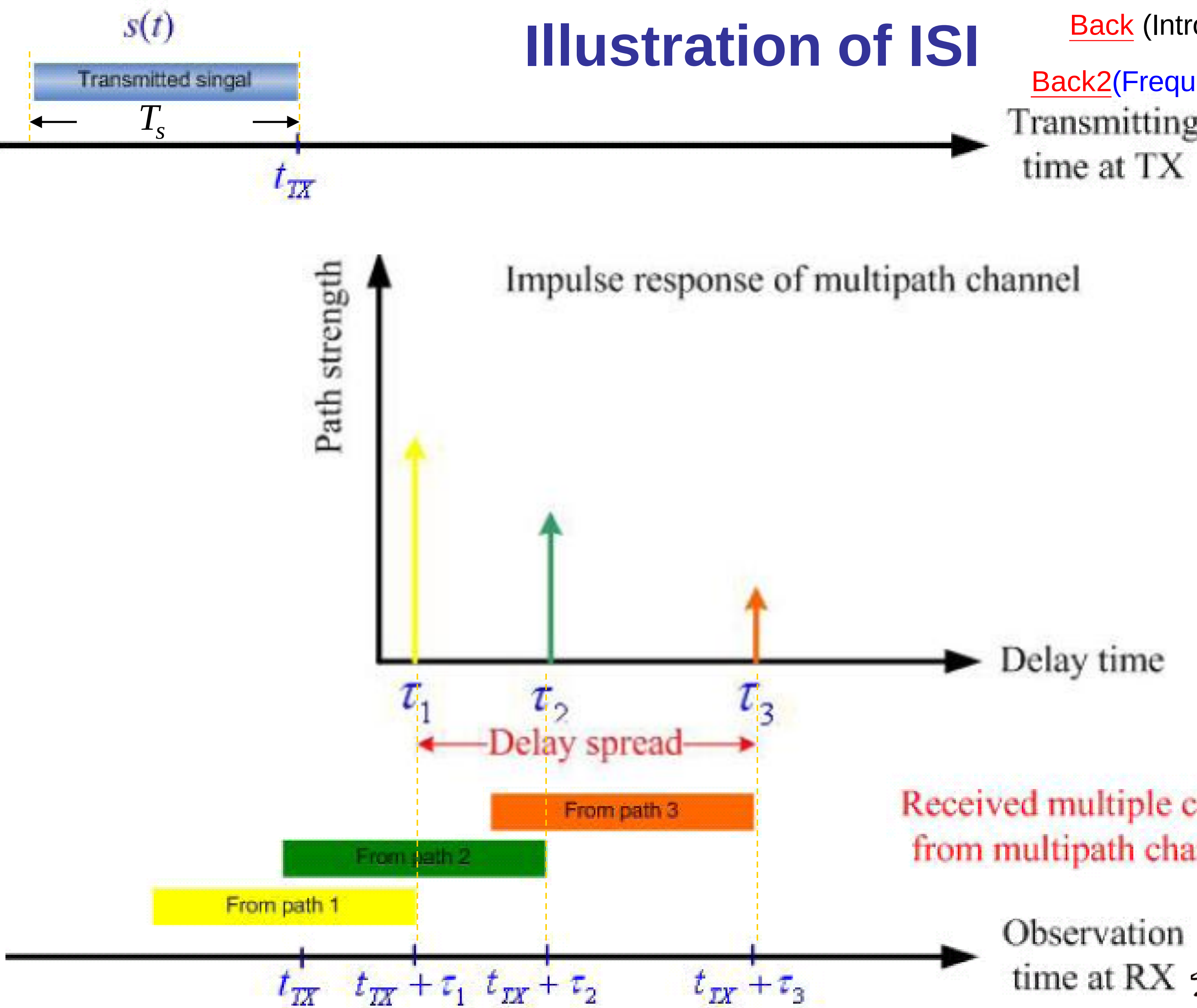

Building
Direct Signal
Reflected Signal
Diffracted Signal
h_b
h_m
d
Transmitter
Receiver

[5,p.59]

Illustration of ISI

Back (Introduction)

Back2(Frequency Selective Fading)



Received multiple copies of $s(t)$ from multipath channel ➔ ISI

Introduction(2/4)

- Small-Scale Fading
  - Multipath in the radio channel creates small-scale fading effects.
    - The carrier wavelength used in mobile radio applications is typically several centimeters.
      - ➔ Small changes in the propagation delays due to MS mobility will cause large changes in the phases of the individually arriving plane waves.
    - When the arriving waves from multipath signals are out of phase, reduction of the signal strength at the receiver can occur. (deconstructive addition)
  - The term "small-scale fading" is used to describe the rapid change in amplitude and phase of a radio signal over a small travel distance or time interval.

Introduction(3/4)

- Factors influencing small-scale fading
  - Multipath propagation
    - The random phase and amplitude of the different multipath components cause signal-strength fluctuations and signal distortion.
  - Speed of the mobile
    - Cause the relative motion between the BS and MS ➔ Doppler shift
    - Create a constantly changing environment ➔ time variant channel
  - Speed of surrounding objects
  - The transmission bandwidth of the signal

Introduction(4/4)

- The small-scale fading manifests itself in two mechanisms:
  - Time-spreading of the signals (or Signal dispersion)
    - Due to multipath propagation
  - Time-variation behavior of the channel
    - Due to the relative motion between BS and MS

Illustration of ISI

Doppler Shift

- **Consider a mobile station (MS) moving at a constant velocity v toward a receiver, along a path.**
- **Let θ be the angle of incidence that the propagation radio wave arrives at MS antenna. (i.e., DOA: direction of arrival)**

➔**The difference of distance Δl between an MS (moving at v) and its BS, after Δt unit time:**

$$\Delta l = v \Delta t \cos(\theta)$$

➔**The phase change in the received signal due to the difference length Δl :**

$$\Delta \phi = 2\pi \frac{\Delta l}{\lambda} = \frac{2\pi v \Delta t \cos(\theta)}{\lambda}, \text{ where } \lambda \text{ is the wave length.}$$

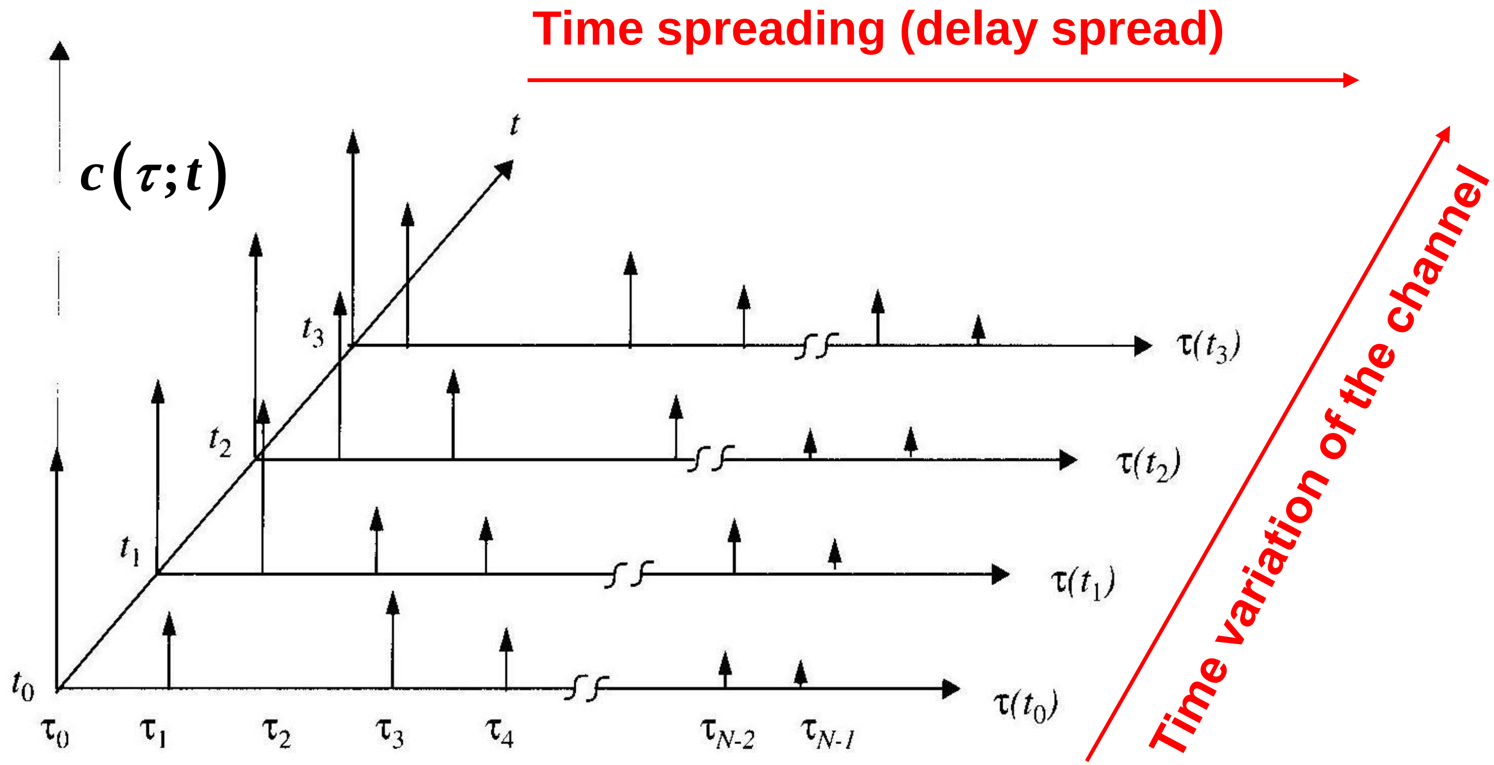


Figure An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

[1, p.184]

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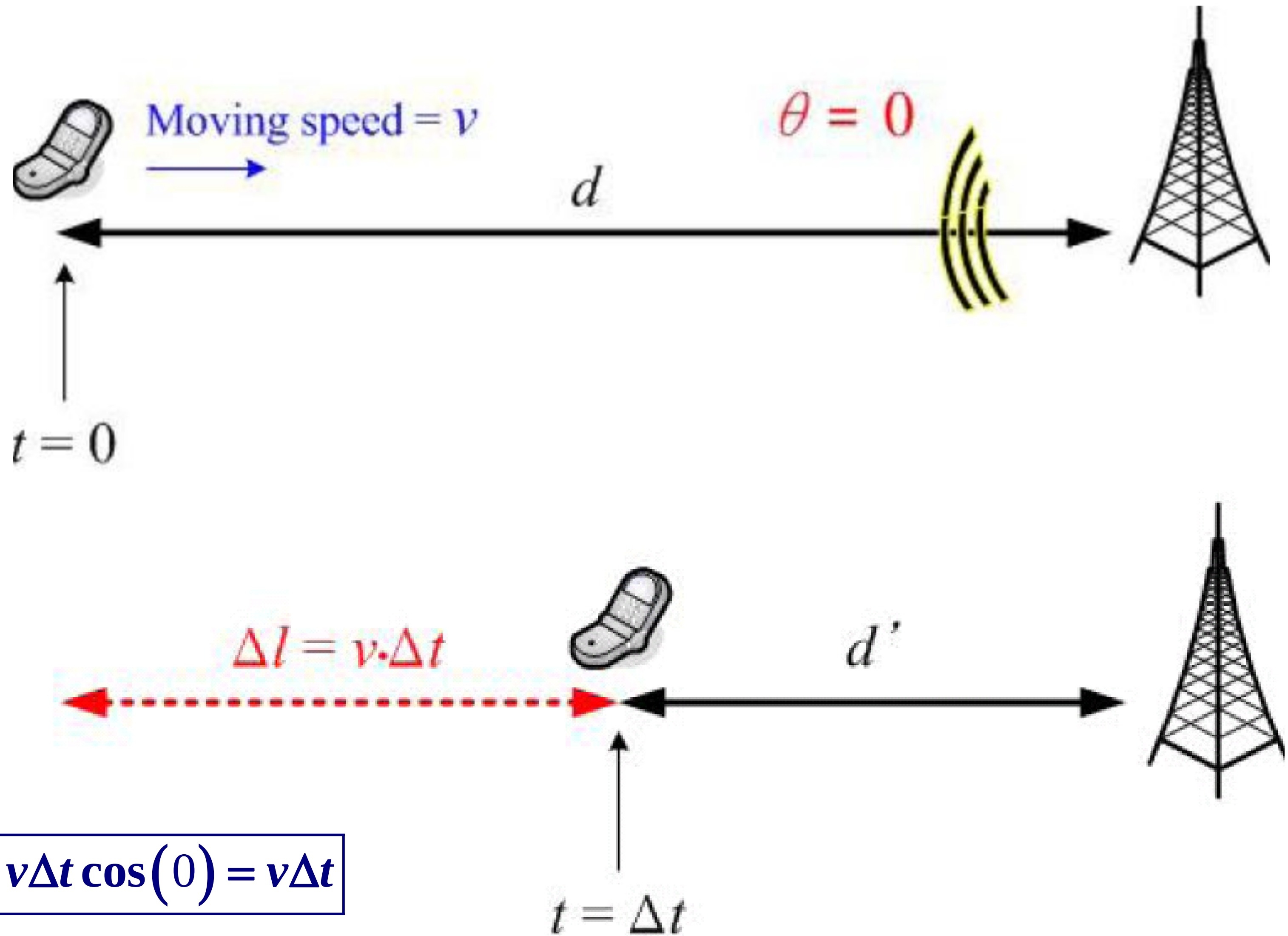
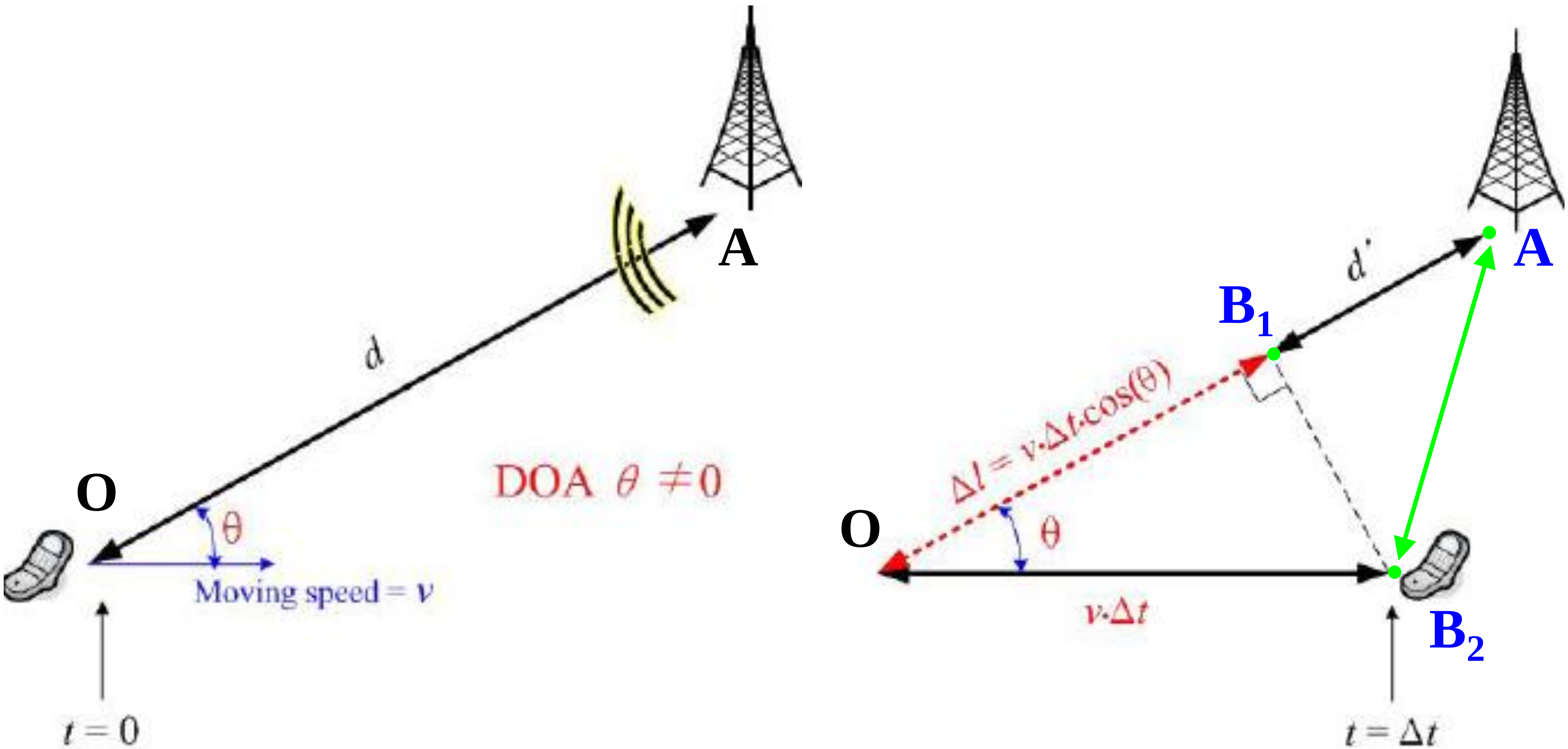
Illustration of θ = 0



$$\boxed{\Delta l = v\Delta t \cos(0) = v\Delta t}$$

Illustration of $\theta \neq 0$



For d' >> 1, $\overline{AB_1}$ will be very close to $\overline{AB_2}$.

$\rightarrow \overline{OA} - \overline{B_2A} \approx \overline{OB_1} \rightarrow \boxed{\Delta l = v\Delta t\cos(\theta)}$

Doppler Shift

- The change in frequency (or Doppler shift, f_D)

$$\Delta\phi = 2\pi f_D \Delta t$$

$$\Rightarrow f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{2\pi v \Delta t \cos(\theta)/\lambda}{2\pi\Delta t}$$

$$= \boxed{\frac{v}{\lambda}\cos(\theta)}$$

$$\equiv f_m \cos(\theta), \text{ where } f_m = \frac{v}{\lambda} \text{ is the maximum Doppler shift.}$$

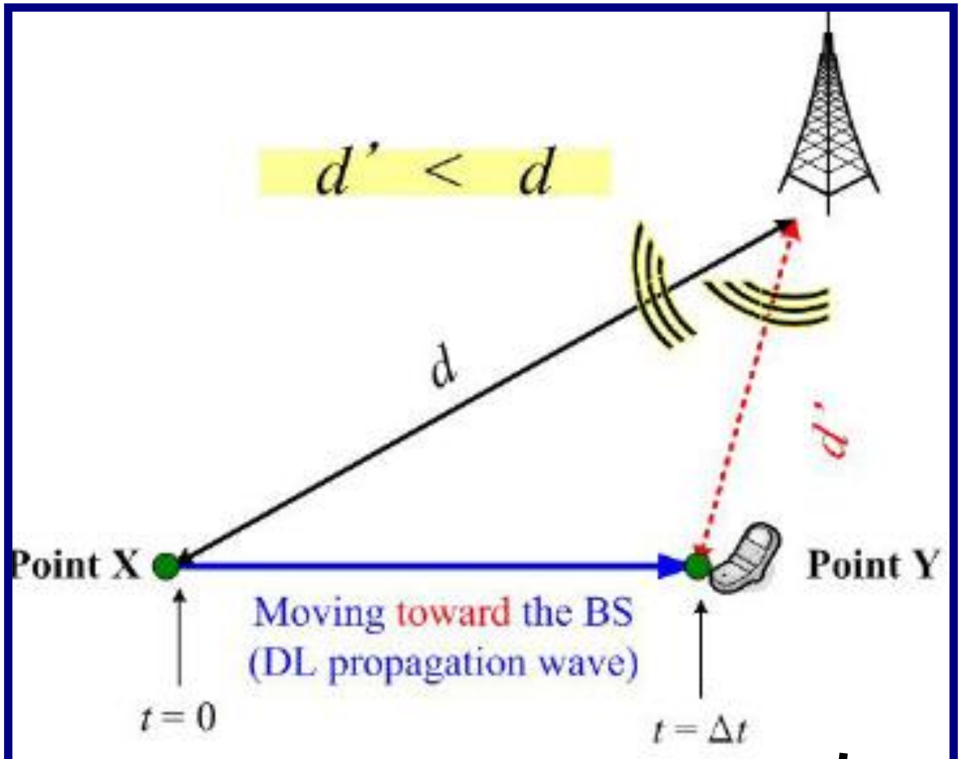
Phase change due to Δl:

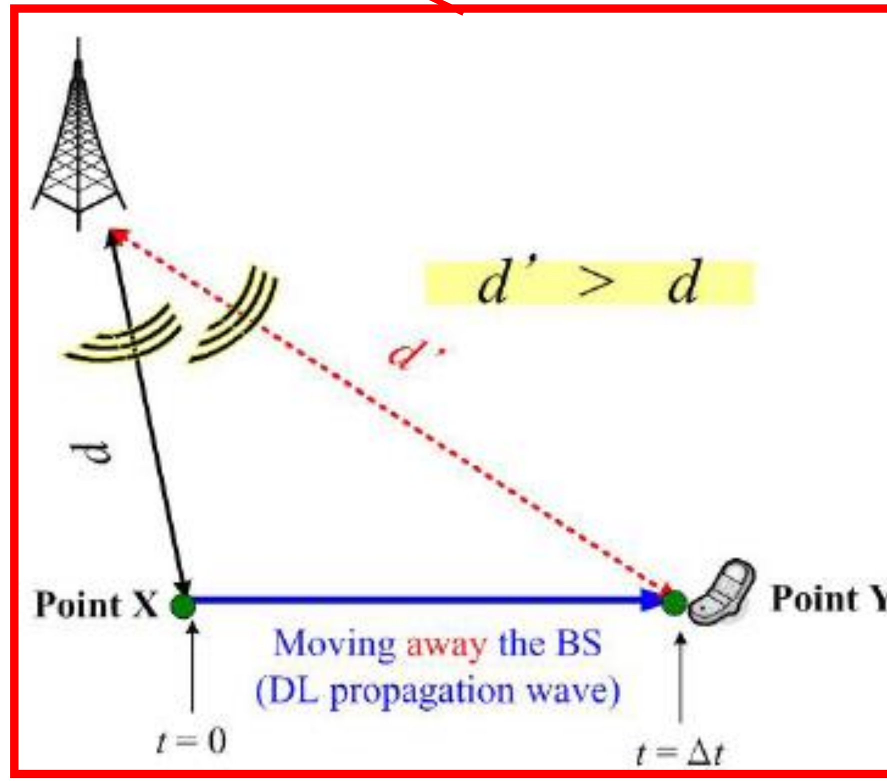
$$\Delta\phi = \frac{2\pi v \Delta t \cos(\theta)}{\lambda}$$

- Notes
  - If the MS is moving toward the direction of arrival of the wave, the Doppler shift is positive (i.e., the apparent received frequency is increased.)
  - If the MS is moving away the direction of arrival of the wave, the Doppler shift is negative (i.e., the apparent received frequency is decreased.)

During the time epoch 0 ~ Δt

- ➔ **The traveling distance of the radio wave is getting longer (from d to d').**
- ➔ **This results that the waves will arrive at Point X before arriving at Point Y.**
- ➔ **This is analogous to the frequency of waves decreases (negative Doppler shift).**





During the time epoch 0 ~ Δt

- ➔ **The traveling distance of the radio wave is getting shorter (from d to d').**
- ➔ **This results that the waves will arrive at Point Y before arriving at Point X.**
- ➔ **This is analogous to the frequency of waves increases (positive Doppler shift).**

Impulse Response Model of a Multipath Channel(1/4)

- Transmitted Bandpass Signal

$$s(t) = \mathrm{Re}\left[\tilde{s}(t)e^{j2\pi f_c t}\right]$$

– $\tilde{s}(t)$ is the equivalent lowpass transmitted signal.

- Noiseless Received Bandpass Signal

$$r(t) = \mathrm{Re}\left[\sum_n \alpha_n(t) e^{j2\pi\left(f_c + f_{D,n}\right)\left(t-\tau_n(t)\right)} \tilde{s}\left(t-\tau_n(t)\right)\right],$$

where n is the resolvable path index

$$= \mathrm{Re}\left[\tilde{r}(t)e^{j2\pi f_c t}\right]$$

– $\alpha_n(t)$: the attenuation factor associated with nth path

– $\tau_n(t)$: the propagation delay associated with nth path

– $\tilde{r}(t)$: the equivalent lowpass received signal

Impulse Response Model of a Multipath Channel(2/4)

- **Noiseless Equivalent Lowpass Received Signal**

$$\boxed{\tilde{r}(t)=\sum_n \alpha_n(t)e^{-j\phi_n(t)}\tilde{s}\left(t-\tau_n(t)\right)}$$

$$-\phi_n(t)=2\pi\left[\left(f_c+f_{D,n}\right)\tau_n(t)-f_{D,n}t\right]$$

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Impulse Response Model of a Multipath Channel(3/4)

- Let $c(\tau;t)$ denote the equivalent lowpass channel.
  - I.e., response (or observe) at t when impulse is applied at $t-\tau$.
- Since $\tilde{r}(t)=\sum_n \alpha_n(t)e^{-j\phi_n(t)}\tilde{s}(t-\tau_n(t))$

$$\equiv \int_{-\infty}^{\infty} c(\tau;t)\tilde{s}(t-\tau)d\tau$$

$$\Rightarrow c(\tau;t)=\sum_n \alpha_n(t)e^{-j\phi_n(t)}\delta(\tau-\tau_n(t))$$

Recall that $\phi_n(t)=2\pi\left[\left(f_c+f_{D,n}\right)\tau_n(t)-f_{D,n}t\right]$ is the phase associated with the nth path.

Impulse Response Model of a Multipath Channel(4/4)

$$\boxed{c(\tau;t)=\sum_n \alpha_n(t)e^{-j\phi_n(t)}\delta\left(\tau-\tau_n(t)\right)}$$

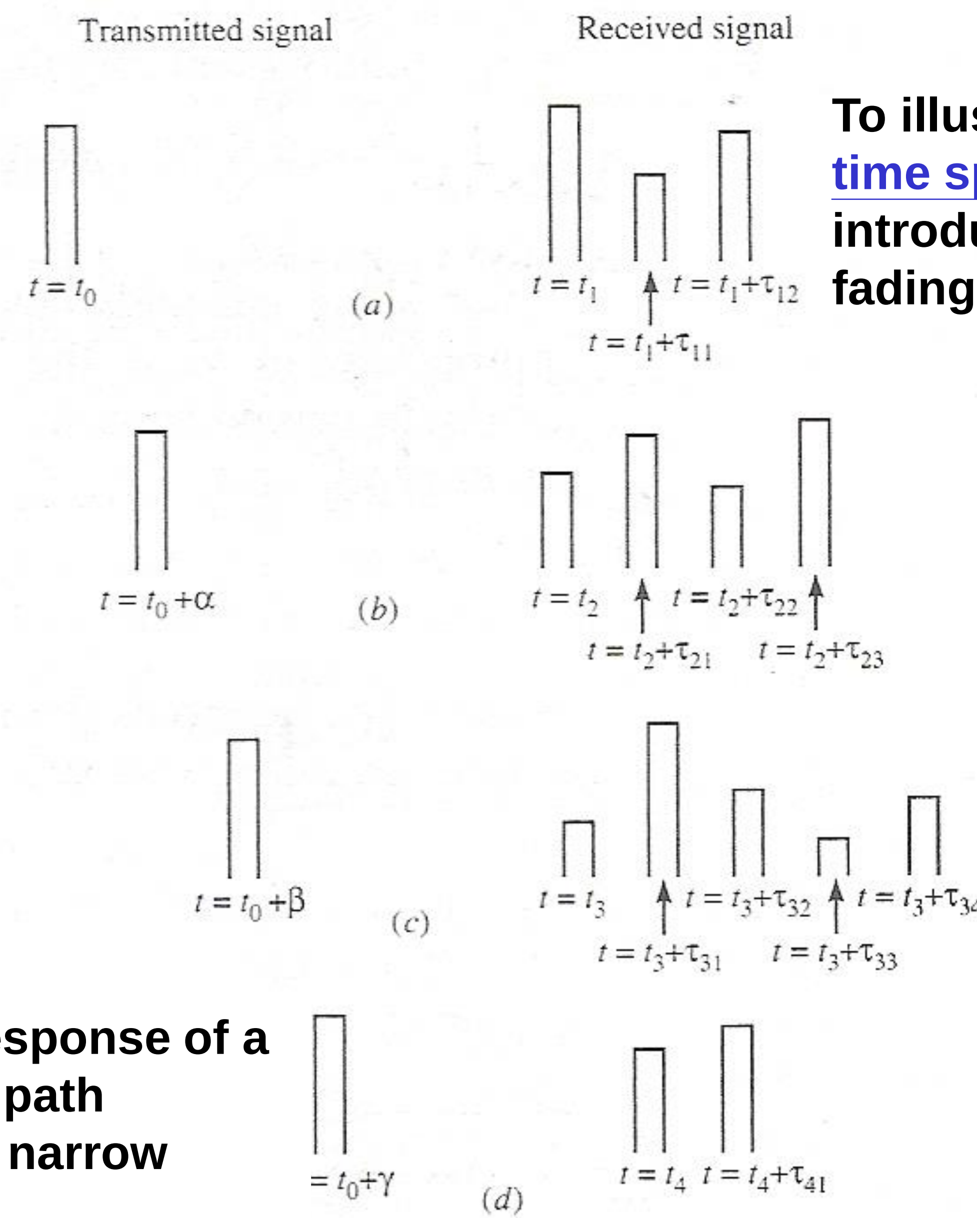
- **Notes**
  - Since f_c is very large, very small changes in the path delays $\tau_n(t)$ will cause a large changes in the phases $\phi_n(t)=2\pi\left[\left(f_c+f_{D,n}\right)\tau_n(t)-f_{D,n}t\right]$.
  - For example, a 900 MHz sinusoid has a wavelength of about 30 cm. (such radio waves propagate at about 30 cm per nanosecond (ns))

    $\Rightarrow$ A path delay change of just 1 ns corresponds to one full wavelength (or 2π radians phase shift)

    $\Rightarrow$ At any time the random phases $\phi_n(t)$ may result in the constructive or destructive addition of multipath components.

    $\Rightarrow$ Thus, multipath fading is primarily due to small variations in the path delays that occur over small spatial distances.

To illustrate both effects of **time spread** & **time variation** introduced by multipath fading channel.



Example of the response of a time-variant multipath channel to a very narrow pulse.

[6, p.759]

Channel Correlation Functions and Spectrum(1/4)

- In statistics, the autocorrelation of a random process $X(t)$ describes the correlation between values of the process at different points in time.

$$R(t_1,t_2)=\frac{1}{2}\frac{E\left\{\left[X(t_1)-m_{X_1}\right]^*\left[X(t_2)-m_{X_2}\right]\right\}}{\sigma_{X_1}\sigma_{X_2}} \in [-1,1]$$

  - m_{X_k} : mean of the random variable $X(t_k)$
  - σ_{X_k} : standard deviation of the random variable $X(t_k)$

- If the process is wide-sense stationary (WSS), the autocorrelation defined above can be further expressed as a function of the time-lag:

$$R(t_2-t_1)=\frac{1}{2}\frac{E\left\{\left[X(t_1)-m\right]^*\left[X(t_2)-m\right]\right\}}{\sigma^2} \in [-1,1]$$

  - A WSS random process only requires that 1st and 2nd moments do not vary with respect to time.

Channel Correlation Functions and Spectrum(2/4)

- **The Wiener-Khinchin theorem relates the autocorrelation function $R(\tau)$ to power spectral density $S(f)$ via the Fourier transform pair:**

$$\begin{cases} R(\tau) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f \tau} df \\ S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f \tau} d\tau \end{cases}$$

- **We now develop a number of useful correlation functions and power spectral density functions that define the characteristics of a multipath fading channel.**

Channel Correlation Functions and Spectrum(3/4)

$$\boxed{c(\tau;t)=\sum_n \alpha_n(t)e^{-j\phi_n(t)}\delta\left(\tau-\tau_n(t)\right)}$$

- **Autocorrelation Function of the Channel**
  - **Assume that the channel is wide-sense stationary with zero mean and unit variance. Then we define its autocorrelation function as:**

$$\phi_c(\tau_1,\tau_2;\Delta t)=\tfrac{1}{2}E\left[c^*(\tau_1;t_1)c(\tau_2;t_2)\right],\ \text{where } \Delta t=t_2-t_1$$

$$=\phi_c(\tau_1;\Delta t)\delta(\tau_1-\tau_2),\ \textit{uncorrelated scattering}\ \text{(UC)}$$

$$=\phi_c(\tau;\Delta t),\ \rightarrow \text{WSSUC}$$

  - **In most radio channel, two different paths are uncorrelated each other. ➔ This is called uncorrelated scattering.**

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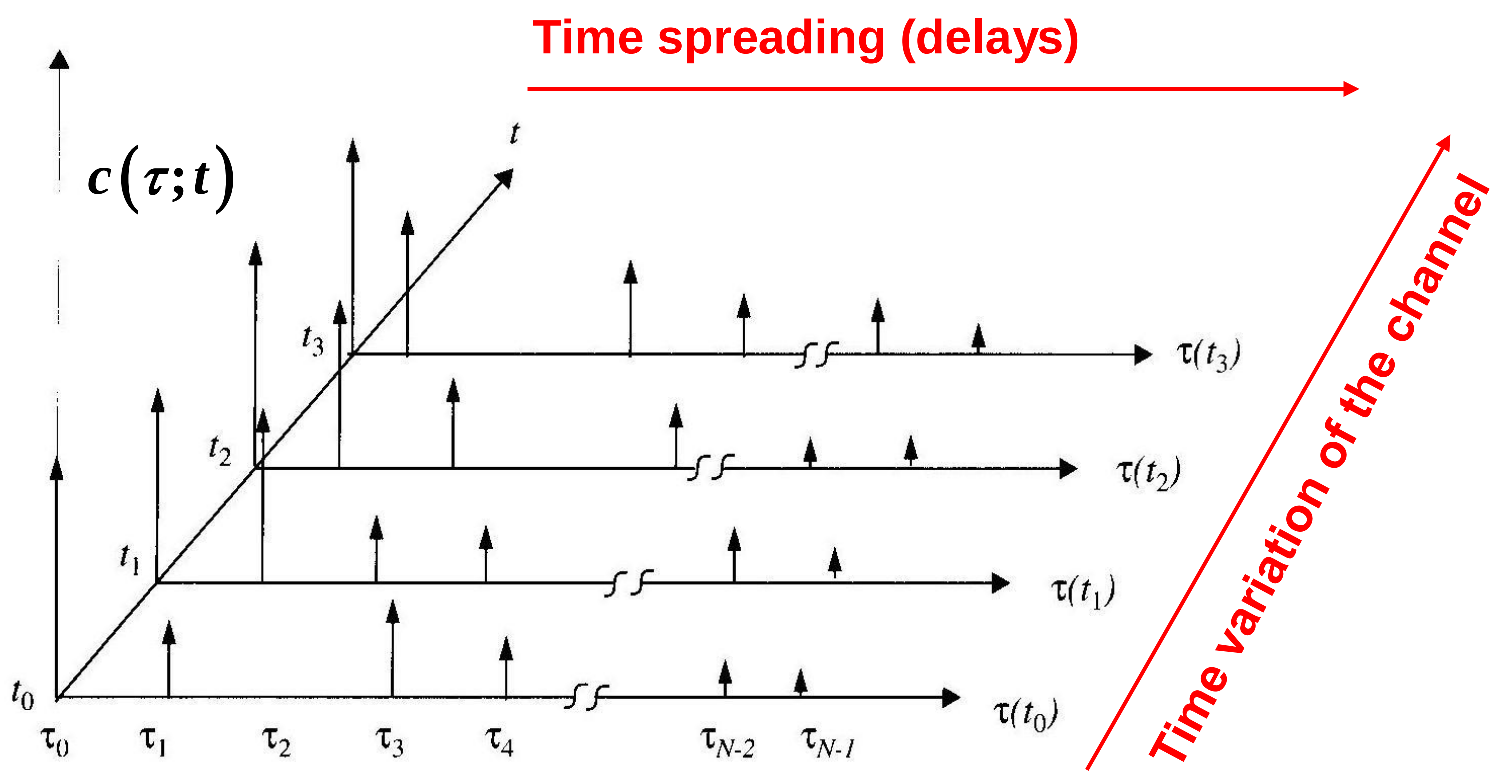

Figure 5.4 An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

[1, p.184]

Channel Correlation Functions and Spectrum(4/4)

- **Equivalent Lowpass Channel**

$$\boxed{c(\tau;t)=\sum_n \alpha_n(t)e^{-j\phi_n(t)}\delta\left(\tau-\tau_n(t)\right)}$$

- **Autocorrelation Function of the Channel**

$$\phi_c(\tau,\tilde{\tau};t,\tilde{t})\equiv \tfrac{1}{2}E\left[c^*(\tau;t)c(\tilde{\tau};\tilde{t})\right]$$

$$=\tfrac{1}{2}E\left\{\left[\sum_n \alpha_n(t)e^{-j\phi_n(t)}\delta\left(\tau-\tau_n(t)\right)\right]^*\left[\sum_m \alpha_m(\tilde{t})e^{-j\phi_m(\tilde{t})}\delta\left(\tilde{\tau}-\tau_m(\tilde{t})\right)\right]\right\}$$

$$=\tfrac{1}{2}E\left\{\sum_n\sum_m \alpha_n^*(t)\alpha_m(\tilde{t})e^{j\left(\phi_n(t)-\phi_m(\tilde{t})\right)}\delta\left(\tau-\tau_n(t)\right)\delta\left(\tilde{\tau}-\tau_m(\tilde{t})\right)\right\}$$

$$\overset{WSSUC}{=}\tfrac{1}{2}E\left\{\sum_n \alpha_n^*(0)\alpha_n(\Delta t)e^{j\left(\phi_n(0)-\phi_n(\Delta t)\right)}\delta\left(\tau-\tau_n\right)\right\}\equiv\phi_c(\tau;\Delta t)$$

UC: **For** $n\neq m,\ E\left[\alpha_n^*(t)\alpha_m(\tilde{t})\right]\overset{UC}{=}E\left[\alpha_n^*(t)\right]E\left[\alpha_m(\tilde{t})\right]=0\ \left(\because\ E[\alpha_n]=0\right)$

Channel Correlation Functions and Spectrum – Time Spread (1/3)

- **Multipath Intensity Profile (MIP)** $\left(\text{Setting } \Delta t = 0\right)$

$$\phi_c(\tau;0) = \tfrac{1}{2} E\left\{\sum_n \alpha_n^*(0)\alpha_n(\Delta t)e^{j(\phi_n(0)-\phi_n(\Delta t))}\delta(\tau-\tau_n)\right\}\Big|_{\Delta t=0}$$

$$= \tfrac{1}{2}\sum_n E\left(\left|\alpha_n\right|^2\right)\delta(\tau-\tau_n) \equiv \phi_c(\tau)$$

$\Rightarrow \phi_c(\tau)$ **is simply the average power of the channel as a function of the time delay** τ.

$\Rightarrow$ **For this reason,** $\phi_c(\tau)$ **is called the MIP of the channel.**

- **Delay Spread** T_m
  - **The range of values of** τ **over which** $\phi_c(\tau)$ **is essentially nonzero is called the delay spread of the channel.**

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An example of multipath intensity profile (MIP)

$$\phi_c(\tau) = \tfrac{1}{2}\sum_n E\left(|\alpha_n|^2\right)\delta(\tau - \tau_n)$$

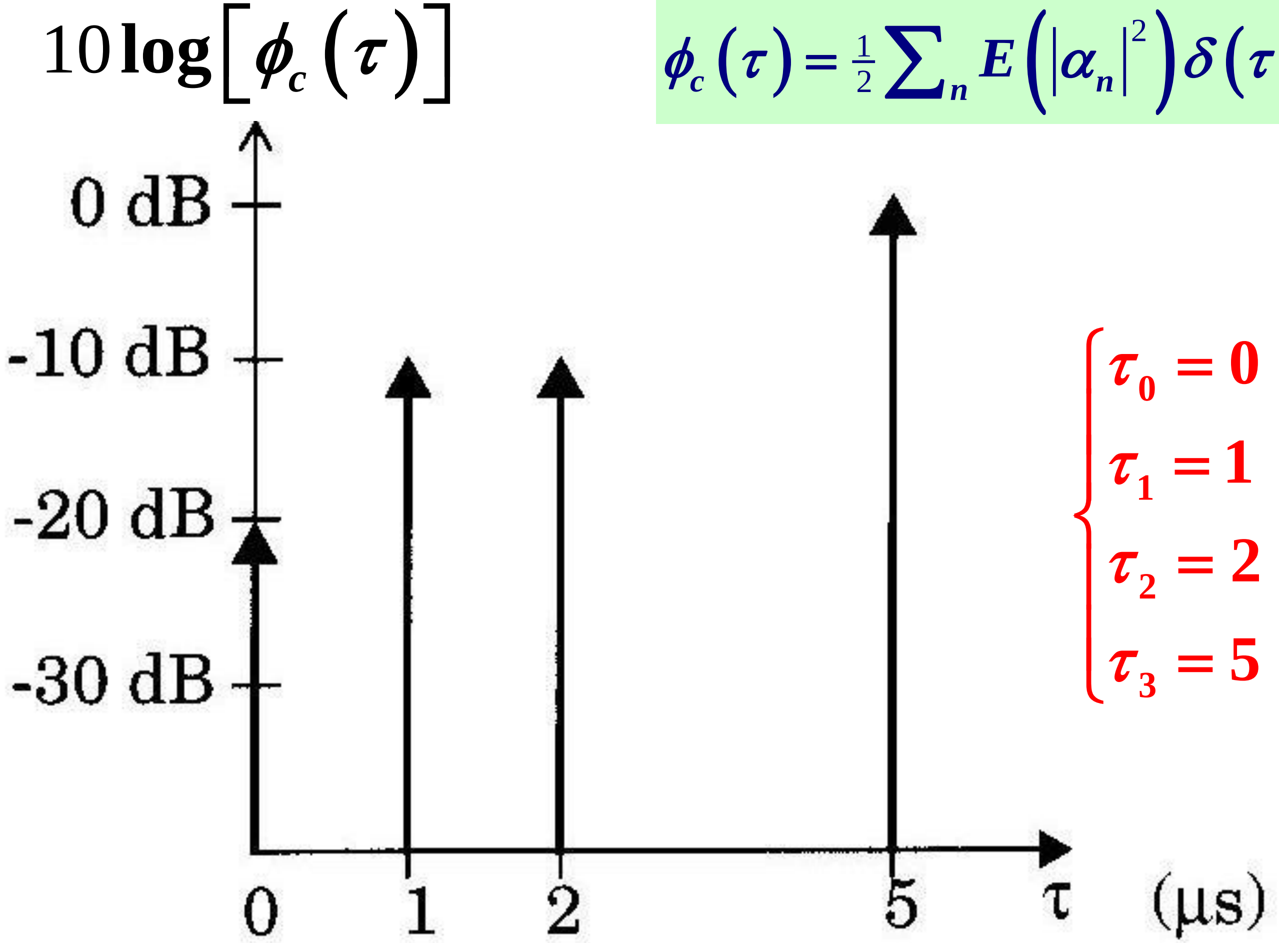


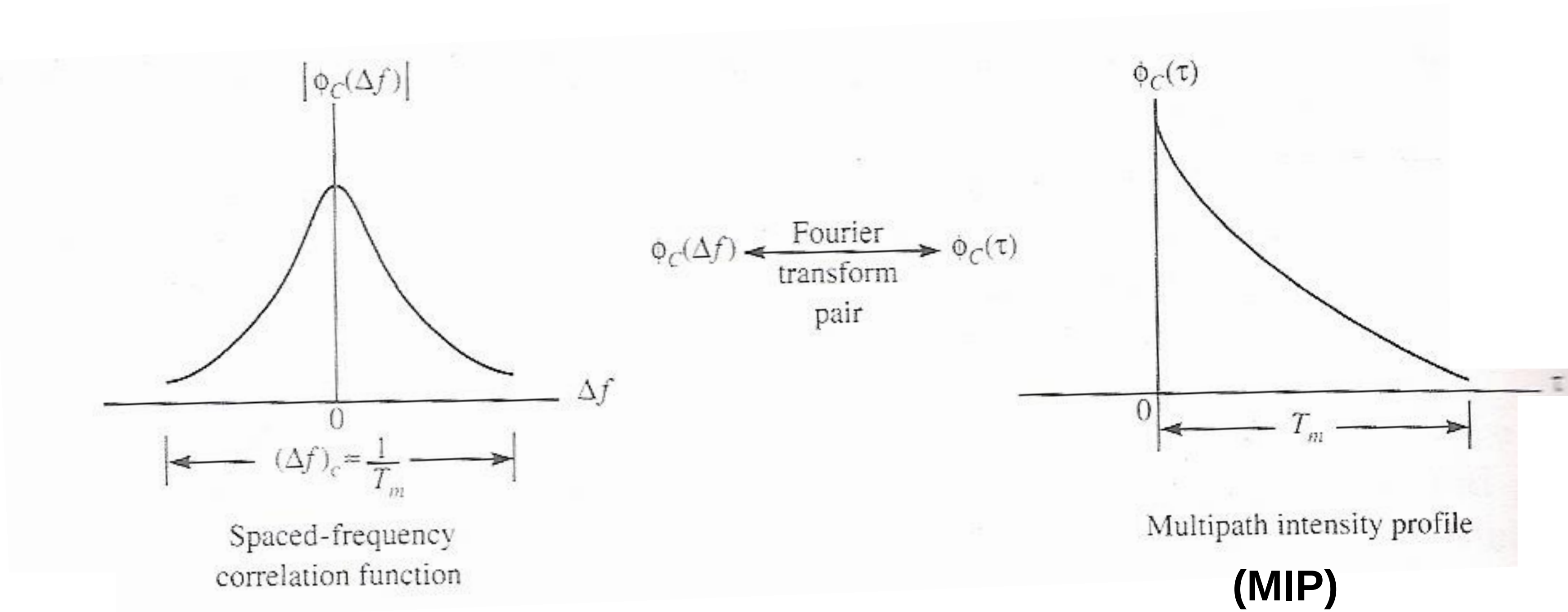
Figure E5.5

$$\begin{cases} \tau_0 = 0 \\ \tau_1 = 1 \\ \tau_2 = 2 \\ \tau_3 = 5 \end{cases}$$

[1,p.22]

Time Spread

- **Delay spread**
- **Coherence bandwidth**



(MIP)
Delay spread

(Coherence bandwidth)

[6, p.764]

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Channel Correlation Functions and Spectrum – Time Spread (2/3)

- **Spaced-Frequency, Spaced-Time Correlation Function of the Channel**
  - Let $C(f;t)$ be the Fourier transform of $c(\tau;t)$ w.r.t. the delay parameter τ.

$$C(f;t)=\int_{-\infty}^{\infty} c(\tau;t)e^{-j2\pi f\tau}d\tau$$

  - Under the assumption of WSS channel, the autocorrelation function of $C(f;t)$:

$$\phi_C(f_1,f_2;\Delta t)\equiv \tfrac{1}{2}E\left[C^*(f_1;t_1)C(f_2;t_2)\right],\ \text{where } \Delta t=t_2-t_1$$

$$=\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E\left[c^*(\tau_1;t_1)c(\tau_2;t_2)\right]e^{-j2\pi(f_1\tau_1-f_2\tau_2)}d\tau_1d\tau_2$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_c(\tau_1;\Delta t)\delta(\tau_1-\tau_2)e^{-j2\pi(f_1\tau_1-f_2\tau_2)}d\tau_1d\tau_2,$$

Autocorrelation function

$\nwarrow$ *uncorrelated scattering* (UC)

$$=\boxed{\int_{-\infty}^{\infty}\phi_c(\tau;\Delta t)e^{-j2\pi\Delta f\tau}d\tau},\ \text{where } \Delta f=f_1-f_2$$

$$\equiv\boxed{\phi_C(\Delta f;\Delta t)},\ \rightarrow \textbf{WSSUC}$$

The UC assumption results that the autocorrelation function of $C(f;t)$ in frequency is a function of only the frequency difference.

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Channel Correlation Functions and Spectrum – Time Spread (3/3)

- **Coherence Bandwidth, $(\Delta f)_c$**
  - Space-Frequency Correlation Function, $\phi_C(\Delta f)$
    - If we let $\Delta t = 0$, $\phi_C(\Delta f; 0) \equiv \phi_C(\Delta f)$, the autocorrelation function of the channel with frequency difference Δf.

    $\Rightarrow$ So, $\phi_C(\Delta f)$ provides us with a measure of the frequency coherence of the channel.
  - The Fourier transform relationship of $\phi_C(\Delta f)$ and the MIP $\phi_c(\tau)$:

    $$\Rightarrow \phi_C(\Delta f) = \int_{-\infty}^{\infty} \phi_c(\tau) e^{-j2\pi\Delta f \tau} d\tau$$

    figure

    Relationship
  - As a result of the Fourier transform relationship between $\phi_C(\Delta f)$ and $\phi_c(\tau)$, the reciprocal of the delay spread T_m is a measure of the coherence bandwidth of the channel, $(\Delta f)_c$. That is, $\boxed{(\Delta f)_c \approx 1/T_m}$.

Channel Correlation Functions and Spectrum – Time Variation (1/3)

- We now turn our attention from the time spread of the channel (the parameter τ) to the time variation of the channel (the parameter Δt).
- Coherence Time, $(\Delta t)_c$
  - Space-time correlation function, $\phi_C\left(\Delta t\right)$
    - If we let $\Delta f = 0$, $\phi_C\left(0;\Delta t\right) \equiv \phi_C\left(\Delta t\right)$, the autocorrelation function of the channel with time difference Δt.

      $\Rightarrow$ So, $\phi_C\left(\Delta t\right)$ provides us with a measure of the time coherence of the channel.
  - Coherence time is the time over which a channel may be considered coherent.

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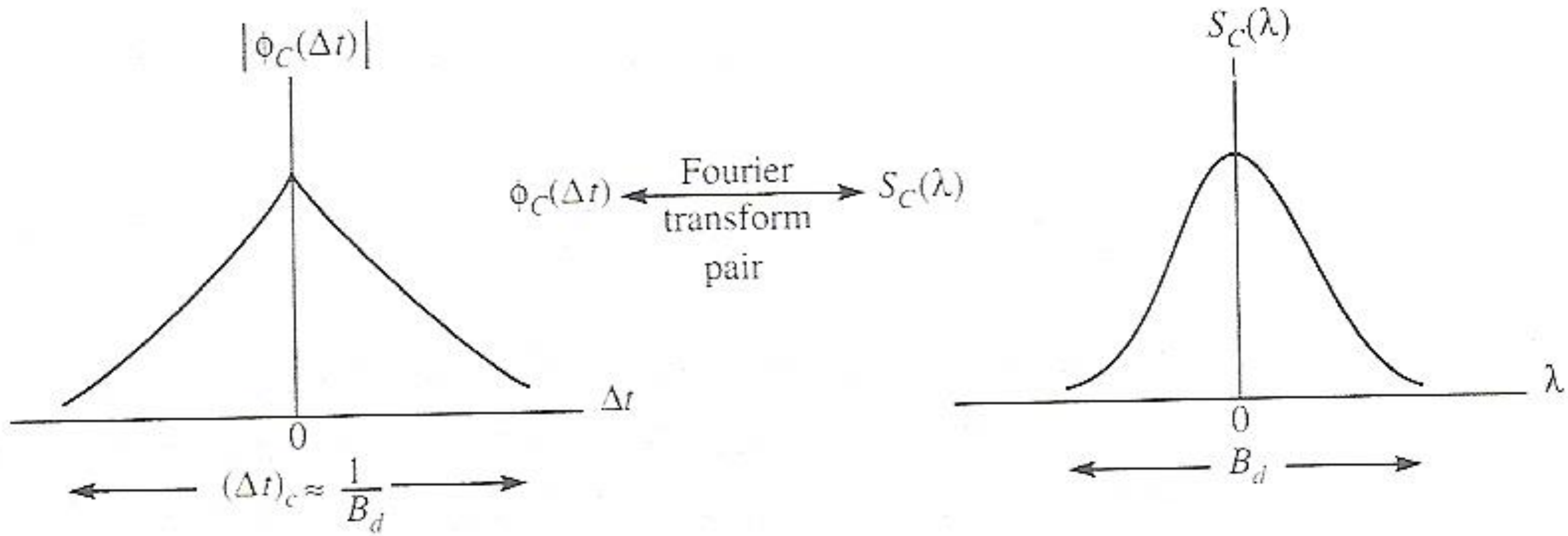
Time Variation

- Coherence time
- Doppler shift



(Coherence time) (Doppler shift)

[6, p.765]

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Channel Correlation Functions and Spectrum – Time Variation (2/3)

- **Space-Frequency Doppler Power Spectrum**
  - **To relate the Doppler effects to the time variations of the channel, we take the Fourier transform of $\phi_C(\Delta f;\Delta t)$ w.r.t. the variable Δt:**

    $$S_C(\Delta f;\lambda)=\int_{-\infty}^{\infty}\phi_C(\Delta f;\Delta t)e^{-j2\pi\lambda\Delta t}d\Delta t$$

    $\Rightarrow$ The variable λ refers to the Doppler frequency.

Channel Correlation Functions and Spectrum – Time Variation (3/3)

- **Doppler Spread, B_d**
  - If we let $\Delta f = 0$, $S_C(0;\lambda) \equiv S_C(\lambda)$, which is called **Doppler power spectrum** of the channel.

    $$\Rightarrow \; S_C(\lambda) = \int_{-\infty}^{\infty} \phi_C(\Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t$$

  - The range of values of λ over which $S_C(\lambda)$ is essentially nonzero is called the **Doppler spread B_d** of the channel.
  - Since $S_C(\lambda)$ is related to $\phi_C(\Delta t)$ by Fourier transform, the reciprocal of $(\Delta t)_c$ is a measure of the Doppler spread. That is,

    $\boxed{(\Delta t)_c \approx 1/B_d}$.   figure

Parameters of Mobile Multipath Channels

- **Time dispersion** (or time spread) parameters describe the **time dispersive nature** of the channel.
  - Delay spread, T_m
  - Coherence bandwidth

$$\left.\begin{array}{l}\text{Delay spread, } T_m \\ \text{Coherence bandwidth}\end{array}\right\} \Rightarrow \boxed{(\Delta f)_c \propto 1/T_m}$$

- **Time variation** parameters describe the **time varying nature** of the channel.
  - Doppler spread, B_d
  - Coherence time

$$\left.\begin{array}{l}\text{Doppler spread, } B_d \\ \text{Coherence time}\end{array}\right\} \Rightarrow \boxed{(\Delta t)_c \propto 1/B_d}$$

Parameters of Mobile Multipath Channels – Delay Spread

- **Delay spread, caused by multipath channel, is the time difference between the arrival moment of the first multipath component and the last one.**
- **Maximum Excess Delay (X dB)**
  - **The temporal delay extent of the multipath that is above a particular threshold, X dB.**
- **Mean Excess Delay**

$$\bar{\tau} = \sum_n \phi_c(\tau_n)\tau_n \Big/ \sum_n \phi_c(\tau_n)$$

  - **Different channels with the same values of maximum excess delay and mean excess delay can exhibit very different profiles of MIP over the delay span.**

  ➔ A more useful measurement of delay spread is RMS delay spread.
- **RMS Delay Spread**

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}, \text{ where } \overline{\tau^2} = \sum_n \phi_c(\tau_n)\tau_n^2 \Big/ \sum_n \phi_c(\tau_n)$$

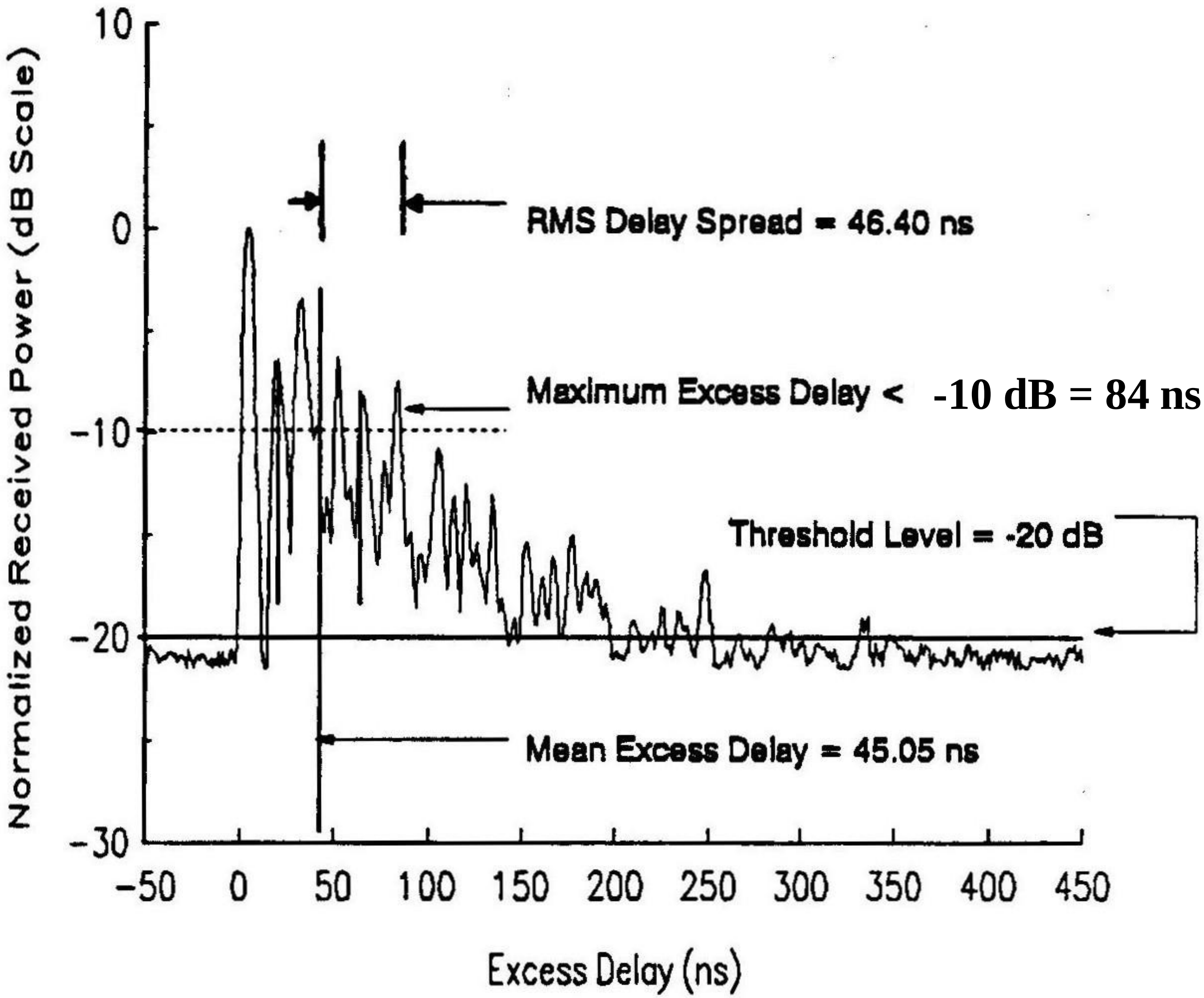


Figure Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

[1,p.200]

Table 5.1 Typical Measured Values of RMS Delay Spread

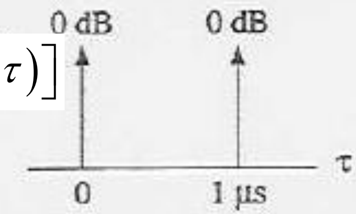
Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10–25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200–310 ns	Averaged typical case	[Cox72]
Suburban	910	1960–2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10–50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70–94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

[1,p.200]

Example 5.4

(a) Compute the RMS delay spread for the following power delay profile:

$10\log\left[\phi_c(\tau)\right]$



(b) If BPSK modulation is used, what is the maximum bit rate that can be sent through the channel without needing an equalizer?

Solution

(a) $\bar{\tau} = \dfrac{(1)(0)+(1)(1)}{1+1} = \dfrac{1}{2} = 0.5\mu s$

$$\overline{\tau^2} = \frac{(1)(0)^2+(1)(1)^2}{1+1} = \frac{1}{2} = 0.5\mu s^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2}-(\bar{\tau})^2} = \sqrt{0.5-(0.5)^2} = \sqrt{0.25} = 0.5\mu s$$

(b) $\dfrac{\sigma_\tau}{T_s} \le 0.1$

The symbol duration $T_s \ge 10\sigma_\tau$ $\Rightarrow$ ISI free

$\Rightarrow$ Without equalizer

$$T_s \ge \frac{\sigma_\tau}{0.1}$$

$$T_s \ge \frac{0.5\mu s}{0.1}$$

$T_s \ge 5\mu s$ $\rightarrow$ $R_s = 1/T_s \le 1/(5\mu) = 0.2\times10^6$ **symbols per second (sps)**

$\max R_s = 0.2\times10^6\text{sps} = 200\text{ksps}$

$\max R_b = 200\text{kbps}$

Parameters of Mobile Multipath Channels – Coherence Bandwidth

- **Coherence bandwidth B_c is a statistical measure of the range of frequencies over which the channel can be considered "flat" (i.e., a channel which passes all spectral components with approximately equal gain and linear phase.)**
- **The RMS delay spread σ_τ and coherence bandwidth B_c are inversely proportional to one another.**
  - **The coherence bandwidth with frequency correlation of above 0.9 is approximately** $Bc \approx 1/(50\sigma_\tau)$
  - **The coherence bandwidth with frequency correlation of above 0.5 is approximately** $Bc \approx 1/(5\sigma_\tau)$

Example 5.5

Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

$10\log\left[\phi_c(\tau)\right]$

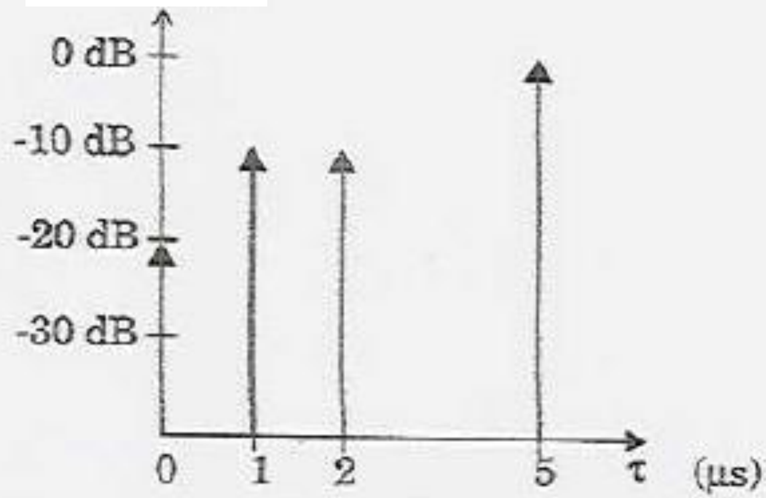


Figure E5.5

[1,p.202]

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Solution

Using the definition of maximum excess delay (10 dB), it can be seen that $\tau_{10\,dB}$ is 5 μs. The rms delay spread for the given multipath profile can be obtained using Equations (5.35)–(5.37). The delays of each profile are measured relative to the first detectable signal. The mean excess delay for the given profile is

$$\bar{\tau} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{[0.01+0.1+0.1+1]} = 4.38\ \mu s$$

The second moment for the given power delay profile can be calculated as

$$\overline{\tau^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)}{1.21} = 21.07\ \mu s^2$$

Therefore the rms delay spread is $\sigma_\tau = \sqrt{21.07-(4.38)^2} = 1.37\ \mu s$

The coherence bandwidth is found from Equation (5.39) to be

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu s)} = 146\ \text{kHz}$$

Since B_c is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds B_c, thus an equalizer would be needed for this channel.

Parameters of Mobile Multipath Channels – Doppler Spread

- **Doppler spread (or Doppler shift) B_d is a statistical measure of the spectral broadening caused by the relative motion between TX and RX.**
  - **Given the mobile speed and wavelength, let f_m be the maximum Doppler shift.**

$$\boxed{f_m = \frac{v}{\lambda}}$$

Parameters of Mobile Multipath Channels – Coherence Time

- Coherence time is a statistical measure of the time duration over which the channel impulse response is essentially invariant.
- The coherence time T_C and maximum Doppler shift f_m are reciprocally related (inversely proportional to one another).
  - $T_C \approx 1/f_m$
    - ➔ Too loose (i.e. the approximate coherence time is too large.)
  - The coherence time with time correlation of above 0.5 is approximately $T_C \approx 9/(16\pi f_m)$
    - Too restrictive (i.e., the coherence time is too small.)
  - A popular rule of thumb is to define the coherence time as the geometric mean of the two above definitions:

$$\boxed{T_C \approx \left[\left(1/f_m\right)\left(\frac{9}{16\pi f_m}\right)\right]^{1/2} = \sqrt{9/16\pi f_m^2}}$$

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Types of Small-Scale Fading

- **Depending on the relation between the signal parameters (such as signal bandwidth, symbol period) and the channel parameters (such as RMS delay spread and Doppler spread), the mobile radio channel can be categorized into different types.**
  - **Multipath delay spread leads to time dispersion and thus frequency selective fading.**
  - **Doppler spread leads to frequency dispersion and thus time selective fading.**

  ➔ The two propagation mechanisms are independent of one another.

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
(Coherence BW)
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

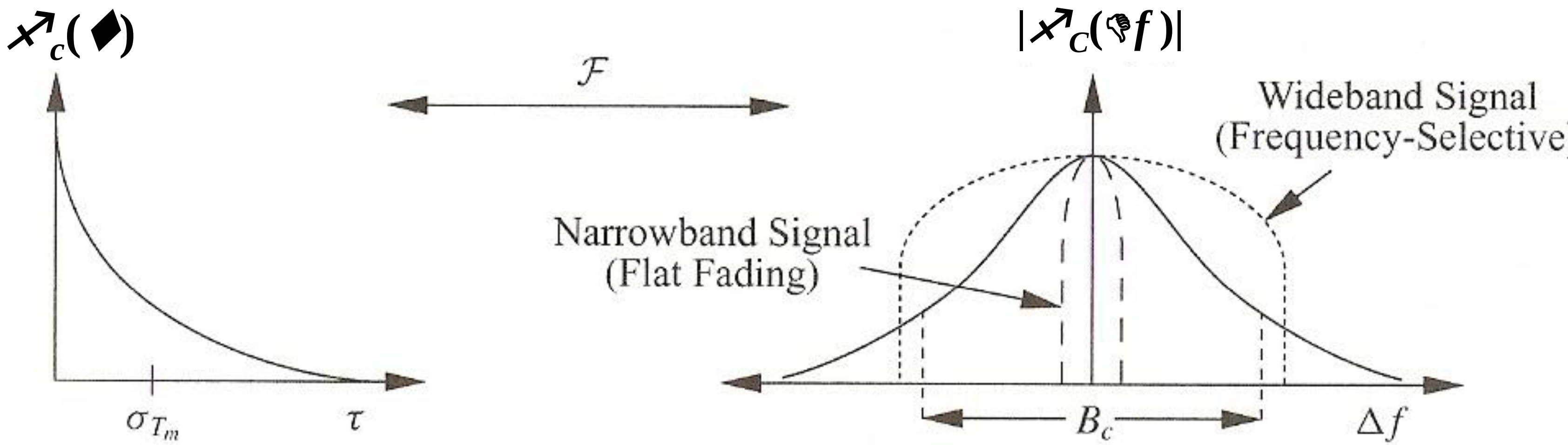
Small-Scale Fading
(Based on Doppler spread) → (or Time variation)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

[1,p.206]


Wideband Signal
(Frequency-Selective)
Narrowband Signal
(Flat Fading)
$\mathcal{F}$
σ_{T_m}
τ
B_c
Δf

[6, p.766]

Types of Small-Scale Fading – Flat Fading (1/2)

- **Flat Fading**
  - **If the radio channel has a constant gain and linear phase response over a bandwidth (coherence bandwidth) which is greater than the bandwidth of the transmitted signal, then the received signal will undergo flat fading.**
    - **➔Flat fading channels are referred to as *narrowband channels*, since the signal bandwidth is *narrow* as compared to the flat bandwidth of channel.**
  - **Over time, the received signal $r(t)$ varies in gain, but the "spectrum of the transmitted signal" is preserved.**
    - **➔Amplitude varying channels. [$r(t) = a(t) \times s(t)$]**
    - **➔Typical flat fading channels cause deep fades, and may require 20 or 30 dB more transmitter power to achieve the same BER as compared to the non-fading channels.**

(100~1000 times transmit power more than that with non-fading channels)

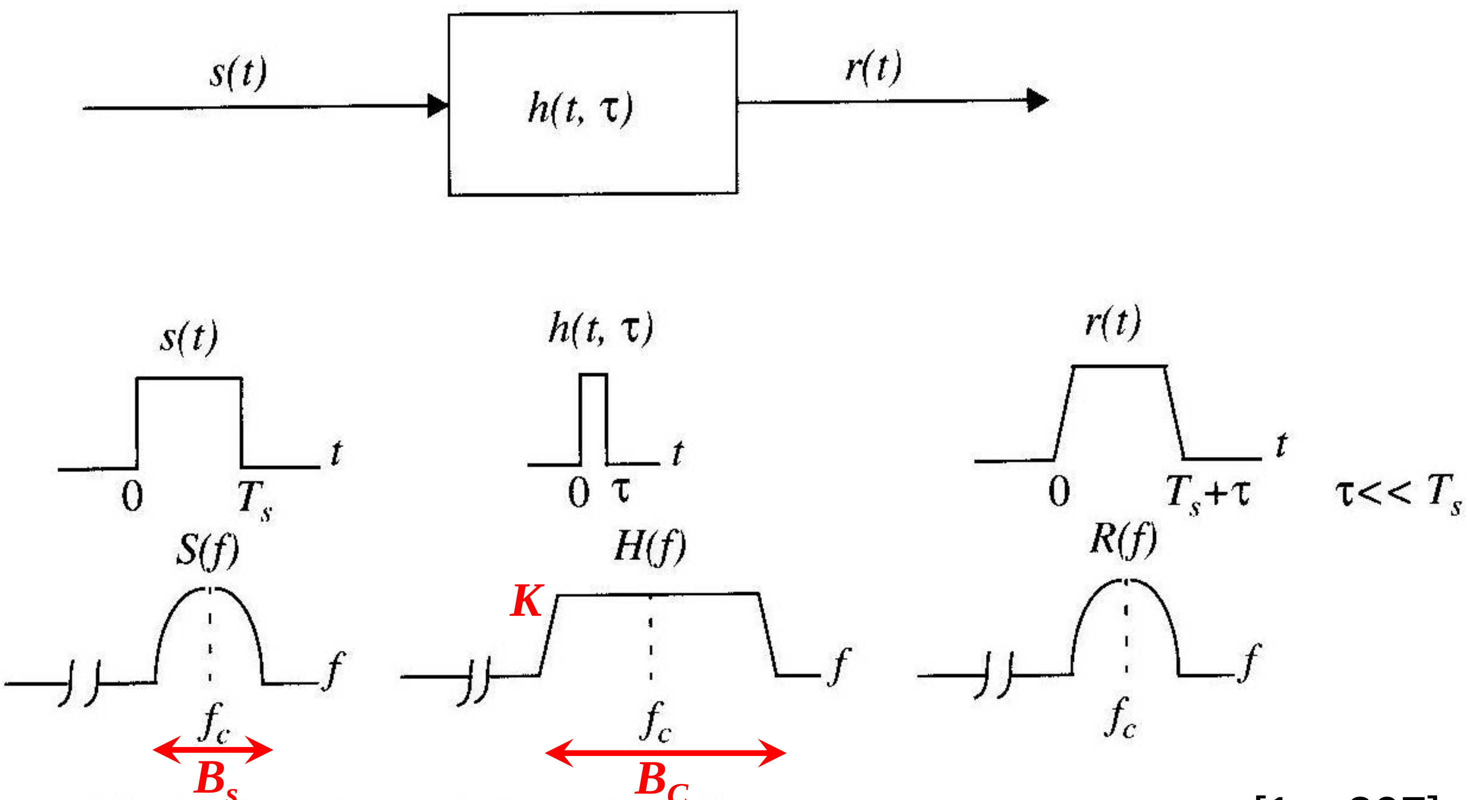


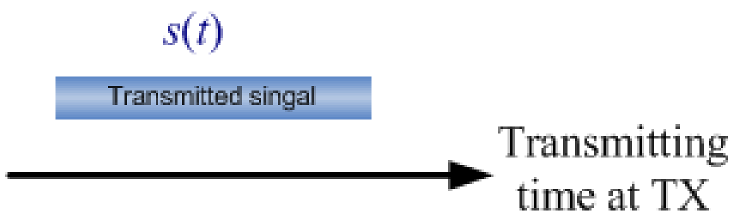
Figure 5.12 Flat fading channel characteristics.

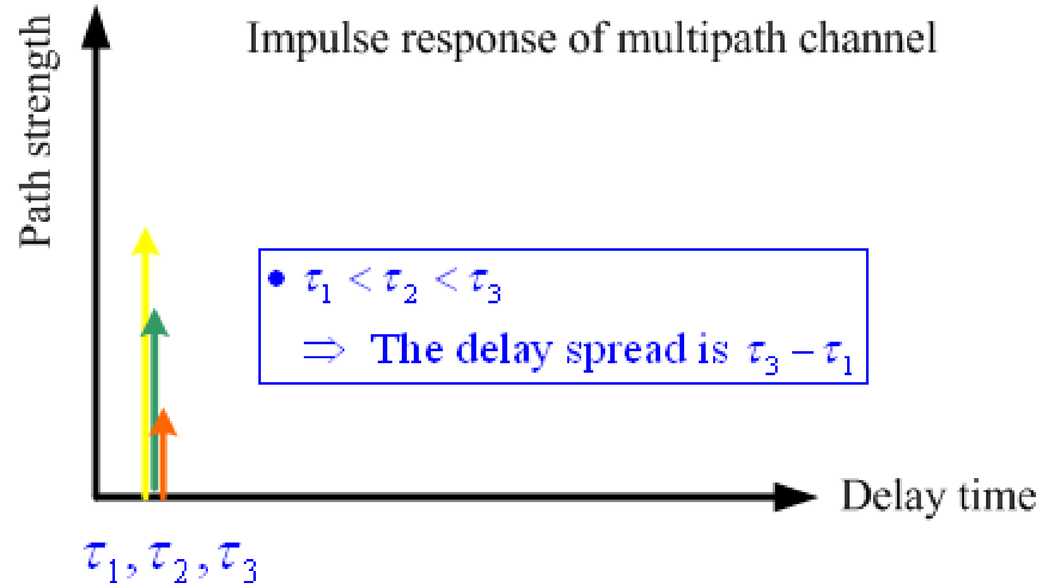
[1,p.207]

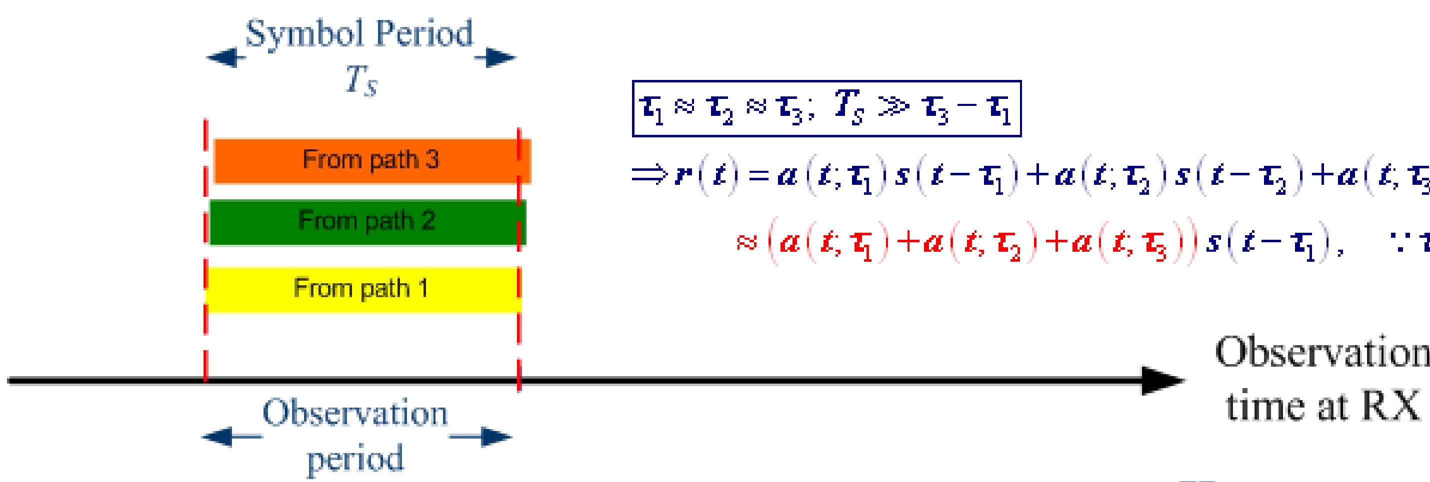
Flat Fading:

Signal bandwidth B_S < **Coherence bandwidth** B_C

$\rightarrow R(f) = H(f)S(f) = KS(f)$







$$\boxed{\tau_1 \approx \tau_2 \approx \tau_3;\ T_S \gg \tau_3 - \tau_1}$$

$$\Rightarrow r(t) = a(t;\tau_1)\,s(t-\tau_1) + a(t;\tau_2)\,s(t-\tau_2) + a(t;\tau_3)\,s(t-\tau_3)$$

$$\approx \big(a(t;\tau_1) + a(t;\tau_2) + a(t;\tau_3)\big)\,s(t-\tau_1), \quad \because \tau_1 \approx \tau_2 \approx \tau_3$$

Back

Types of Small-Scale Fading – Flat Fading(2/2)

- Flat Fading (Cont.)
  - To summarize, a signal undergoes flat fading if

$$\begin{cases} \text{Signal bandwidth } B_S \ll \text{ Coherence bandwidth } B_C \\ \text{Symbol period } T_S \gg \text{ RMS delay spread } \sigma_\tau \end{cases}$$

- $B_S \downarrow \;\propto\; 1/T_S \;\Rightarrow\; T_S \uparrow \;\Rightarrow\; R_S \downarrow (\text{Symbol rate})$

  In general, low data rate experiences flat fading with high probability.

- $B_C \propto 1/\sigma_\tau$

Illustrate ISI

Types of Small-Scale Fading – Frequency Selective Fading(1/2)

- **Frequency Selective Fading**
  - **The spectrum $S(f)$ of the transmitted signal has a bandwidth which is greater than the coherence bandwidth B_C of the channel.**
    - **The gain of the fading channel is different for different frequency components.**
  - **A channel is typically described as wideband (wideband channel) if the signal bandwidth significantly exceeds the channel's coherence bandwidth.**

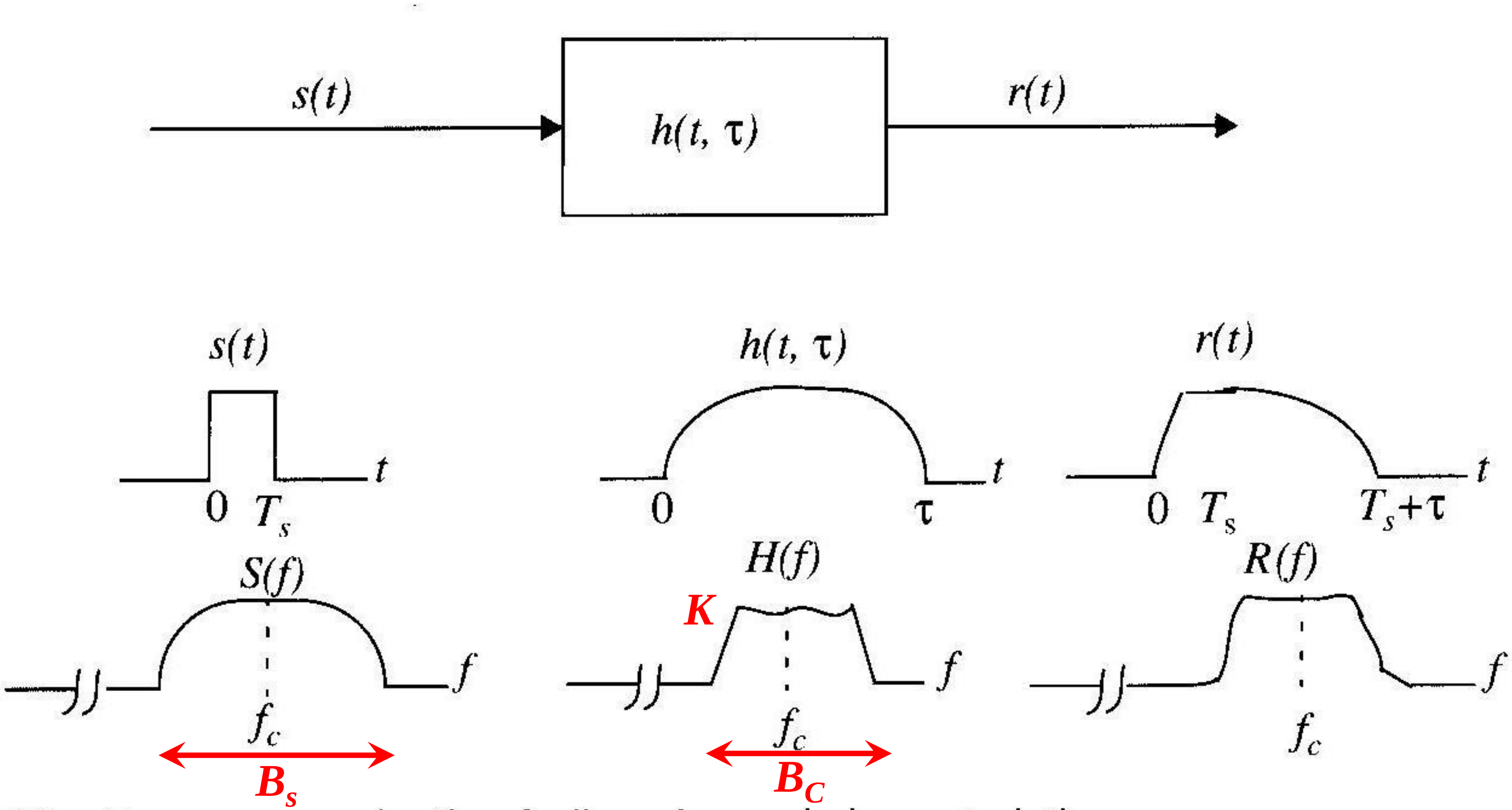


Figure 5.13 Frequency selective fading channel characteristics.

[1,p.208]

Frequency Selective Fading:

Signal bandwidth B_S > Coherence bandwidth B_C

$\rightarrow R(f) = H(f)S(f) \neq KS(f)$

Types of Small-Scale Fading – Frequency Selective Fading(2/2)

- Frequency Selective Fading (Cont.)
  - To summarize, a signal undergoes frequency selective fading if

$$\begin{cases} \text{Signal bandwidth } B_S > \text{ Coherence bandwidth } B_C \\ \text{Symbol period } T_S < \text{ RMS delay spread } \sigma_\tau \end{cases}$$

- $B_S \propto 1/T_S \,;\; B_C \propto 1/\sigma_\tau$

Illustration of ISI

$\Rightarrow$ A common rule of thumb is that

- a channel is flat fading if $T_S \geq \sigma_\tau$.
- a channel is frequency selective fading if $T_S < \sigma_\tau$.

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Types of Small-Scale Fading – Fast Fading

- Fast Fading
  - The channel impulse response changes rapidly **within the symbol duration**.
    - That is, the coherence time of the channel is smaller than the symbol period of the transmitted signal.
    - In practice, fast fading only occurs for very low data rates (very large symbol duration).
  - A signal undergoes fast fading if

$$\begin{cases} \text{Symbol period } T_S > \text{ Coherence time } T_C \\ \text{Signal bandwidth } B_S < \text{ Doppler shift } B_D \end{cases}$$

- $B_S \propto 1/T_S \,;\; B_D \propto 1/T_C$

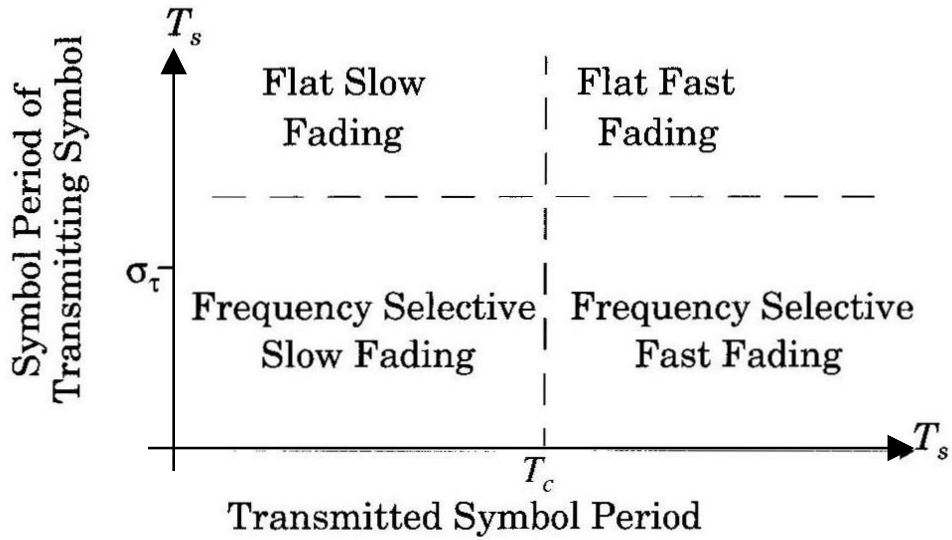
Types of Small-Scale Fading – Slow Fading

- Slow Fading
  - The radio channel may be static over more than one symbol period.
    - That is, the coherence time of the channel may cover the duration of more than one symbols (generally, hundreds symbols.)
  - A signal undergoes slow fading if

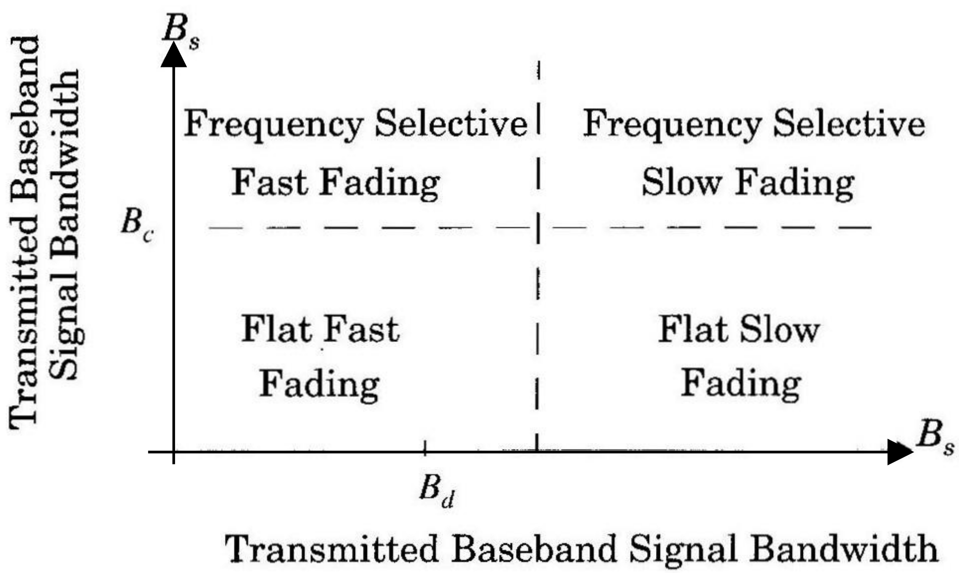
$$\begin{cases} \text{Symbol period } T_S \ll \text{ Coherence time } T_C \\ \text{Signal bandwidth } B_S \gg \text{ Doppler shift } B_D \end{cases}$$

- $B_S \propto 1/T_S$; $B_D \propto 1/T_C$

(a)



(b)



[1,p.210]

Figure 5.14 Matrix illustrating type of fading experienced by a signal as a function of: (a) symbol period; and (b) baseband signal bandwidth.

Received Envelope and Phase Distribution(1/11)

- **Rayleigh Fading**
  - **Rayleigh fading is used to describe the envelope (amplitude) of each multipath component is with Rayleigh distribution.**
  - **The construction of each multipath component:**
    - **Each multipath component is the composite of a large number of plane waves.**
    - **Thus the received complex envelope of each multipath component $g(t) = g_I(t) + jg_Q(t)$ can be treated as a complex Gaussian random process. (The law of large number)**
    - **Furthermore, $g_I(t)$ and $g_Q(t)$ are i.i.d. Gaussian random variables at any time t, with identical value of variance, σ^2.**

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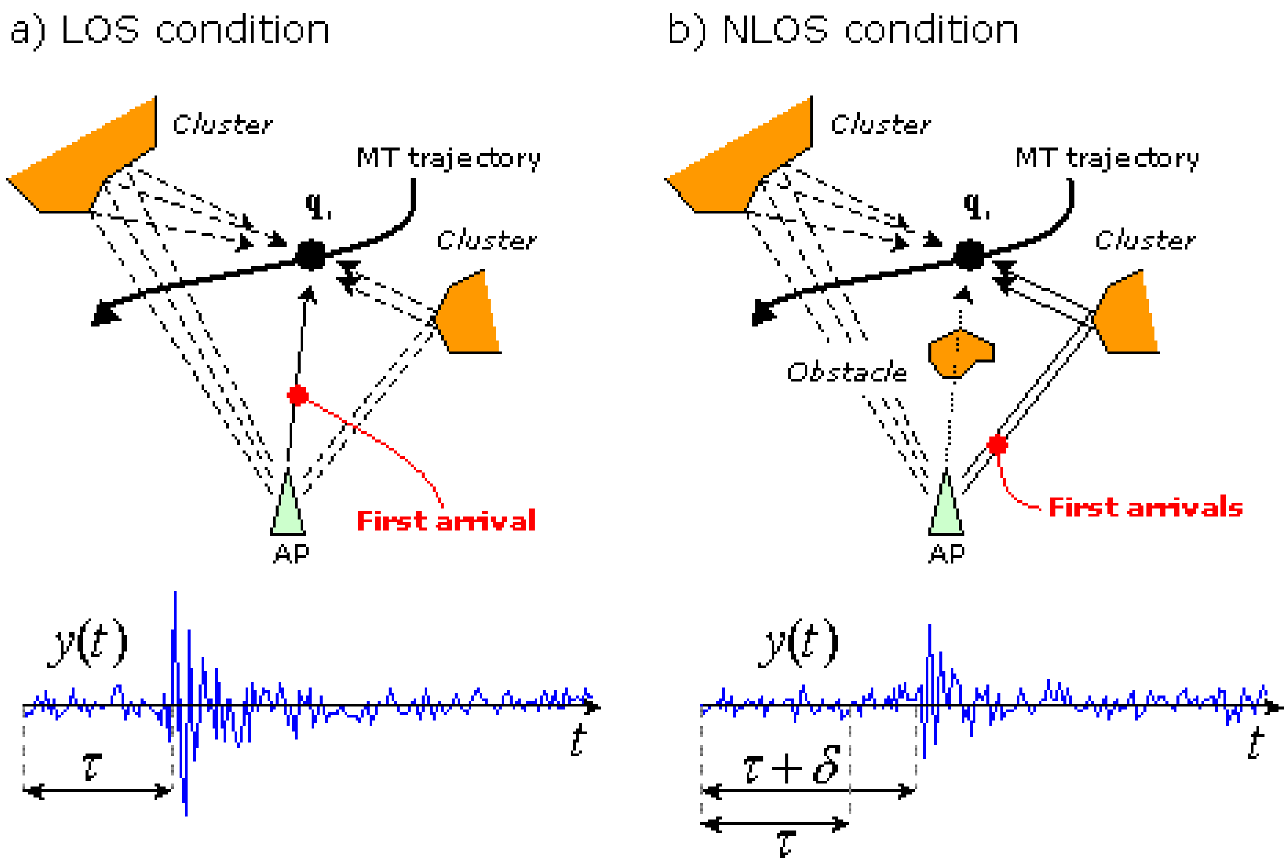

Ref.
http://risorse.dei.polimi.it/dsp/tlc/position.htm

Received Envelope and Phase Distribution(2/11)

- **Rayleigh Fading (Cont.)**
  - **If $g_I(t)$ and $g_Q(t)$ are zero-mean Gaussian random processes, the envelope $X(t)$ of $g(t)$ has a Rayleigh distribution at any time t.**

$$X(t)=|g(t)| \Rightarrow \boxed{\begin{array}{l} f_X(x)=\dfrac{x}{\sigma^2}\exp\left\{-\dfrac{x^2}{2\sigma^2}\right\}, \quad x\geq 0 \\ E(X)=\sigma\sqrt{\dfrac{\pi}{2}};\ Var(X)=\sigma^2\left(2-\dfrac{\pi}{2}\right) \end{array}}$$

  - **The instantaneously power at any time t is exponentially distributed:**

$$Y(t)=X^2(t)=|g(t)|^2 \Rightarrow \boxed{f_Y(y)=\dfrac{1}{2\sigma^2}\exp\left\{-\dfrac{y}{2\sigma^2}\right\}, \quad y\geq 0}$$

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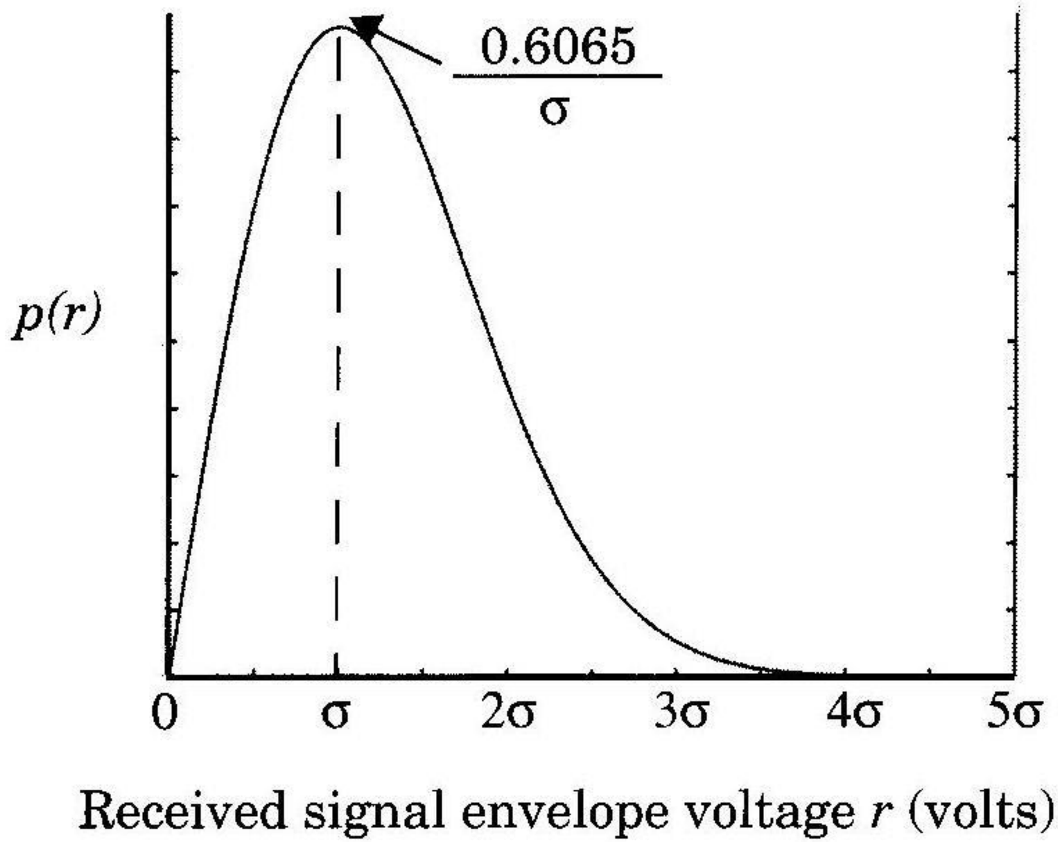

Figure 5.16 Rayleigh probability density function (pdf).

[1,p.212]

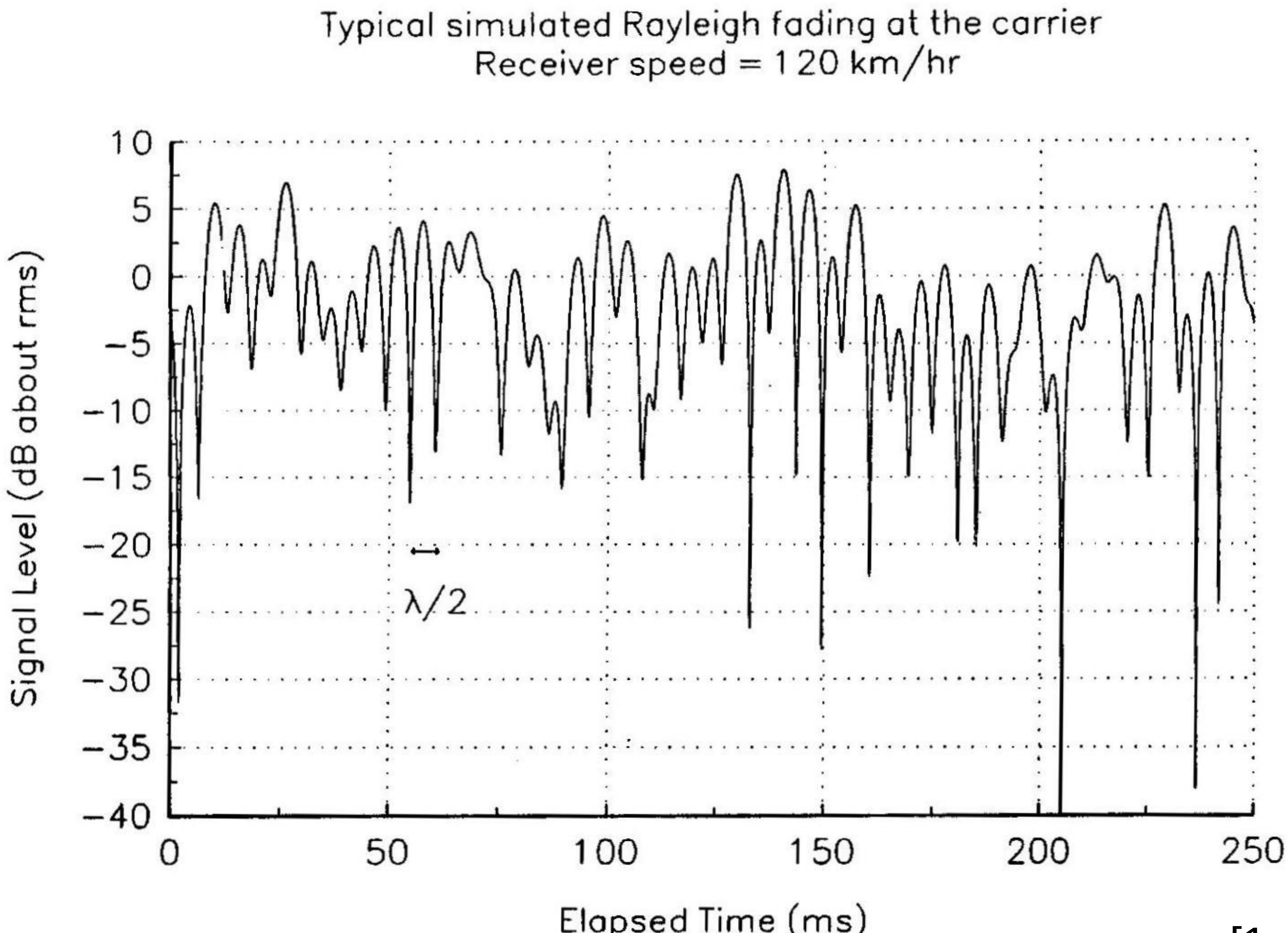

Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

[1,p.211]

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Received Envelope and Phase Distribution(3/11)

- Rician Fading
  - If there exists an LOS path, at any time t, $g_I(t)$ and $g_Q(t)$ are Gaussian random variables with non-zero means $m_I(t)$ and $m_Q(t)$, and the same variance σ^2.
  - The envelope $X(t)$ of $g(t)$ has a Rician distribution at any time t.

$$X(t) = |g(t)| \;\Rightarrow$$

$$\boxed{f_X(x) = \frac{x}{\sigma^2}\exp\left\{-\frac{x^2+s^2}{2\sigma^2}\right\} I_0\left(\frac{xs}{\sigma^2}\right), \quad x \ge 0}$$

where $s^2 = m_I^2(t) + m_Q^2(t)$, and $I_0(\cdot)$ is the modified Bessel function of the first kind and zero-order.

Received Envelope and Phase Distribution(4/11)

- **Rician Fading (Cont.)**
  - **The Rice factor, K, is defined by $K = s^2/(2\sigma^2)$.**
    - **With $K = 0$, the Rician distribution degenerates to a Rayleigh distribution.**
    - **With K approaches to infinite, the channel does not exhibit any fading at all.**

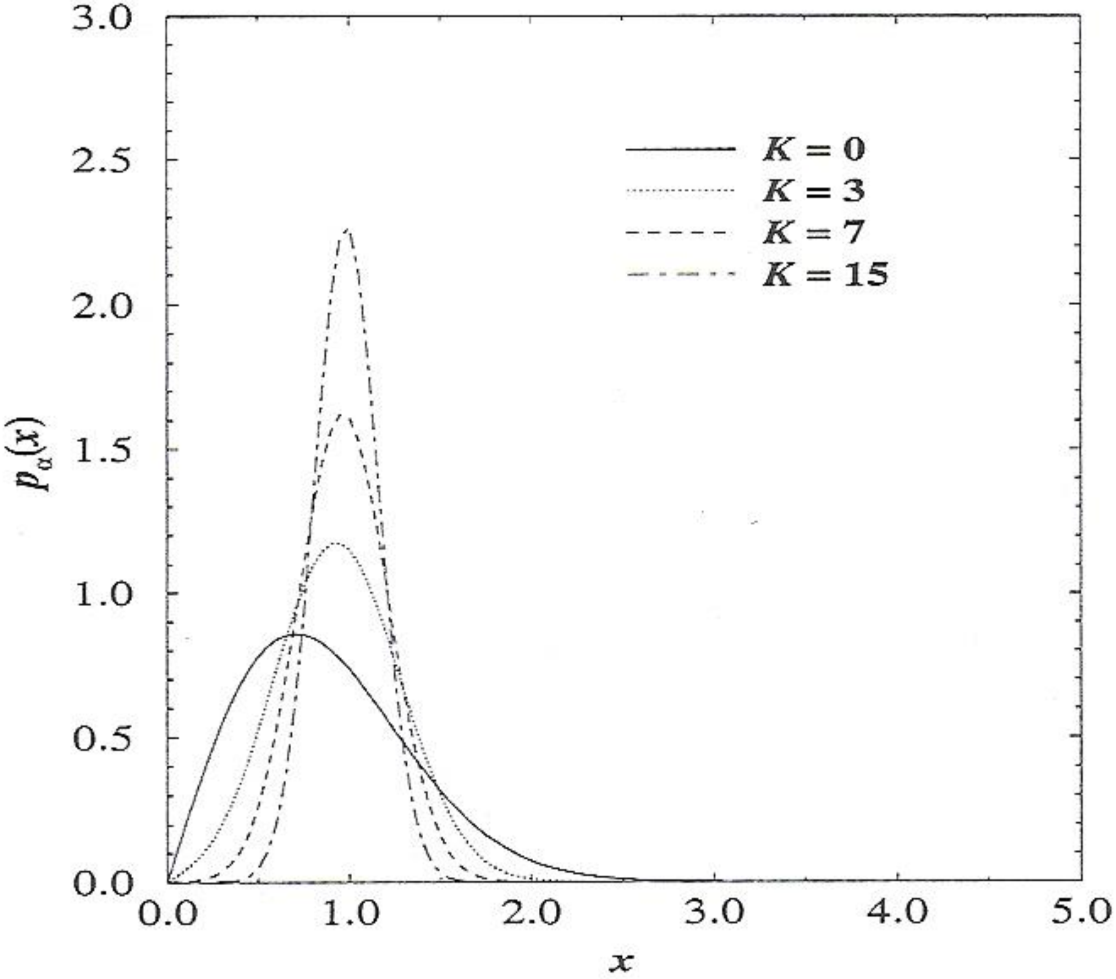

Figure 2.8. The Rice pdf for several values of K with $\Omega_p = 1$. $\sigma^2 = 0.5$

[2,p.53]

Received Envelope and Phase Distribution(5/11)

- Nakagami Fading
  - The Nakagami fading model was initially proposed because it **matched empirical results for HF wave propagation**.
  - The Nakagami distribution describes the magnitude of the received envelope by the distribution:

$$\boxed{f_X(x) = \frac{2m^m x^{m-1}}{\Gamma(m)\Omega^m} \exp\left\{-\frac{mx^2}{\Omega}\right\}, \quad m \geq 0.5, \ x \geq 0}$$

- m is called the "shape factor", $\Omega \equiv E\left(x^2\right)$, and $\Gamma(\cdot)$ is the Gamma function.

Received Envelope and Phase Distribution(6/11)

- Nakagami Fading (Cont.)
  - The Nakagami distribution can model fading conditions that are either more or less severe than Rayleigh fading.
    - With $m = 1$, the Nakagami distribution becomes the Rayleigh distribution.
    - With $m < 1$, the Nakagami fading causes more severe performance degradation than Rayleigh fading.
      - When $m = 1/2$, it becomes a one-sided Gaussian distribution.
    - With $m > 1$, the Nakagami fading reduces the fluctuations of the signal strength compared to Rayleigh fading.
      - When $m \rightarrow \infty$, the distribution becomes an impulse (no fading).

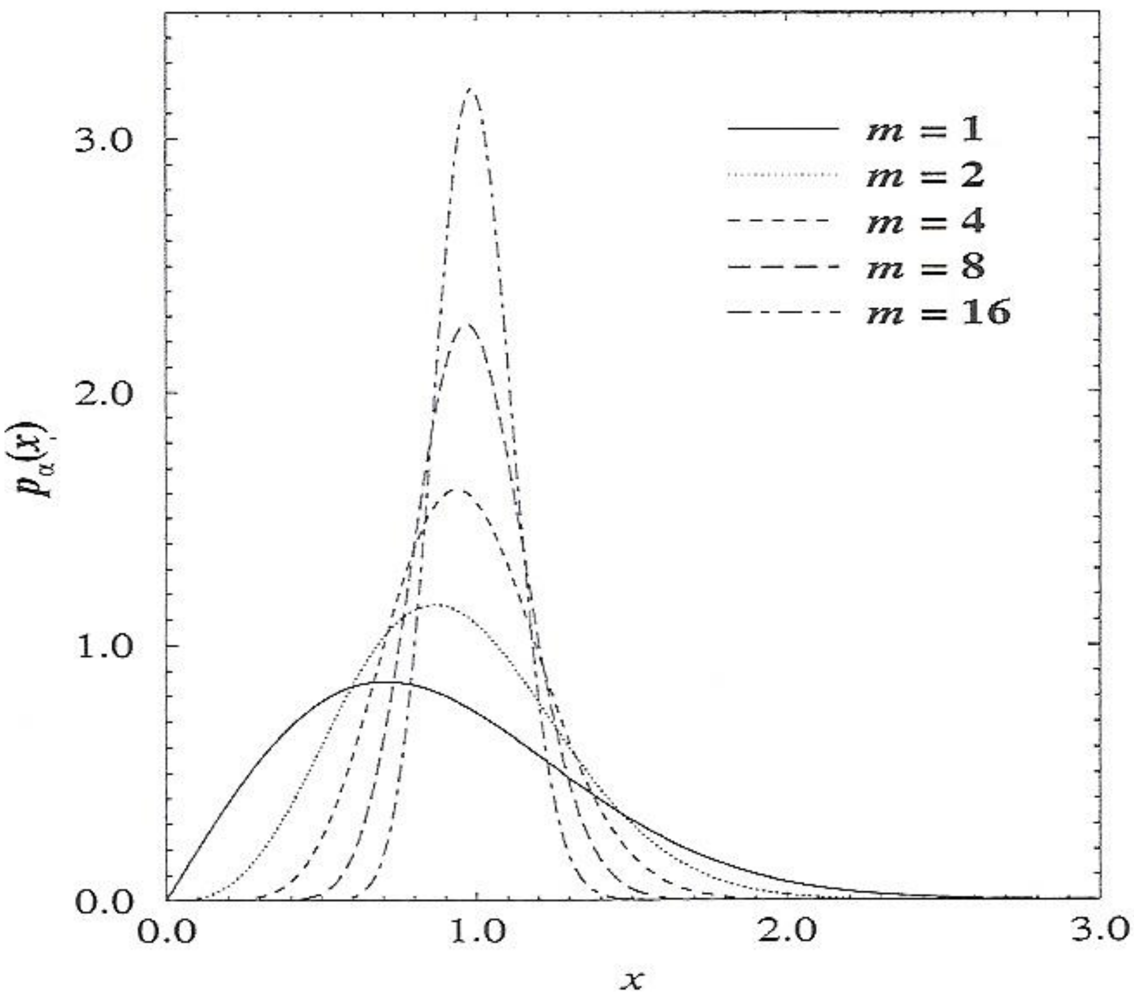

Figure 2.9. The Nakagami pdf for several values of m with $\Omega_p = 1$.

[2,p.54]

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Received Envelope and Phase Distribution(7/11)

- Nakagami Fading (Cont.)
  - The Rician and Nakagami distribution can be closely approximated by using the following relation between the Rice factor K and the Nakagami shape factor m:

$$K=\frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}},\ \text{for } m>1,\ \text{and}\ \ m=\frac{(K+1)^2}{(2K+1)}$$

    - The Nakagami distribution often leads to convenient closed form analytical expressions that are otherwise unattainable.
    - However, it becomes highly inaccurate for the tails. Thus, as bit errors occur during deep fades, these performance measures are mainly determined by the tail of the pdf.

Received Envelope and Phase Distribution(8/11)

- **Nakagami Fading (Cont.)**
  - **If the envelope is Nakagami distributed, the corresponding instantaneous power is Gamma distributed.**

$$Y(t) = X^2(t) \Rightarrow$$

$$\boxed{f_Y(y) = \left(\frac{m}{\Omega}\right)^m \frac{y^{m-1}}{\Gamma(m)} \exp\left\{-\frac{my}{\Omega}\right\},\ y \ge 0}$$

Received Envelope and Phase Distribution(9/11)

- **Other Important Roles in Nakagami Distribution**
  - **It describes the amplitude of received signal after maximum ratio diversity combining (MRC).**
    - **After k-branch MRC with Rayleigh-fading signals, the resulting signal is Nakagami with $m = k$.**
    - **It follows that MRC combining of m-Nakagami fading signals in k branches gives a Nakagami signal with shape factor mk.**
  - **The sum of multiple i.i.d. Rayleigh-fading signals have a Nakagami distributed signal amplitude.**
    - **The distribution of the power sum of n i.i.d. Rayleigh-fading signals is the n-th convolution of the exponential distribution, which is a Gamma distribution.**
    - **It may not be fully appropriate to speak of the envelope of such a power sum, but if one defines the amplitude to be proportional to the square root of the power, then one finds that the amplitude has a Nakagami distribution.**

Received Envelope and Phase Distribution(10/11)

- Other Important Roles in Nakagami Distribution (cont.)
  - Nakagami fading occurs for multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves.
    - Within any one cluster, the phases of individual reflected waves are random, but the delay times are approximately equal for all waves.
      - As a result the envelope of each cumulated cluster signal is Rayleigh distributed.
      - The average time delay is assumed to differ significantly between clusters.
    - If the delay times also significantly exceed the bit time of a digital link, the different clusters produce serious intersymbol interference, so the multipath self-interference then approximates the case of co-channel interference by multiple incoherent Rayleigh-fading signals. (i.e. frequency selective fading)

Received Envelope and Phase Distribution(11/11)

Envelope Phase

- **The phase of the received complex envelope $g(t) = g_I(t) + jg_Q(t)$ is**

$$\phi(t) = \tan^{-1}\left(\frac{g_Q(t)}{g_I(t)}\right)$$

- **For Rayleigh fading, $g_I(t)$ and $g_Q(t)$ are i.i.d. zero-mean Gaussian random variables at any time.**
- **For Rician fading channels, the phase is not uniformly distributed and takes on a more complicated form.**

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Thank you