教育部「5G行動寬頻人才培育跨校教學聯盟計畫」
5G行動網路協定與核網技術聯盟中心

課程: 5G系統層模擬技術

第十一週：Precoder and Receiver

國立中正大學
National Chung Cheng University

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Outline

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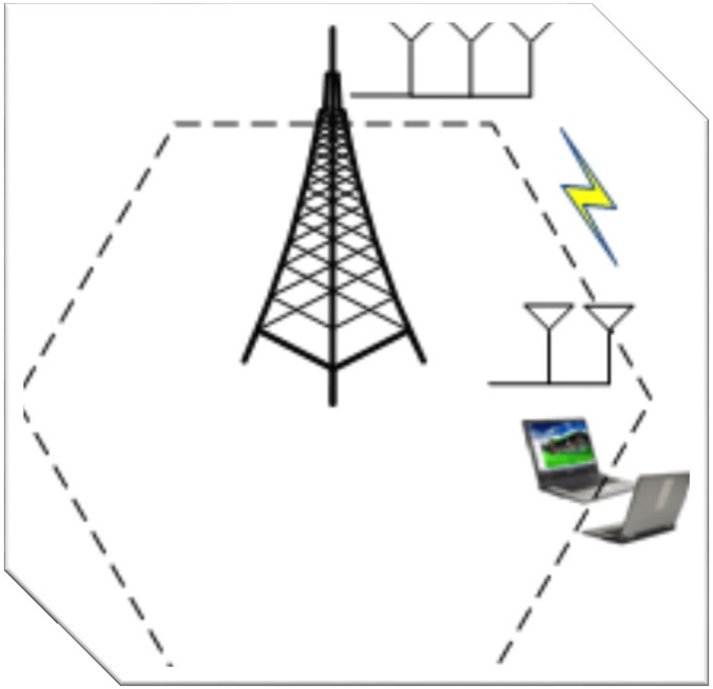
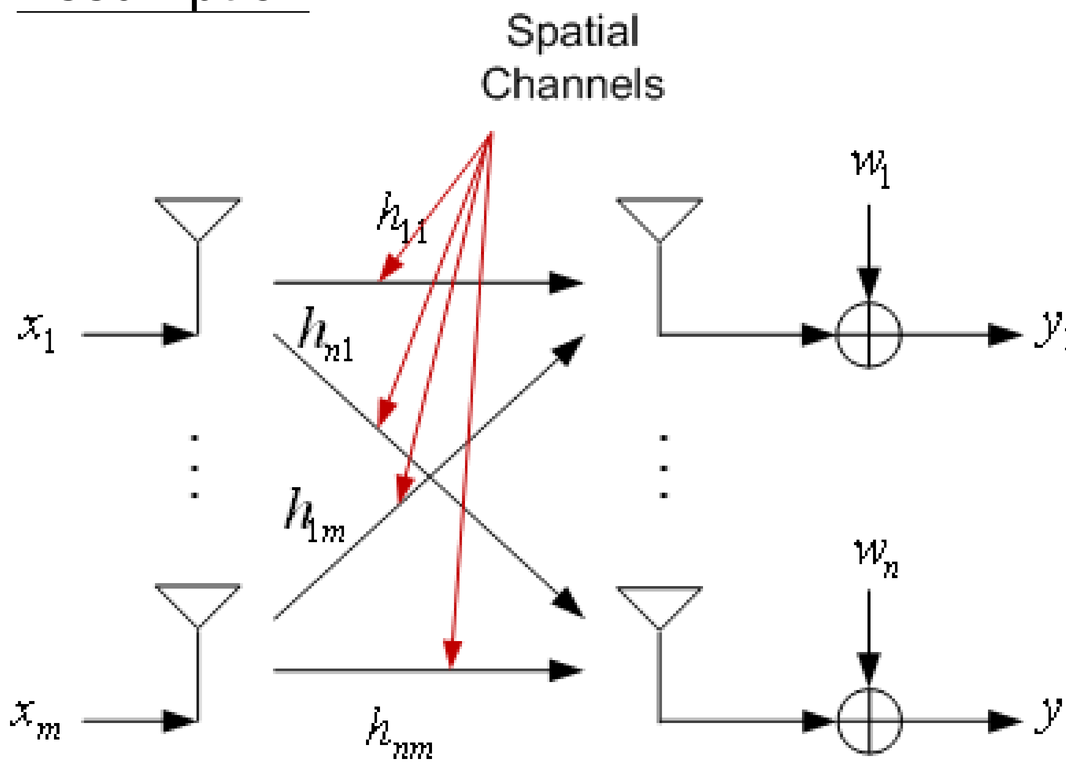
Usage of Multiple Antennas

- Single antenna vs. multiple antennas
  - There is only one spatial channel between transmitter and receiver in the single antenna system.
  - Using multiple antennas can create **multiple spatial channels** between transmitter and receiver.
- Multiple antenna systems are also called **MIMO** (multiple input, multiple output) systems.



Spatial Channel Under Flat Fading Assumption



- Flat fading assumption can be obtained in OFDM systems. Thus MIMO needs to be incorporated with OFDM technique.

Gains from Exploiting the Multiple Spatial Channels (1/3)

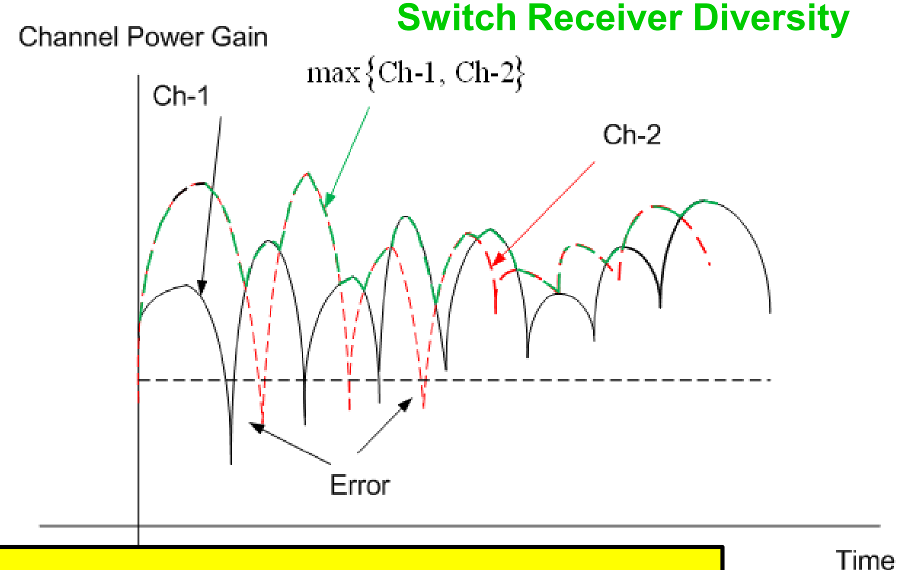
- **Diversity Gains** (by Enlarging the Antenna Spacing)
  - The transmit/receive diversity gain is obtained by transmitting the same signal and receiving a transmit signal via multiple antennas over independent spatial channels, respectively.
  - Transmit/receive diversity is used to mitigate the effects of small-scale fading.

– No channel state information at transmitter (CSIT) is needed for transmit diversity. (Nevertheless, CSIR is necessary for diversity combining.)

– STBC

– Transmission mode 2 in LTE

Switch Receiver Diversity



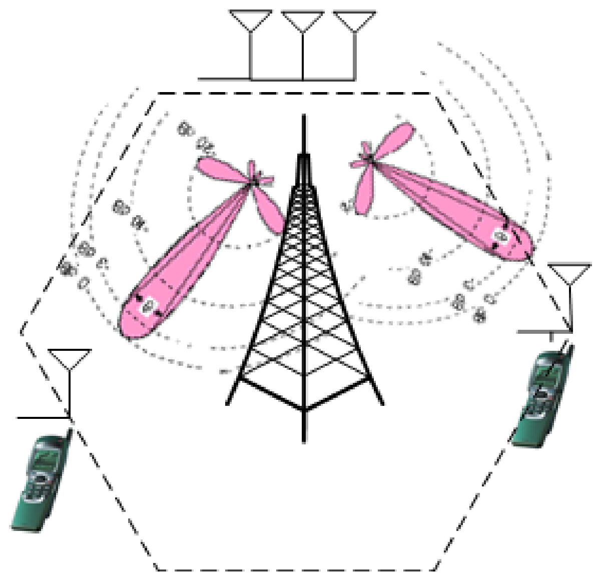
✓ At eNB side, the antenna spacing is about 10λ.
✓ However, at UE side, the antenna spacing is only about 0.5λ because of *richly scattered environment*.

Gains from Exploiting the Multiple Spatial Channels (2/3)

- **Array Gain** (Antenna Spacing Is Not Greater than 0.5λ)
  - To concentrate transmit power in one or more directions to allow multiple users to be served simultaneously. (Anti-blockage → Multiple beams for a UE)
  - Space division multiple access, multi-user MIMO

– Beamforming (achieved by large-scale antenna system)

– AoD (angle of departure), i.e., downlink beam information of UE is needed at gNB.

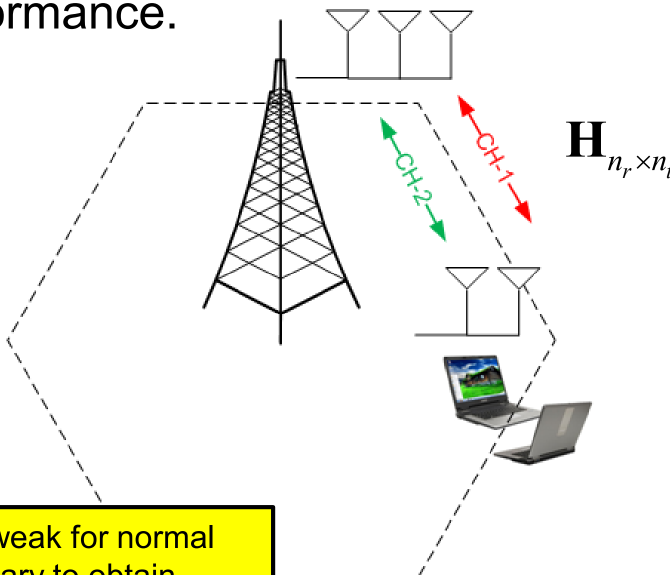


The surface area of the sphere=$4\pi R$^2, the total transmission power of the ideal omnidirectional antenna is spread over the area of $4\pi R$^2.

Because UE is moving, beam management is necessary.

Gains from Exploiting the Multiple Spatial Channels (3/3)

- **Spatial Multiplexing Gain** (by Enlarging the Antenna Spacing to Obtain Independent Spatial Channel, i.e., to Increase the Degree of Freedom)
  - To transmit different signals over different spatial channels to increase data rates (improve spectral efficiency).
  - Full CSIT **H** is needed at gNB for the best performance.
    - If **H** is known, the optimal precoding at transmitter side is via the SVD.
    - It is formidable for FDD systems due to a huge signaling overhead.
    - One common practice is to use codebook-based precoding.



- In the mm-wave frequency band, the strength of the received signal will be too weak for normal communication. Therefore, under a huge antenna array, beamforming is necessary to obtain antenna gain.
- Q: In 5G, how to have beamforming (array gain) and spatial multiplexing (spatial multiplexing gain) at the same time?

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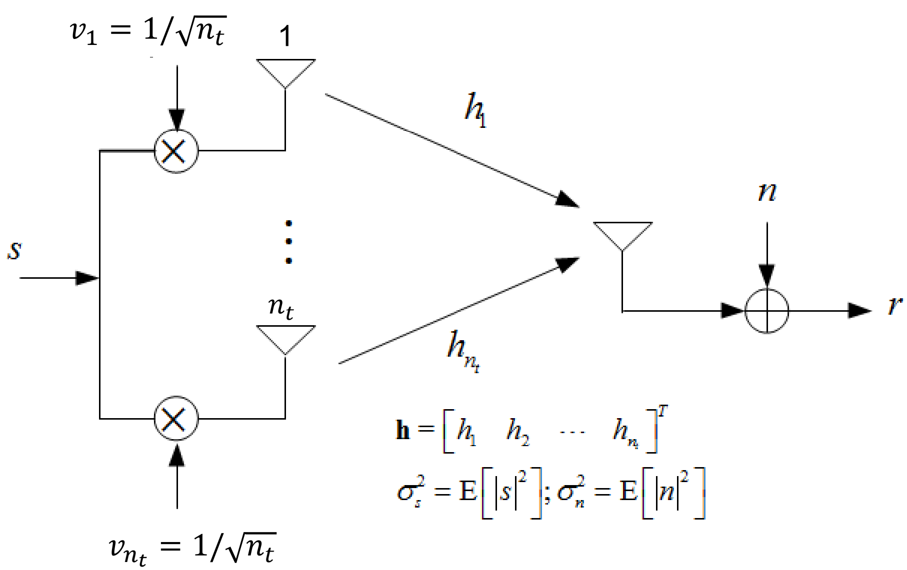
Precoding for Downlink MISO (2/2)

- Case B: **h** unknown at transmitter
  - No diversity gain (power gain)
  - Number of antenna port: 1 (only one antenna to UE, UE 需要估的通道數目為1個)



- $v_i = \dfrac{1}{\sqrt{n_t}}$: Uniform power allocation
- $r = \dfrac{1}{\sqrt{n_t}} s \left(\sum_{i=1}^{n_t} h_i \right) + n \rightarrow$ The only one channel needed to be estimated is $\sum_{i=1}^{n_t} h_i$.
- For $\{h_i\}$ i.i.d. variable with zeo-mean and $E\left[|h_i|^2\right] = 1 \rightarrow \text{SNR} = 1 \cdot \dfrac{\sigma_s^2}{\sigma_n^2}$
- Fading channel $\rightarrow$ AWGN channel

Precoding一般又被稱為transmit beamforming, 但這不是很正確的說法, 因為這裡的precoding並不在意要在特定的訊號方向上形成beams。
Precoding is generally called transmit beamforming, but this is not a very correct statement, because precoding here does not care about forming beams in a specific signal direction.

Precoding for Downlink MISO (1/2)

- Case A: **h** known at transmitter
  - Diversity gain (power gain of n_t)
  - Number of ports (LTE terminology) is n_t
  - (i.e., n_t antennas to UE, UE 需要估的通道數目為n_t個)

1. The phase of the channel is compensated at the transmitting end in advance.
2. Channel gain reflects the power distribution.
3. The total power remains unchanged.

[redacted]

- $\mathbf{v} = \begin{bmatrix} v_1 & \cdots \end{bmatrix}$: precoding vector

$$v_i = \frac{h_i^*}{\sqrt{\mathbf{h}^H \mathbf{h}}} \rightarrow E\left[\|\mathbf{v}\|^2\right] = E\left[\mathbf{v}^H \mathbf{v}\right] = 1$$

- $r = \dfrac{\mathbf{h}^H \mathbf{h}}{\sqrt{\mathbf{h}^H \mathbf{h}}} s + n \rightarrow \text{SNR} = \dfrac{\sigma_s^2 E\left[\mathbf{h}^H \mathbf{h}\right]}{\sigma_n^2}$
- For $\{h_i\}$ i.i.d. variable with zero-mean and

$$E\left[|h_i|^2\right] = 1 \rightarrow \text{SNR} = n_t \cdot \frac{\sigma_s^2}{\sigma_n^2}$$

這裡的一根天線可以是一個virtual antenna：由*M*個antenna elements (per polarization) 接到一個TXRU所形成的。 one port → one virtual antenna (see p. 20)

Precoding for Downlink SU-MIMO (1/4)

- **H** is known at transmitter.
- Channel capacity is achieved through SVD precoding.

$$\mathbf{H}_{n_r \times n_t} = \mathbf{U}\Lambda\mathbf{V}^*, \quad \mathbf{U}_{n_r \times n_r} = \left[\mathbf{u}_1 \quad \mathbf{u}_2 \quad \cdots \qquad\qquad \cdots\right.$$

$$\mathbf{HH}^* = \mathbf{U}\Lambda^2\mathbf{U}^* \rightarrow \mathbf{HH}^*\mathbf{U} = \mathbf{U}\Lambda^2; \quad \mathbf{H}^*\mathbf{H} = \mathbf{V}\Lambda^2\mathbf{V}^* \rightarrow \mathbf{H}^*\mathbf{HV} = \mathbf{V}\Lambda^2$$

– The superscrip $*$ denotes the Hermitian operation.

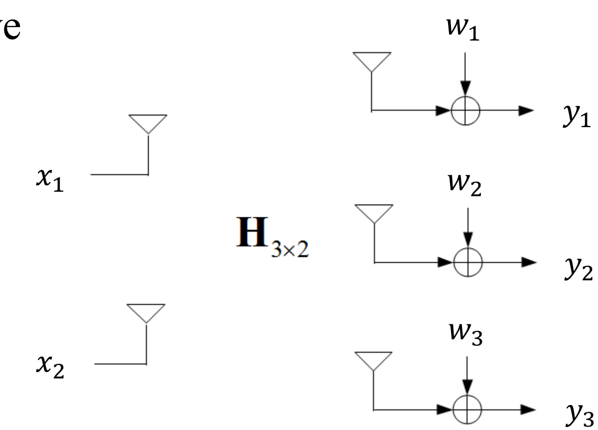
– $\mathbf{U}, \mathbf{V}$: unitary matices;

– $\Lambda_{n_r \times n_t}$: a rectangular matrix whose diagonal elements are nonnegative real numbers and whose off-diagonal elements are zero.

– The diagonal elements, $\lambda_1 \geq \lambda_2 \geq \cdots$ are the ordered singular values of the matrix **H**.

– $n_{\min} = \min\{n_t, n_r\}$: full rank

– λ_i^2: the eigenvalues of the matrix $\mathbf{HH}^*$ and $\mathbf{H}^*\mathbf{H}$



Precoding for Downlink SU-MIMO (2/4)

- MIMO system with SVD precoding

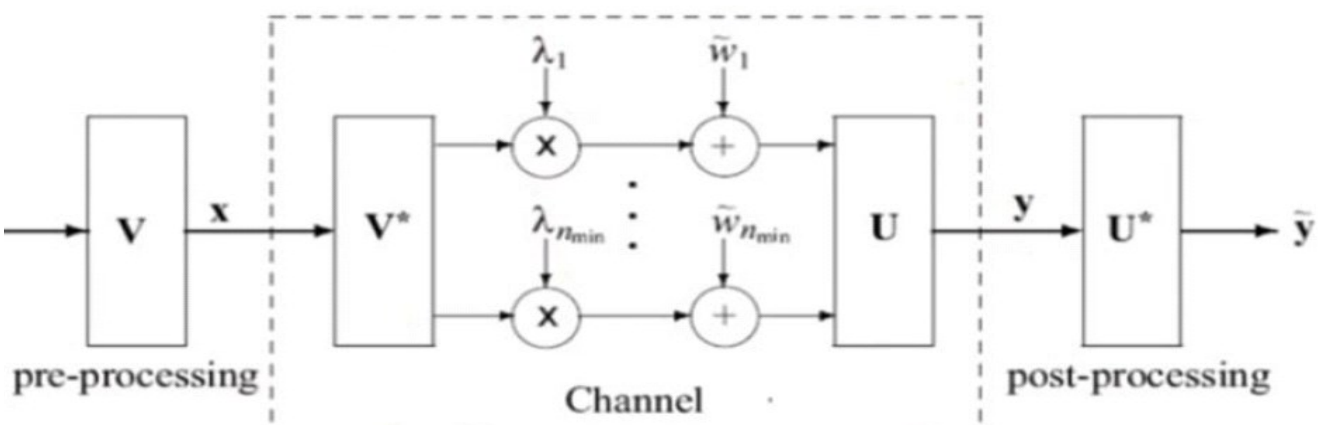
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} = \mathbf{U}\Lambda\mathbf{V}^{*}\mathbf{x} + \mathbf{w}$$

$$\rightarrow \tilde{\ } \qquad \mathbf{U}^{*}\left(\mathbf{U}\Lambda\mathbf{V}^{*}\mathbf{x} + \mathbf{w}\right)$$

$$= \Lambda\mathbf{V}^{*}\left(\underbrace{\mathbf{V}\tilde{\ }}_{\mathbf{x}\equiv\mathbf{V}\tilde{\ }} \quad \underbrace{\qquad}_{\tilde{\ }} \quad \Lambda\tilde{\ } \quad \tilde{\ }\right.$$

It is formidable for FDD systems due to a huge signaling overhead.
→ One common practice is to use codebook-based precoding.
(Feasibility vs. Performance degradation)

$$\begin{bmatrix} \tilde{\ } \\ \tilde{\ } \\ \vdots \\ \end{bmatrix} \qquad \tilde{\ } \qquad \tilde{\ } \qquad \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

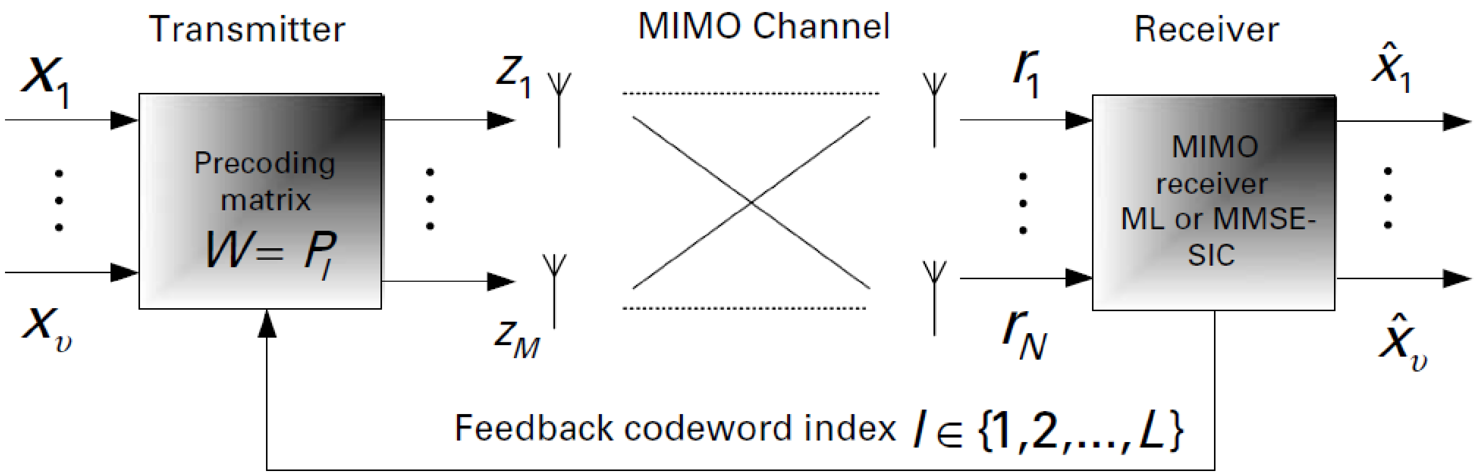


Precoding for Downlink SU-MIMO (3/4)

- Codebook-based precoding in LTE
  - Based on measurements on the cell-specific reference signals (**CRS**s), the terminal selects a suitable transmission rank and corresponding precoder matrix.
  - Information about the selected rank and precoder matrix is reported to the network in the form of a Rank Indication (**RI**) and a Precoder-Matrix Indication (**PMI**), respectively.
    - RI is the number of the transmission layers.
  - To limit the signaling on both uplink and downlink only a limited set of precoder matrices, the codebook, is defined for each transmission rank for a given number of antenna ports.

Precoding for Downlink SU-MIMO (4/4)

- Code book: a set of precoding matrices known at both the eNB and the UE denoted by $\{\mathbf{W}_i\}, i = 1, \cdots, 2^L$.
- The receiver observes a channel realization, selects the best precoding matrix, and feeds back the PMI to the transmitter.



[1]R1-1709232, "WF on Type I and II CSI Codebooks," 3GPP TSG-RAN WG1-89, Hangzhou, China, May 2017

Summary

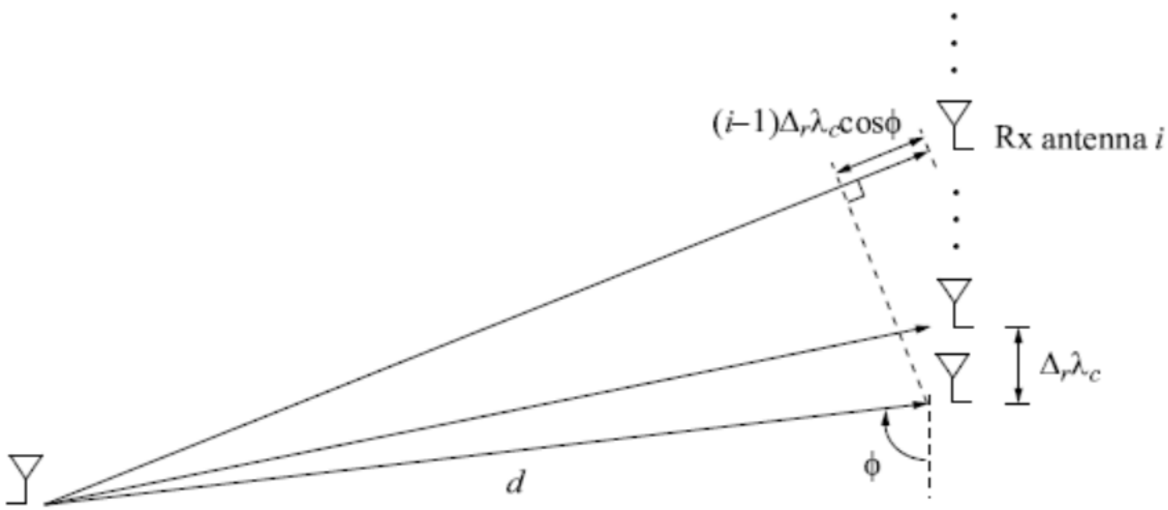
- Precoding is designed to maximize receive performance with channel being taken into consideration.
- No cares are about antenna configuration (e.g., including antenna pattern, antenna element spacing, etc).
- It is not necessary to form beams in space; the precoders $\mathbf{v} = \mathbf{h}^*$ in example 1 (downlink MISO) and $\mathbf{V}$ in example 2 (downlink MIMO) are not necessary forming beams in space.
- Precoding can be implemented in digital domain or analog domain. Digital-domain implementation might be easier though.

Antenna configuration → The antenna structure of different companies will be different (and the antenna pattern is different), so even if the coefficient of the beamformer is the same, the beam formed will be different.
Precoding doesn't care antenna configuration (i.e., its design has nothing to do with antenna configuration), so precoding at the transmitter cannot be regarded as transmitter beamforming.

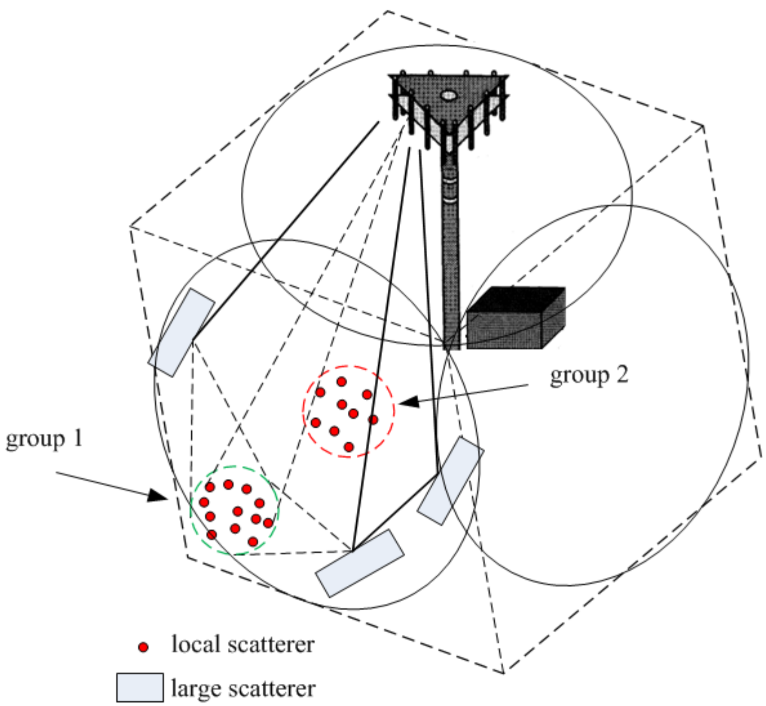
Outline

Beamforming

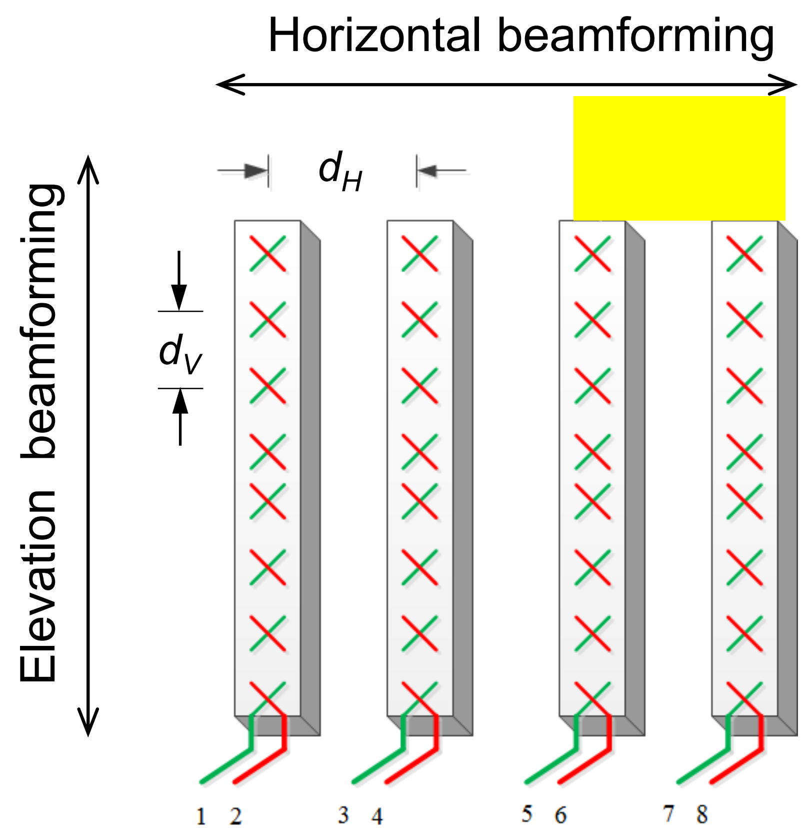
- Take linear array with the line-of-sight channel as example.
- SIMO
  - There is only free space without any reflectors or scatters, and only a direct signal path between each antenna pair.



Example Linear Arrays



Channel sparsity illustration



d : Array Spacing

- *M* antennas per polarization in each column are connected to a single **TXRU** (transceiver unit) to form a virtual antenna.
- Elevation beamforming → Vertical sectorization

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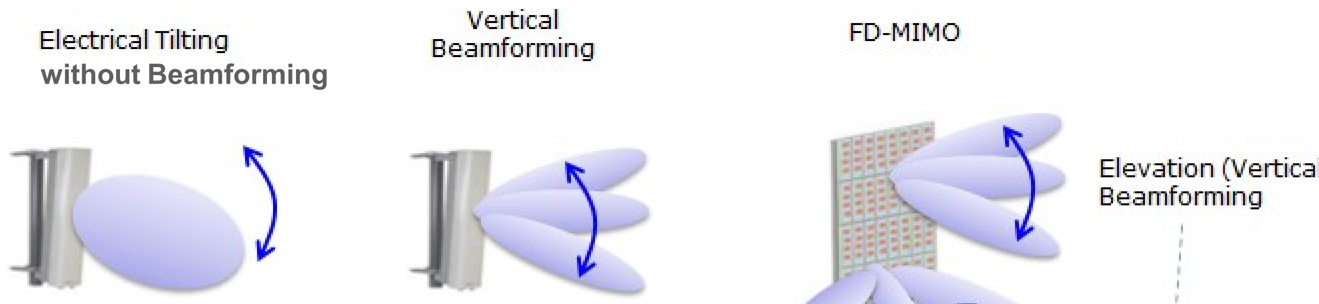
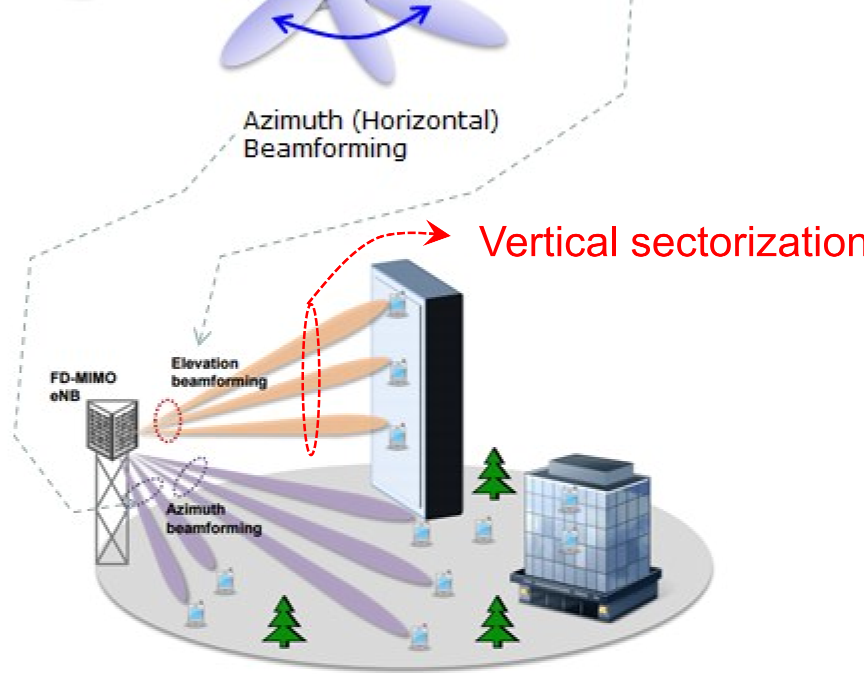
FD-MIMO





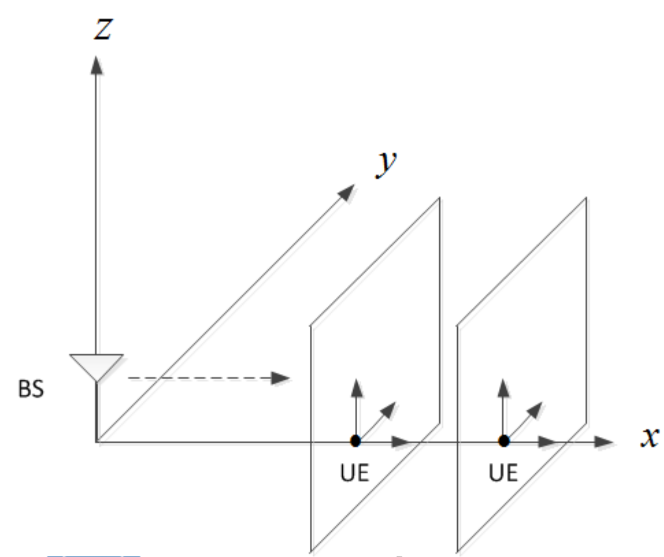
- FD-MIMO stands for full dimension multiple-input multiple-output.
- FD-MIMO means that the antenna system that can form a beam in both horizontal and vertical direction so that it can cover anywhere in 3D spaces.

[2]https://www.sharetechnote.com/html/5G/5G_MassiveMIMO_FD_MIMO.html

Plane-wave Assumption (1/2)

- The antenna separation is $\Delta_b \lambda_c$, where λ_c is the carrier wavelength and Δ_b is the normalized receive antenna separation, normalized to the unit of the carrier wavelength.
- Assume that the dimension of the antenna array is much smaller than the distance between the transmitter and the receiver.
- Then we have plane-wave propagation.

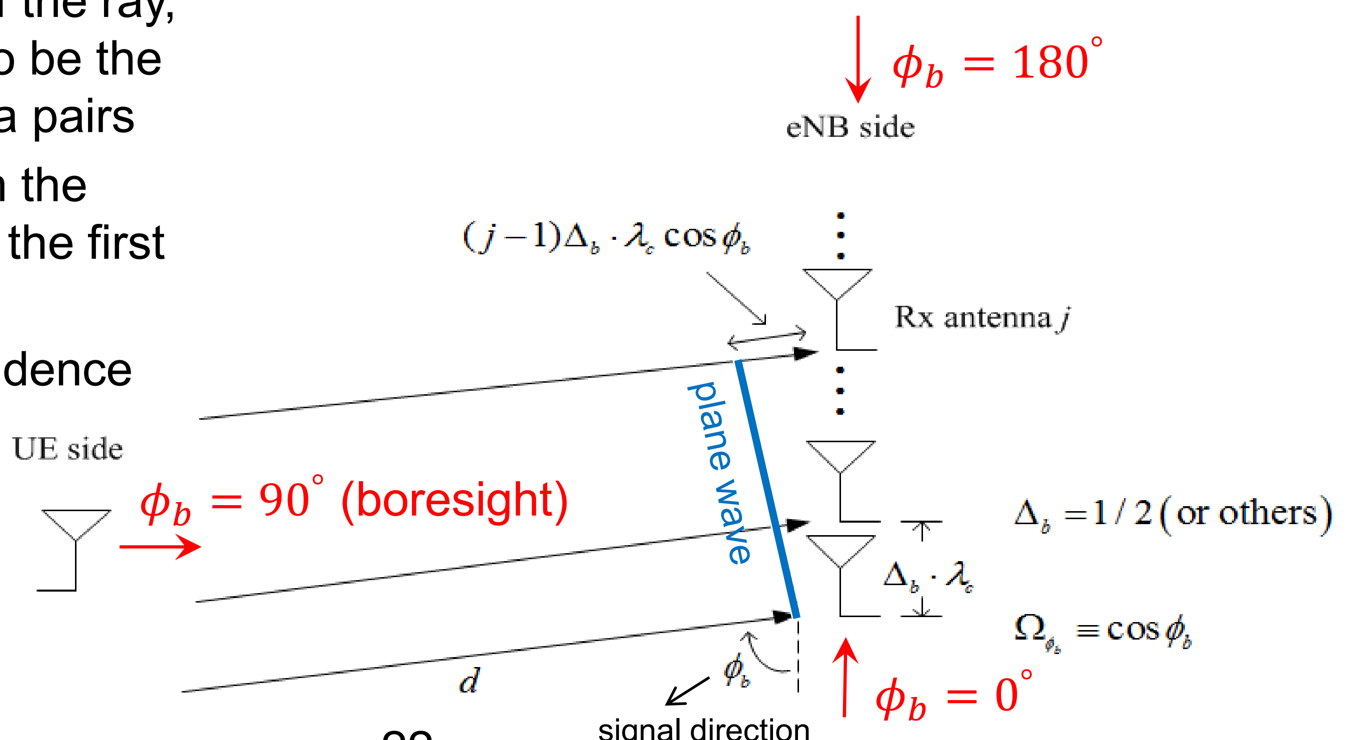
傳遞中之球體的電磁波接觸到接收天線(因為TX&RX相距很遠，電磁波球體會是極大的)，因為接收天線的dimension夠小，所以天線陣列所接觸到的電磁波會是一平面。故可將電磁波視為平面波。但當接收天線的dimension變得夠大 (如整個電視牆大小的panel antenna)，其所接觸到電磁波處就不再是一個平面，而是curve，故看到的電磁波也不再是plane-wave 。



Plane-wave Assumption (2/2)

- The continuous-time impulse response $h_i(\tau)$ between the transmit antenna and the ith receive antenna is given by $h_i(\tau) = a \cdot \delta(\tau - d_i/c)$.

  - d_i: propagation distance
  - c: the speed of light
  - a: the attenuation of the ray, which we assume to be the same for all antenna pairs
  - d: the distance from the transmit antenna to the first receive antenna
  - ϕ_b: the angle of incidence



Narrowband Assumption (1/2)

- The assumption that $\left(\Delta_b \cdot \lambda_c \cdot (n_r - 1)\right)/c \cdot \ll 1/W$ means that the received signals at different antennas are only different in phase for a ray, where W denotes the signal bandwidth. (For narrowband signals, the delay is approximated by a phase shift.)
- The baseband channel gain at receive antenna i:

$$h_i = ae^{-j2\pi f_c d_i/c} = ae^{-j2\pi d_i/\lambda_c}, i = 1,2, \dots, n_r$$

  - $d_i = d + (i-1)\Delta_b \lambda_c \cos\phi_b$
  - n_r is the number of receive antennas.
  - $\Omega_{\phi_b} \equiv \cos\phi_b$ is the directional cosine w.r.t the receive antenna array.

Narrowband Assumption (2/2)

- Given the transmitted signal $x(t)$, the received signal at array is $\mathbf{y}(t) = \mathbf{h}(t) \circledast x(t) + \mathbf{w}(t)$, where channel model is

$$\mathbf{h}(\tau) \equiv \begin{bmatrix} h_1(\tau) \\ h_2(\tau) \\ \vdots \\ h_{n_r}(\tau) \end{bmatrix} = a e^{-\frac{j2\pi d}{\lambda_c}} g_b(\Omega_{\phi_b}) \begin{bmatrix} 1 \\ e^{-j2\pi\Delta_b\Omega_{\phi_b}} \\ \vdots \\ e^{-j2\pi(n_r-1)\Delta_b\Omega_{\phi_b}} \end{bmatrix} \delta\left(\tau - \frac{d}{c}\right)$$
$$= a_b g_b(\Omega_{\phi_b}) \mathbf{e}_r(\Omega_{\phi_b}) \delta(\tau - d/c)$$

  - $a_b = \sqrt{n_r} a e^{-j2\pi d/\lambda_c}$ is the amplitude gain of the ray.
  - $g_b(\Omega_{\phi_b})$ is the antenna gain of antenna element (antenna radiation pattern)
  - $\mathbf{e}_r(\Omega_{\phi_b}) = \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 \\ e^{-j2\pi\Delta_b\Omega_{\phi_b}} \\ \vdots \\ e^{-j2\pi(n_r-1)\Delta_b\Omega_{\phi_b}} \end{bmatrix}$ is the unit spatial signature in the directional cosine Ω_{ϕ_b}.
  - The delay time of d/c associated with the ray is identical across the antenna elements, since the delay difference is approximated by a phase.

Receiver Beam-forming

- If the signal arrives from a single direction ϕ_0, then the optimal receiver projects the received signal onto the vector $\mathbf{e}_r(\Omega_{\phi_0})$.
  - Because of $\mathbf{e}_r(\Omega_{\phi_0})^H \mathbf{e}_r(\Omega_{\phi_0}) = 1$, $\mathbf{e}_r(\Omega_{\phi_0})^H \mathbf{y}(t) = a_b g_b(\Omega_{\phi_b}) x(t -$

antenna length

$$\left|\mathbf{e}_b^H(\Omega_{b_0})\mathbf{e}_b(\Omega_b)\right| = \left|\mathrm{P}(\Omega_b - \Omega_{b_0})\right|$$

$$\Omega \equiv \Omega_b - \Omega_{b_0} \rightarrow \mathrm{P}(\Omega) = \frac{1}{n_r}\exp\left[-j\pi\Delta_b\Omega(n_r - 1)\right]\frac{\sin(\pi L_r \Omega)}{\sin\left(\frac{\pi L_r \Omega}{n_r}\right)}, \quad L_r = n_r \Delta_b$$

Beam Pattern

- The beam pattern associated with the vector $\mathbf{e}_r(\Omega_{\phi_0})$ is the polar plot

$$\left(\phi, \left|\mathrm{P}(\Omega)\right|\right), \phi \in [0, 2\pi)$$

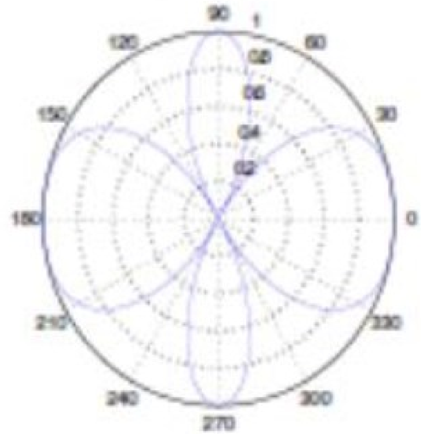
在傳統的MIMO，為了讓天線之間的通道是獨立的，$\Delta_b > 1/2$；但對beamforming而言，反而會產生多個main lobes，所以進行beamforming時，必需是$\Delta_b \leq 1/2$ 。

In traditional MIMO, in order to make the channels between antennas independent, $\Delta_b > 1/2$；
But for beamforming, multiple main lobes will be generated instead, so when beamforming is performed, it must be $\Delta_b \leq 1/2$

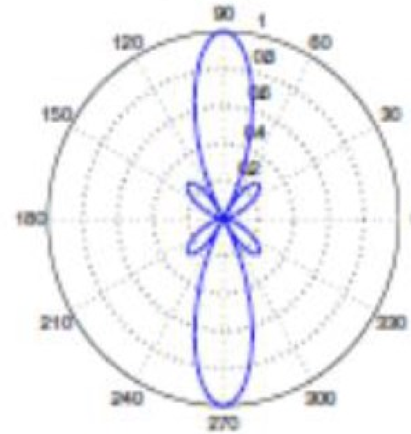
- For normalized antenna separation $\Delta_b \leq 1/2$, there is a single main lobe around 90°; otherwise, there is an additional pair of main lobe.
- The array length $L_r = n_r \Delta_b$ determines the beam width. The main lobe has a directional cosine width of $2/L_r$.
- n_r determines the total number of orthogonal beams that can be formed. That is, the total number of orthogonal beams is n_r.

形成的beam的粗細與array length $L_r = n_r \Delta_b$有關，所以當Δ_b固定時，beam的粗細就與只與天線的數目有關。

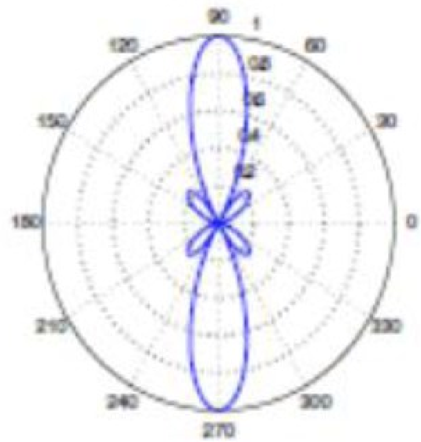
$L_r = 2,\ n_r = 2\ \ (\Delta_b = 1)$



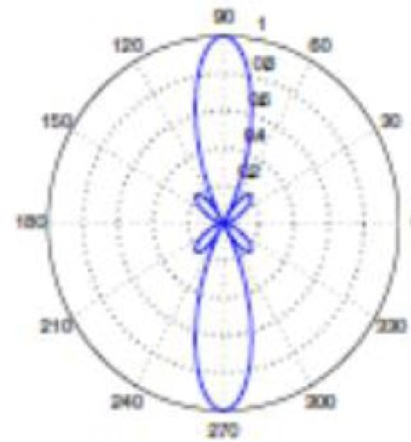
$L_r = 2,\ n_r = 4\ \ (\Delta_b = 1/2)$



$L_r = 2,\ n_r = 6\ \ (\Delta_b = 1/3)$

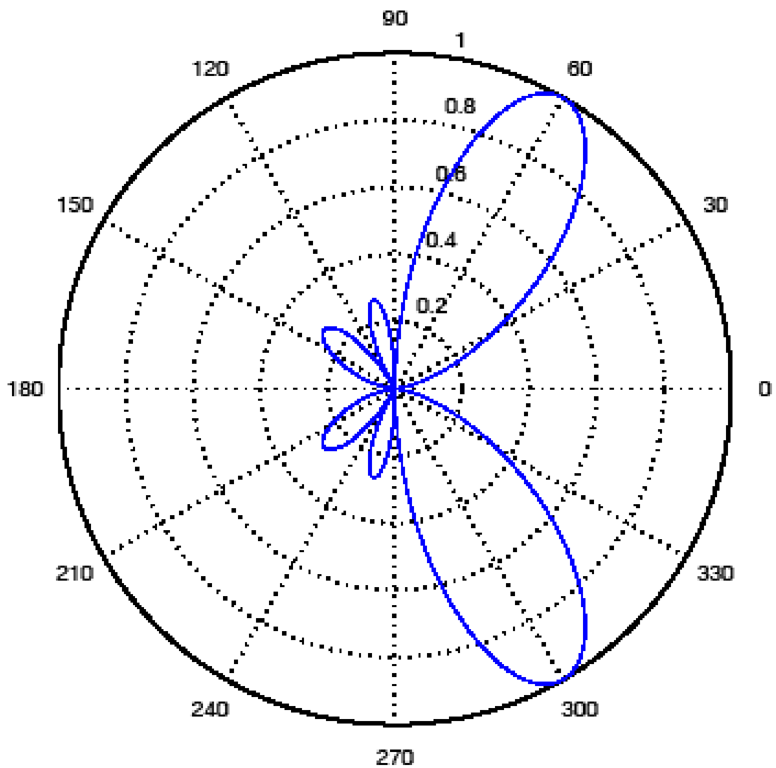


$L_r = 2,\ n_r = 32\ \ (\Delta_b = 1/16)$

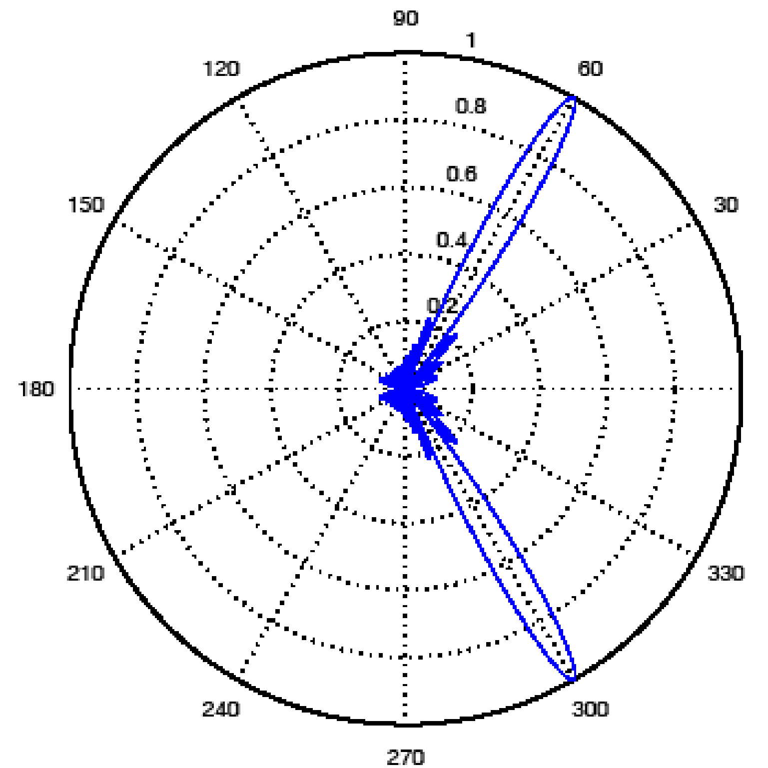


Receive beamforming patterns aimed at 90°, with antenna array length $L_r = 2$ and different numbers of receive antennas n_r.

$n_r = 4, L_r = 2$



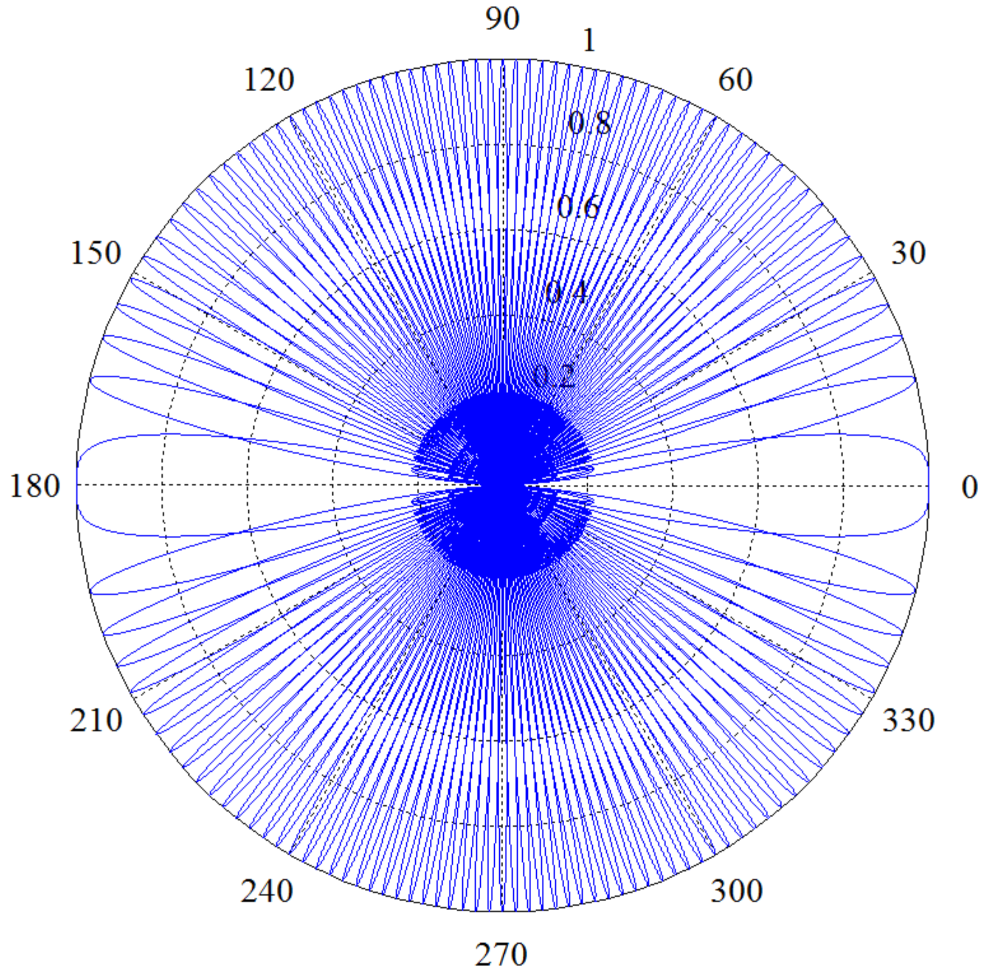
$n_r = 16, L_r = 8$



Signal direction $\phi = 60^{\mathrm{o}}$; normalized antenna separation $\Delta_r = 1/2$

- $\mathrm{P}_j(\Omega_b) \equiv \mathbf{e}_b^H\left(\frac{j-1}{L}\right)\mathbf{e}_r(\Omega_b), j=1,\cdots$
- n_r orthogonal beam pattern of $\mathbf{e}_r\left(\frac{j-1}{L}\right)$:

$\left(\phi_b, \left|\mathrm{P}_j(\Omega_b)\right|\right), \phi_b \in [0, 2\pi)$ for $j=1,\cdots$



$n_r = 64, \Delta_b = \frac{1}{2}, L_r = 32;\ g_b(\Omega_b) = 1\ \forall \Omega_b$

DFT Beambook (1/4)

- A beambook is a set of beams used to cover the intended spatial coverage.
- Orthonormal basis for the received beam space

$$S \equiv \left\{ \mathbf{e}_r(0), \mathbf{e}_r\left(\frac{1}{L_r}\right), \cdots \left(\frac{-1}{L_r} \right) \right\}$$

This basis provides the representation of the received signals in the angular domain.

$$\text{where} \quad \mathbf{e}_r\left(\frac{j-1}{L_r}\right) = \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 \\ e^{-j2\pi\Delta_b(j-1)/L_r} \\ \vdots \\ e^{-j2\pi(n_r-1)\Delta_b(j-1)/L_r} \end{bmatrix} \quad 1 \begin{bmatrix} 1 \\ e^{-j2\pi(j-1)/n_r} \\ \vdots \\ e^{-j2\pi(n_r-1)(j-1)/n_r} \end{bmatrix}$$

$$j = 1, \cdots \quad ; \quad \cos\phi_j = \frac{j-1}{L_r}; \quad L_r = n_r\Delta_b$$

The base in S is irrelevant the antenna spacing Δ_b, antenna length L_r,…and other antenna parameters, so the same beambook applied to different antenna configurations will form different beams.

DFT Beambook (2/4)

- Prove: n_r orthogonal beams

$$\mathbf{e}_r\left(\frac{l-1}{L_r}\right)=\frac{1}{\sqrt{n_r}}\begin{bmatrix} 1 \\ e^{\frac{-j2\pi(l-1)}{n_r}} \\ \vdots \\ e^{\frac{-j2\pi(n_r-1)(l-1)}{n_r}} \end{bmatrix}, \quad l=1,2,\ldots,n_r$$

$$\rightarrow \mathbf{e}_r^H\left(\frac{j-1}{L_r}\right)\mathbf{e}_r\left(\frac{l-1}{L_r}\right)=\frac{1}{n_r}\sum_{m=0}^{n_r-1} e^{\frac{j2\pi(j-l)m}{n_r}}=\begin{cases} 1, & j=l \\ 0, & j\neq l \end{cases}$$

DFT Beambook (3/4)

- For normalized antenna spacing $\Delta_b \leq 1$, it results in $n_r \geq L_r$ because of $L_r = n_r \Delta_b$.
- Thus, the directional cosine $\cos\phi = {}^{n_r - k}/_{L_r} > 1$ for $k = 1,2,\ldots,n_r - L_r - 1$ is contradictive.
- In this case, actually the directional cosine $\cos\phi = -k/L_r < 0$, where $\phi \in [90^\circ, 180^\circ]$.

$$\mathbf{e}_r\left(\frac{n_r - k}{L_r}\right) = \frac{1}{\sqrt{n_r}}\begin{bmatrix} 1 \\ e^{\frac{-j2\pi\Delta_b(n_r-k)}{L_r}} \\ \vdots \\ e^{\frac{-j2\pi(n_r-1)\Delta_b(n_r-k)}{L_r}} \end{bmatrix} \quad 1 \begin{bmatrix} 1 \\ e^{\frac{-j2\pi\Delta_b(-k)}{L_r}} \\ \vdots \\ e^{\frac{-j2\pi(n_r-1)\Delta_b(-k)}{L_r}} \end{bmatrix} \; \left(-k\right.$$

DFT Beambook (4/4)

- Angular Domain Transformation as DFT

$$\mathbf{U}_r = \left[\mathbf{e}_r(0) \quad \mathbf{e}_r\left(\frac{1}{L_r}\right) \quad \cdots \quad \left(\frac{-1}{L_r}\right)\right]$$

$$= \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{\frac{-j2\pi}{n_r}} & \cdots & e^{\frac{-j2\pi\cdot(n_r-1)}{n_r}} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & e^{\frac{-j2\pi(n_r-1)}{n_r}} & \cdots & e^{\frac{-j2\pi\cdot(n_r-1)\cdot(n_r-1)}{n_r}} \end{bmatrix}$$

* $\mathbf{U}_r$: A DFT matrix; a unitary matrix; that is

* $\mathbf{U}_r^H \mathbf{U}_r = \mathbf{U}_r \mathbf{U}_r^H = \mathbf{I}$; $\mathbf{U}_r^H$: An IDFT matrix

Hamming Beambook

$$\mathbf{e}_r(\Omega) = \frac{1}{\sqrt{n_r}}\begin{bmatrix} 1 & e^{-j2\pi\Delta_r\Omega} & \cdots & e^{-j2\pi(n_r-1)\Delta_r\Omega} \end{bmatrix}^T$$

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_r \end{bmatrix}^T = \begin{bmatrix} -\frac{n_r-1}{2} & -\frac{n_r-2}{2} & \cdots & \frac{n_r-1}{2} \end{bmatrix}^T$$

$$\mathbf{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_{n_r} \end{bmatrix}^T, h_i = \left\{ 0.08 + 0.92\cos^2\left(\frac{\pi w_i}{n_r}\right) \right\}$$

$$S_{r,h} \equiv \left\{ \mathbf{h} \circ \mathbf{e}_r(0), \mathbf{h} \circ \mathbf{e}_r\left(\frac{1}{L_r}\right), \cdots, \mathbf{h} \circ \mathbf{e}_r\left(\frac{n_r-1}{L_r}\right) \right\}$$

$\circ$: Hadamard product

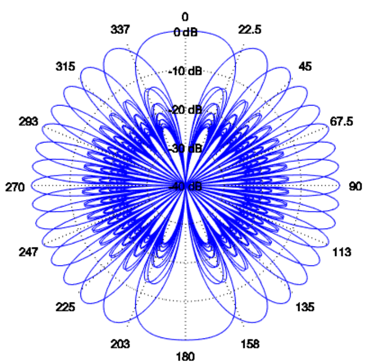
- Beams are not orthogonal anymore.

Comparison of Different Beambooks

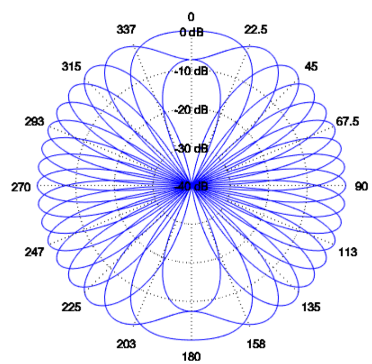
- Antenna (radiation) pattern of antenna element: **omni-directional**
- $n_r = 16;\ \Delta_r = 1/2$

n_r：天線的總數目。
BW (main lobe的width)越大，空間的鑑別力就差。
The larger the BW (width of the main lobe), the poorer the spatial discrimination.

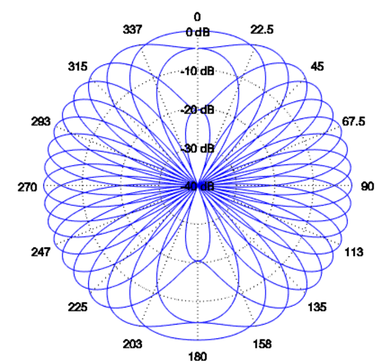
Beambook	BW（null-to-null）	Sidelobe（dB）
DFT	good $2\frac{2}{n_r}$	-13 bad
Hamming	$4\frac{2}{n_r}$	-39.5
Blackman Harris	bad $6\frac{2}{n_r}$	-56.6 good



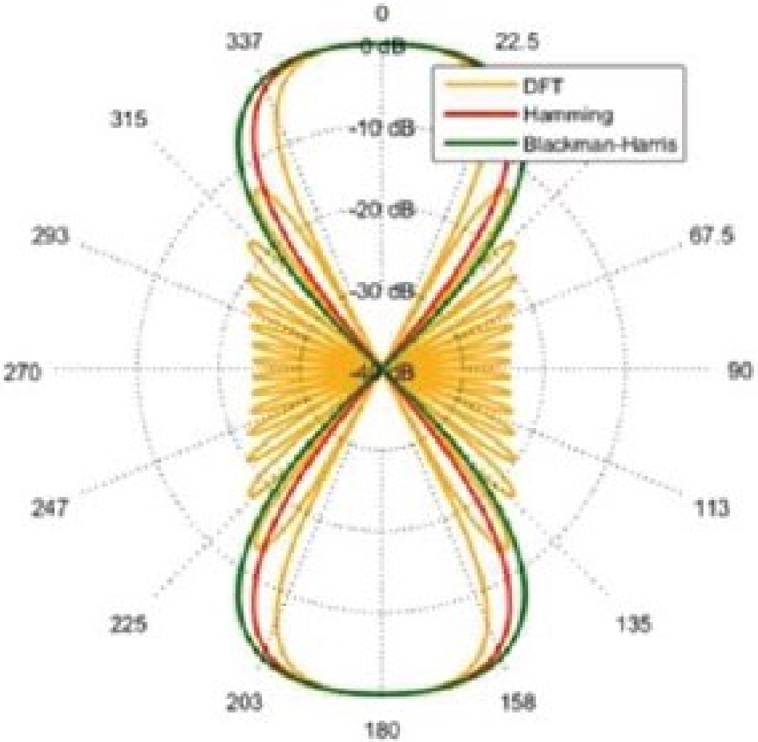
DFT

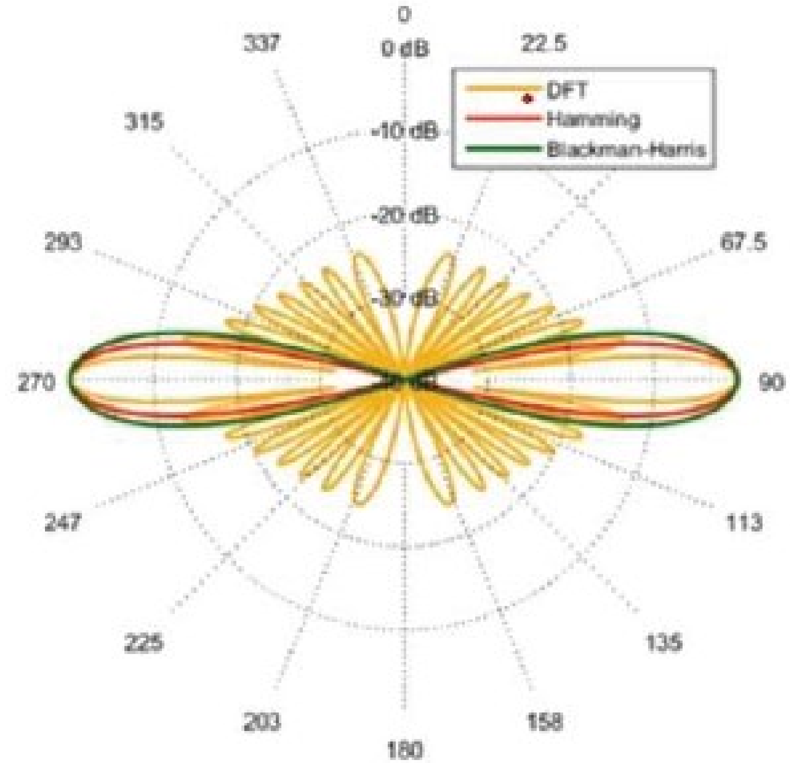


Hamming



Blackman-Harris



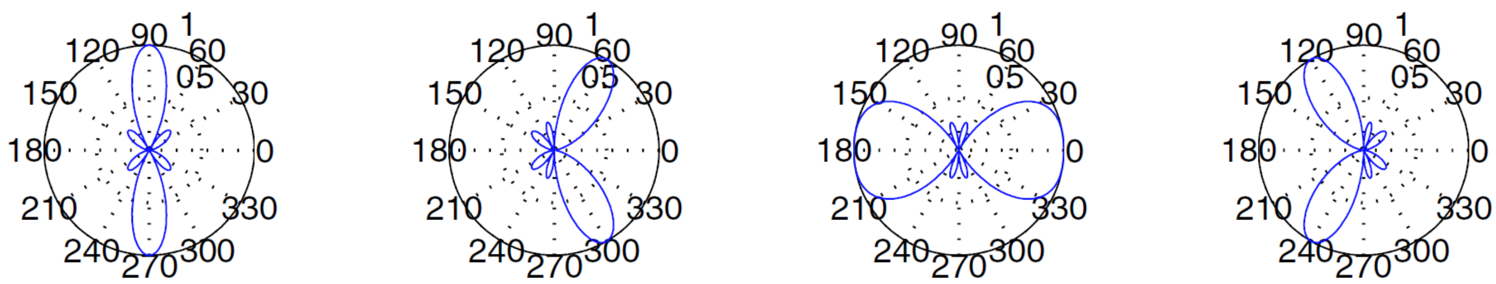


以0°的beam pattern為例，與DFT的beam pattern比較，Hamming與Blackman-Harris的beam pattern的side lobe幾乎沒有。
但他們二者之main lobe的寬度變大了。

Dependency on Antenna Spacing (1/4)

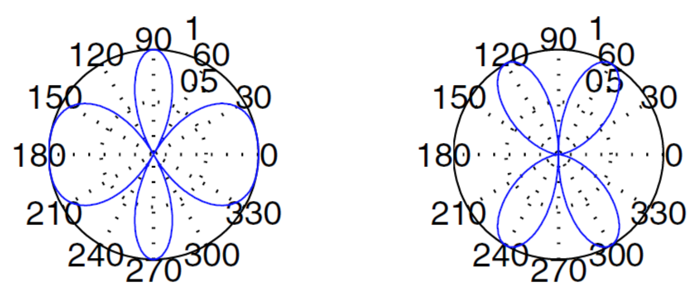
- When $\phi = 0^{\circ}$ $(k = 2)$，the main lobe will also appear in 180°. Because： 180° beam can also receive 0°beam，but it is a negative sign. 。
- When $k = 3$，$\frac{k}{L_r} > 1$→ beam會落在90°~180°。

$k = 0$ $k = 1$ $k = 2$ $k = 3$



(a) $L_r = 2$, $n_r = 4$, $\Delta_r = 1/2$ (critically-spaced)

Ref. Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory, Harry L. Van Trees, April 2004



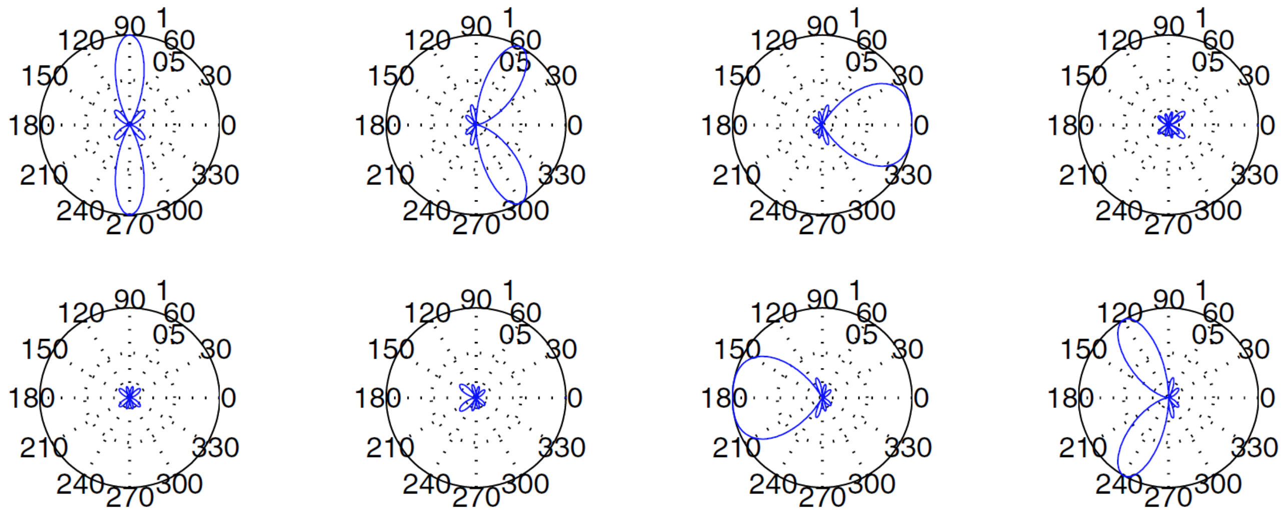
There will be grating lobes.

(b) $L_r = 2$, $n_r = 2$, $\Delta_r = 1$ (sparsely-spaced)

Cheng University

Dependency on Antenna Spacing (2/4)

$n_r = 8$，仍然會有8個beams，但有些beam的main lobe不見了。
there will still be 8 beams, but the main lobe of some beams is missing.



(c) $L_r = 2, n_r = 8$

[3]Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory, Harry L. Van Trees, April 2004

Figure 7.12: Receive beamforming patterns of the angular basis vectors. Independent of the antenna spacing, the beamforming patterns all have the same beam widths for the main lobe, but the number of main lobes depends on the spacing. (a) Critically-spaced case; (b) Sparsely-spaced case; (c) Densely-spaced case.

Dependency on Antenna Spacing (3/4)

- Antennas are critically spaced at half the wavelength ($\Delta_r = 1/2$). In this case, each of the basis vector $\mathbf{e}_r(k/L_r)$ has a single pair of main lobe around the angles $\pm$arccos(k/L_r).
- Antennas are sparsely spaced($\Delta_r > 1/2$). In this case, some of the basis vectors have more than one pair of main lobe.
- Antennas are densely spaced (Δ_r< 1/2). In this case, some of the basis vectors have no main lobes.

Dependency on Antenna Spacing (4/4)

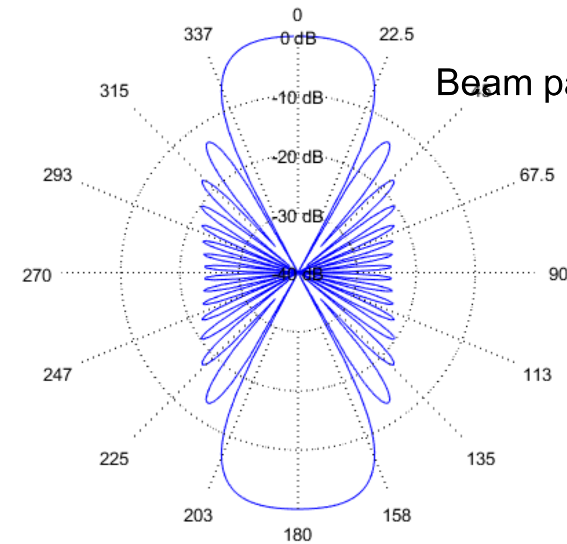
- A main lobe are at angle ϕ for which:

$$\cos\phi = k / L_r \bmod \left(1 / \Delta_r\right), k = 0, \cdots$$

  - In the critically-spaced case, $\frac{1}{\Delta_r} = 2$ and k/L_r is between 0 and 2; there is a unique solution $\cos\phi = \frac{k}{L_r}$.
  - In the sparsely-spaced case, $\frac{1}{\Delta_r}$ < 2 and for some values of k, there are multiple solutions: $\cos\phi = \frac{k}{L_r} + \frac{m}{L_r}$ for integers m.
  - In the densely-spaced case, $\frac{1}{\Delta_r}$ > 2, and for k satisfying $L_r < k < n_r - L_r$, there is no solution.
  - Only in the critically-spaced antennas there is a one-to-one correspondence between the angular windows and the angular basis vectors.

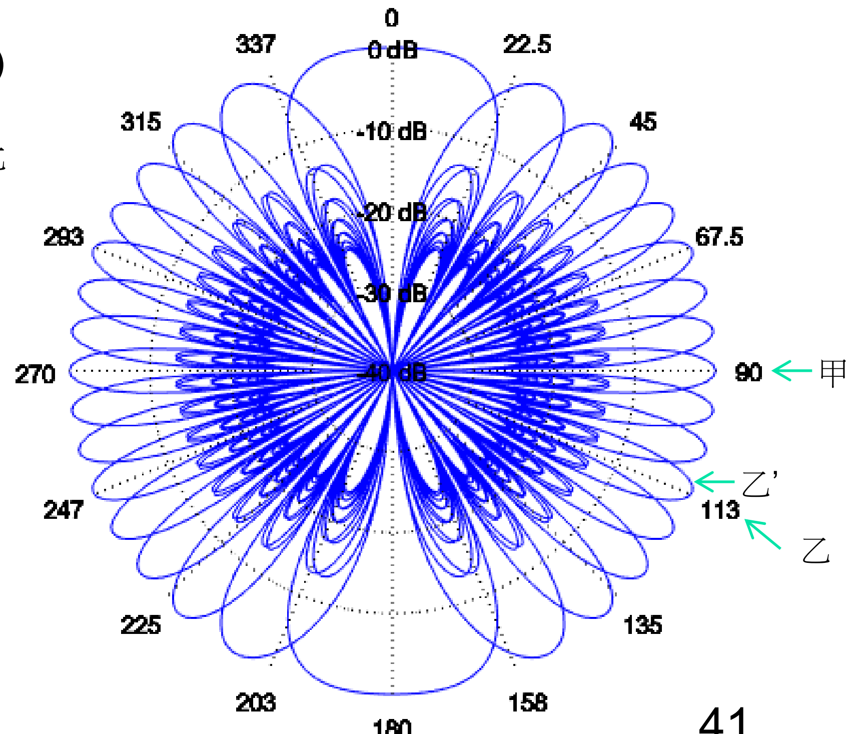
Beam Pattern without Antenna Pattern

- Example beambook (DFT)
  - Antenna (radiation) pattern of antenna element: **omni-directional**
  - $n_r = 16;\ \Delta_r = 1/2$



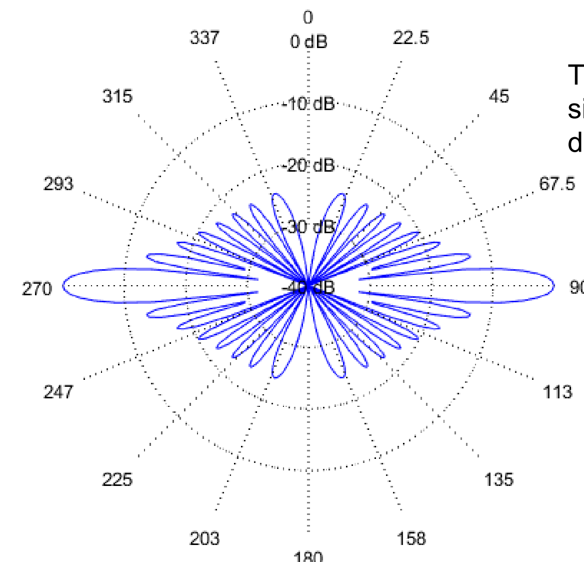
Beam pattern for 0°

甲beam(at90°)與乙beam(at113°)是彼此正交的。但乙若稍微偏移至乙’，就不再與甲完全正交。完全正交是發生在特定的訊號方向。



Beam patterns for all 16 beams

The 1st sidelobe: -13 dB



Beam pattern for 90°

Beam Pattern Incorporated with Antenna Radiation Pattern (1/2)

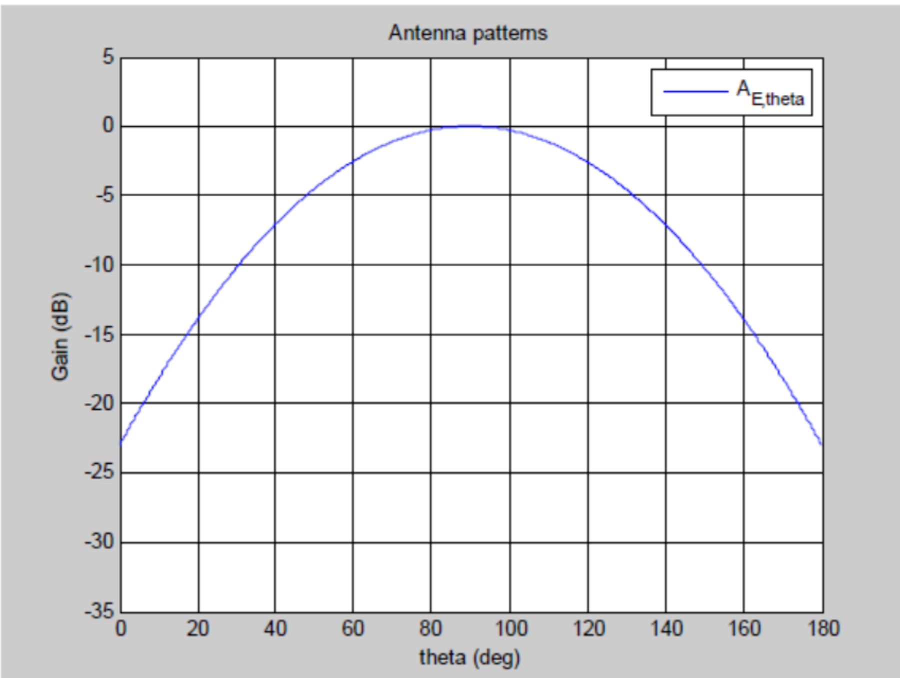
- 3-Sector BS antenna radiation pattern for above 6 GHz

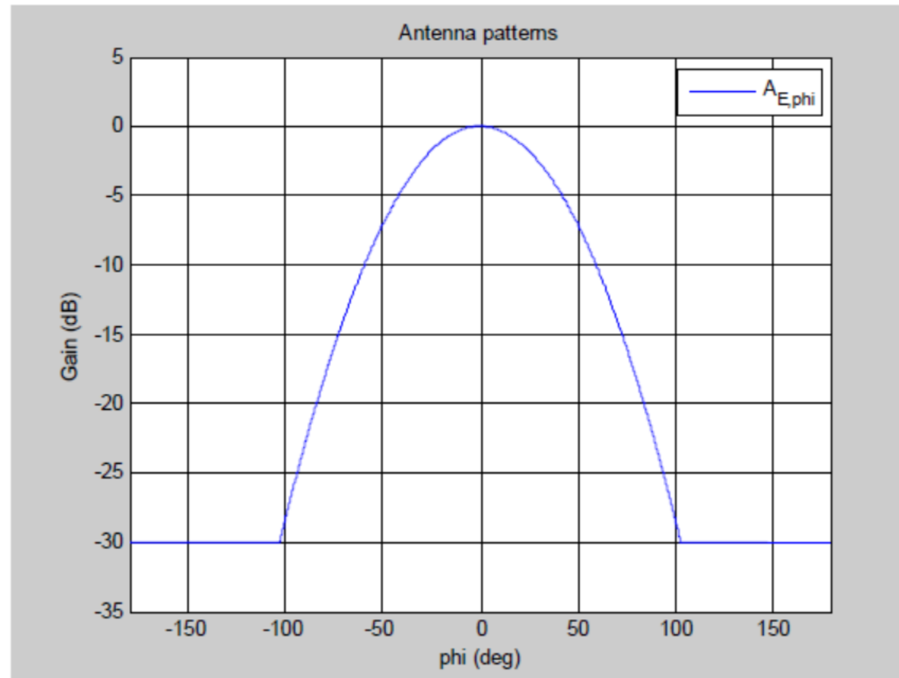
Parameter	Values
Antenna element vertical radiation pattern (dB)	$A_{E,V}(\theta'') = -min\left[12\left(\frac{\theta''-90^0}{\theta_{3dB}}\right)^2, SLA_V\right], \theta_{3dB} = 65^0, SLA_V = 30$
Antenna element horizontal radiation pattern (dB)	$A_{E,H}(\varphi'') = -min\left[12\left(\frac{\varphi''}{\varphi_{3dB}}\right)^2, A_m\right], \varphi_{3dB} = 65^0, A_m = 30$
Combining method for 3D antenna element pattern (dB)	$A''(\theta'',\varphi'') = -min\{-[A_{E,V}(\theta'') + A_{E,H}(\varphi'')], A_m\}$
Maximum directional gain of an antenna element $G_{E,max}$	8 dBi

[4] 3GPP TR 36.873 V12.7.0, 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Study on 3D channel model for LTE (Release 12)

  - Boresight: 90° and 0° for vertical and horizontal radiation pattern, respectively.

Beam Pattern Incorporated with Antenna Radiation Pattern (2/2)

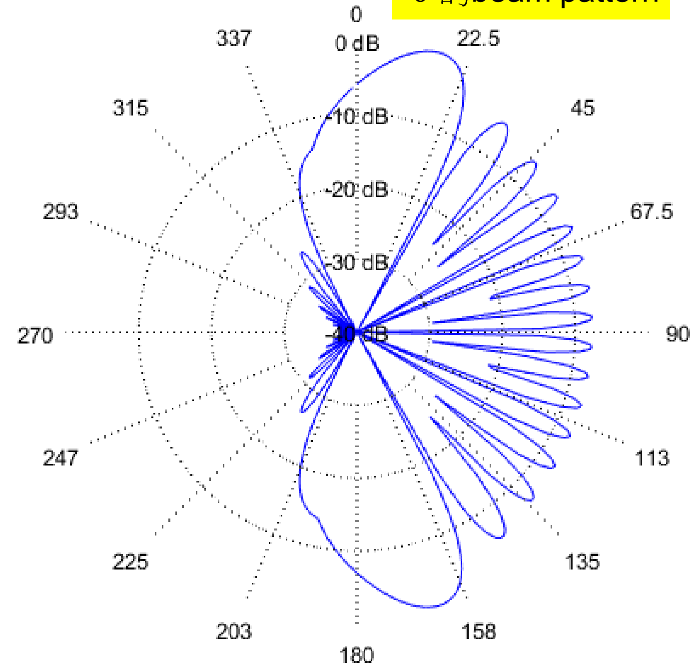




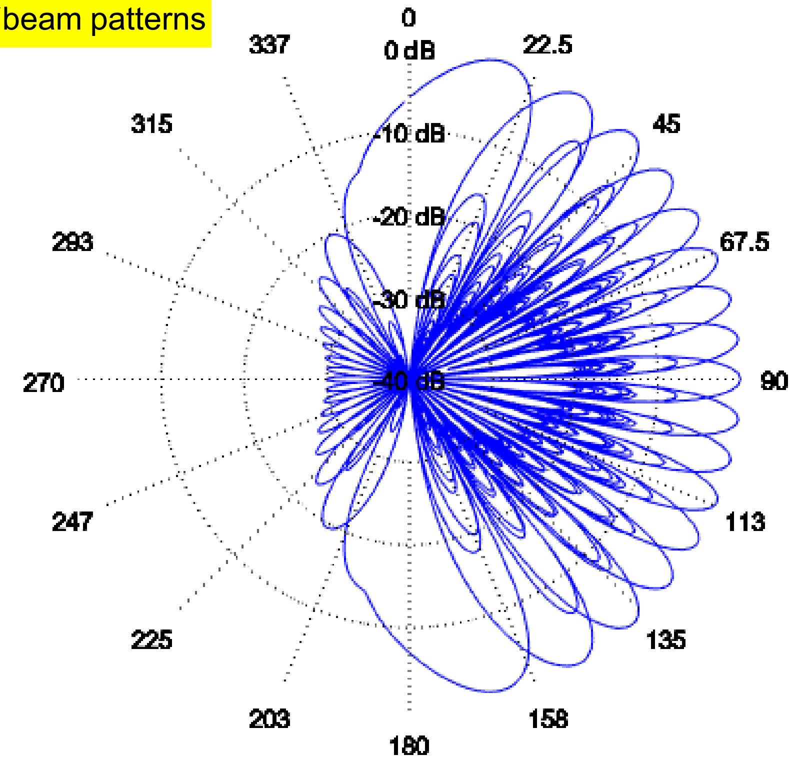
[5]3GPP TR 37.840 V12.1.0 (2013-12) 3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Study of Radio Frequency (RF) and Electromagnetic Compatibility (EMC) requirements for Active Antenna Array System (AAS) base station (Release 12)

- Example beambook (**DFT**)
  - Incorporated with 3GPP antenna element vertical radiation pattern
  - $n_r = 16; \Delta_r = 1/2$

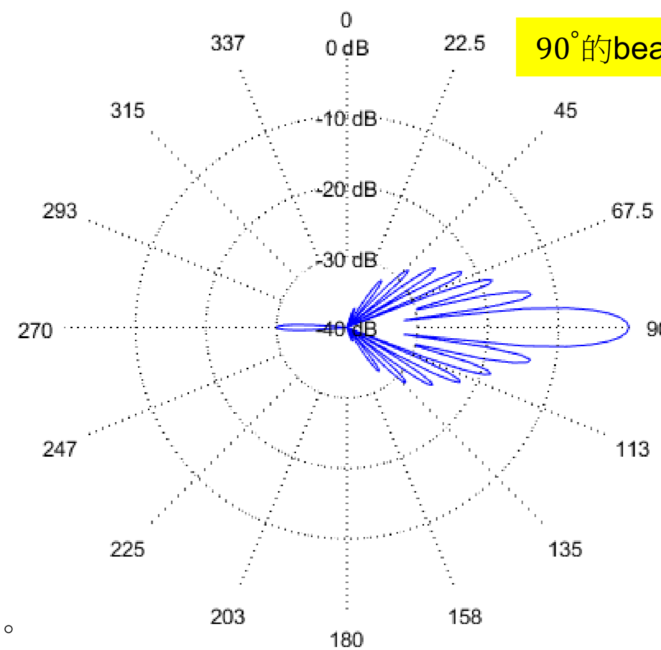
0°的beam pattern



所有16個beam patterns



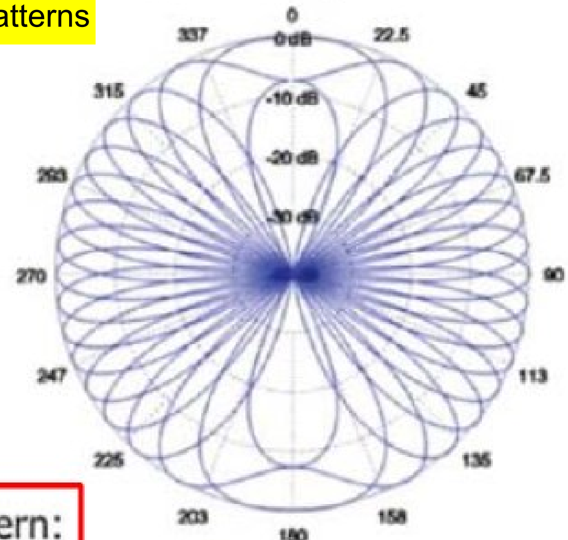
90°的beam pattern

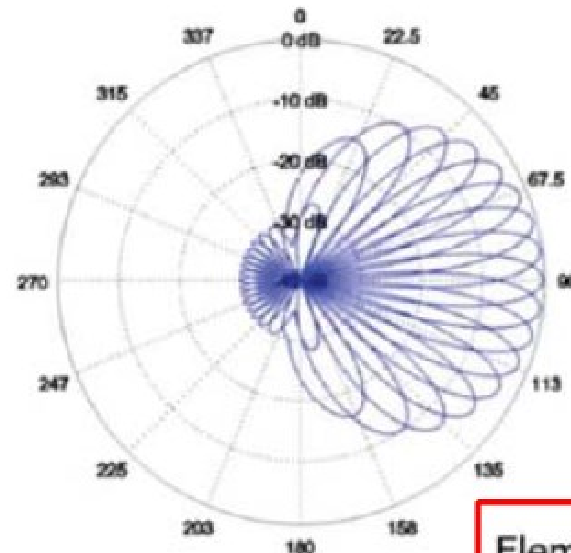


因為antenna pattern的coverage範圍是在0°~180°，beam pattern出現在0°~180°，另一面180°~360°就幾乎不存在。

- Example beambook (**Hamming**)
  - $n_r = 16; \Delta_r = 1/2$

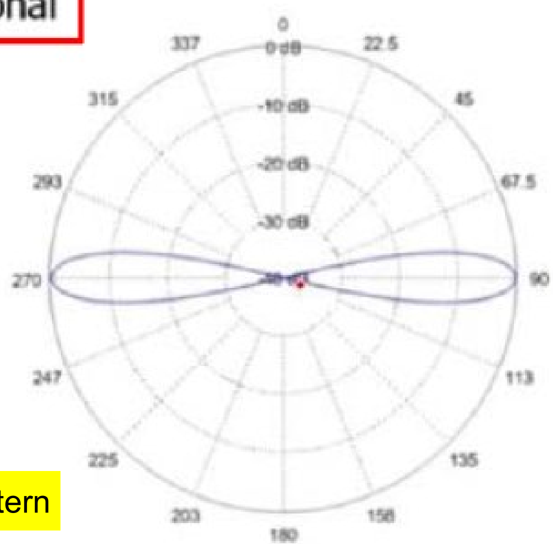
所有16個beam patterns

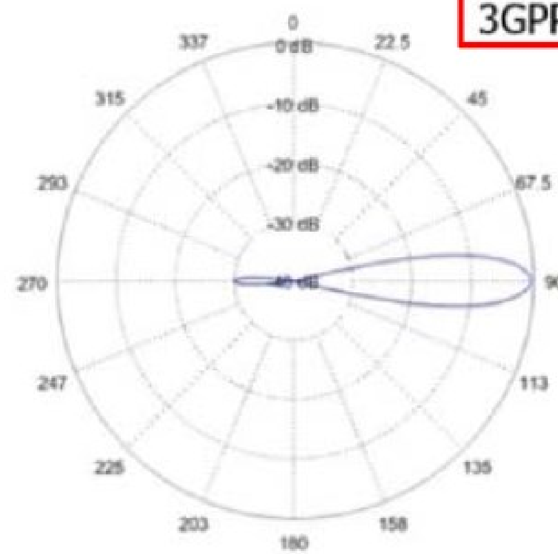




Element pattern: Omni-directional

Element pattern: 3GPP





90°的beam pattern

Outline

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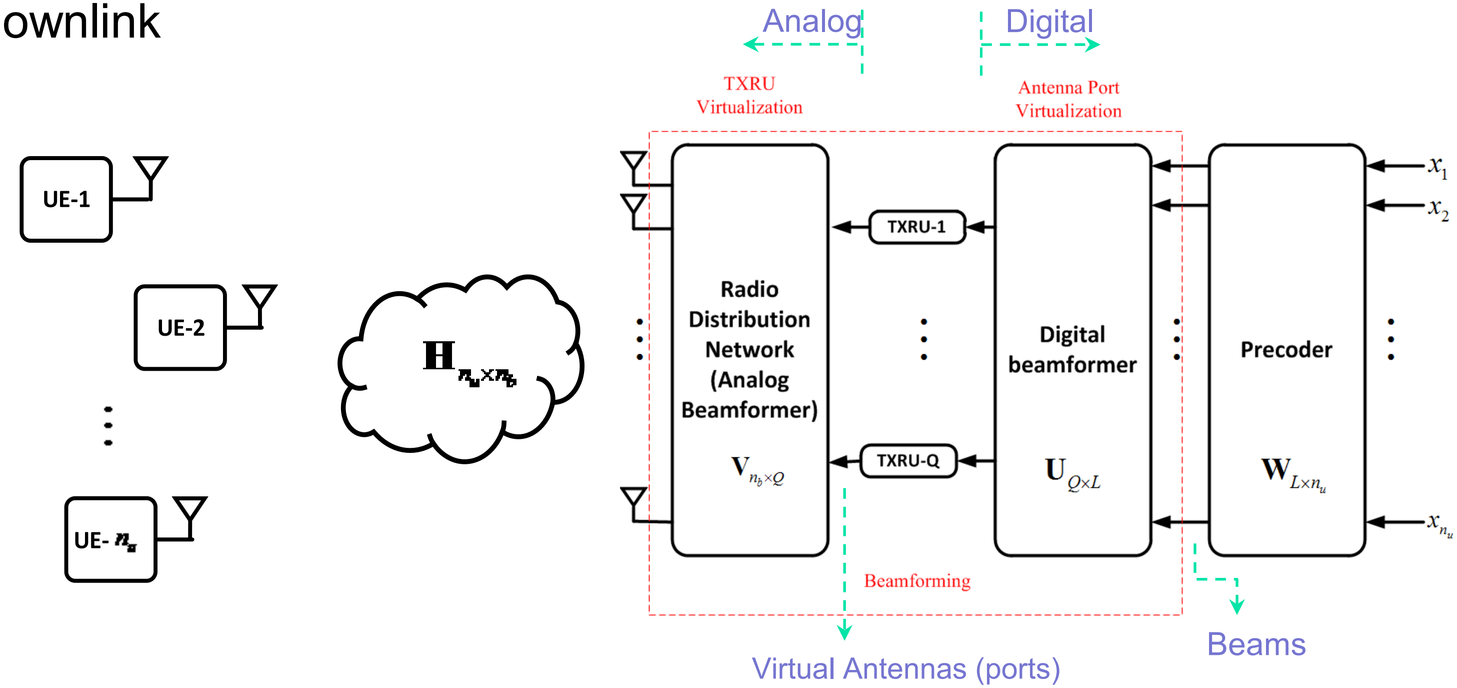
General Architecture ([6]3GPP 36.897)

- Downlink



$$\mathbf{r}_{n_u \times 1} = \mathbf{H}_{n_u \times n_b} \cdot \mathbf{V}_{n_b \times Q} \cdot \mathbf{U}_{Q \times L} \cdot \mathbf{W}_{L \times n_u} \cdot \mathbf{x}_{n_u \times 1} + \mathbf{n}_{n_u \times 1}$$

n_u : Number of UEs

$n_b \geq Q$: Number of antennas at eNB

$Q \geq L$: Number of RF chains

$L \geq n_u$: The set of beams used by n_u users

$\mathbf{H}_{n_u \times n_b}$: Channel matrix

$\mathbf{V}_{n_b \times Q}$: RF chain virtualization matrix

$\mathbf{U}_{Q \times L}$: Antenna port virtualization matrix

$\mathbf{W}_{L \times n_u}$: Precoding matrix

$\mathbf{x}_{n_u \times 1}$: Data vector

$\mathbf{n}_{n_u \times 1}$: AWGN vector

All-digital Architecture (1/5)

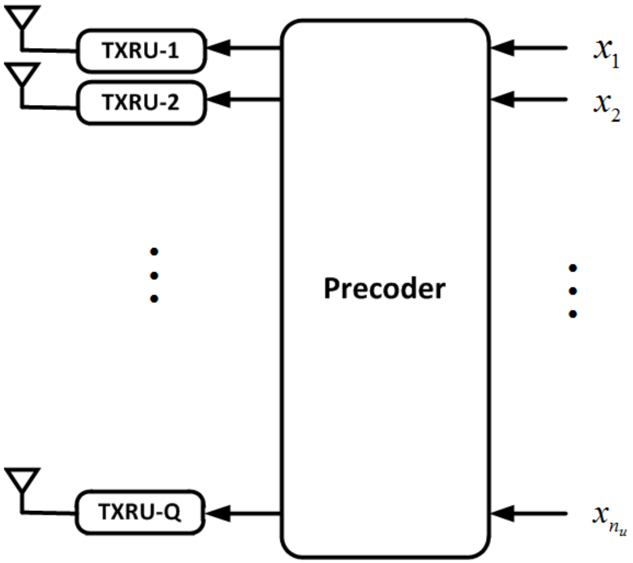
- $Q = n_b$: number of TXRUs equals to that of antenna-elements
  - High flexibility; narrow beam width
  - Very costly for a large number of antenna-elements and large bandwidth (1024 antenna elements at 70 GHz in 3GPP)

E.g., 500MHz頻寬 → A/D: 4倍oversampling, 13bits -> 26G bits/sec

All-digital Architecture (2/5)

- Case 1: antenna-space processing (no explicit digital beamformer)
  - Total of $n_b \times n_u$ spatial (antenna) channels needed to be estimated in $\mathbf{H}_{n_u \times n_b}$.
  - No exploitation of the characteristics channel sparsity at high-frequency band

You need to estimate $\mathbf{H}$ to perform MU-MIMO precoder in antenna-space. Even if we can accurately estimate the $\mathbf{H}$ for precoder, the UE can only receive its own signal without MU interference. Due to the high path loss of mm-wave, the UE's received signal strength is still very weak→ we need to use a giant antenna to obtain additional antenna gain to increase the signal strength at the receiving end→ beam-domain processing

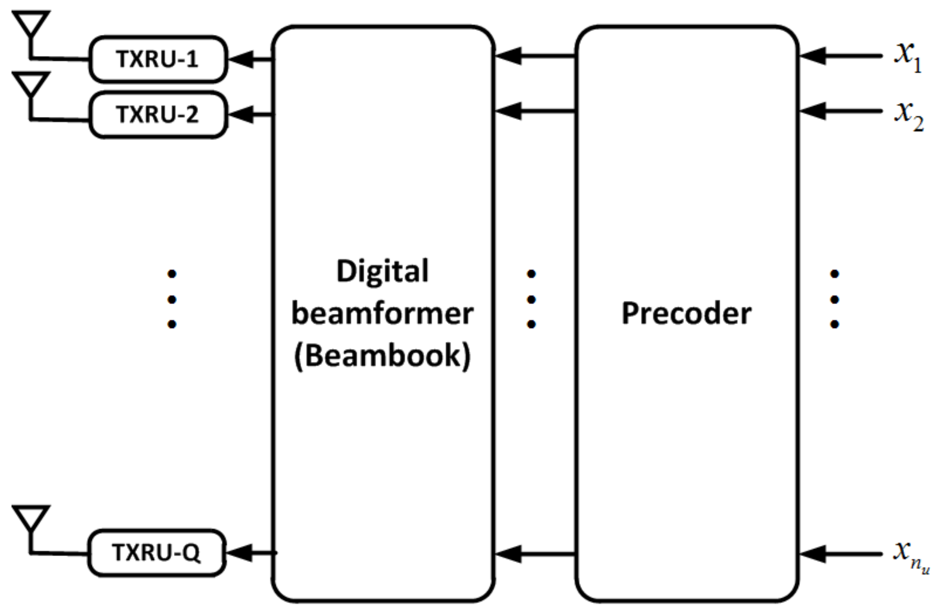


All-digital Architecture (3/5)

- Case 2: beam-space processing (which explicit digital beamformer)
  - Separate digital beam-former and digital precoder
  - Precoding is done in beam-space

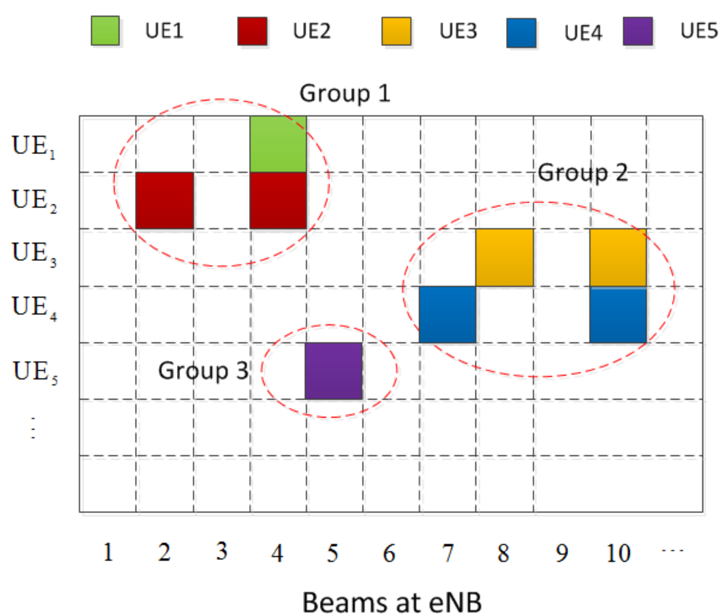
– Much smaller number of beam-channels (beam-link in 3GPP) due to the salient feature of channel sparsity at high-frequency band

– Using a fixed beambook is a common assumption for NR in 3GPP



All-digital Architecture (4/5)

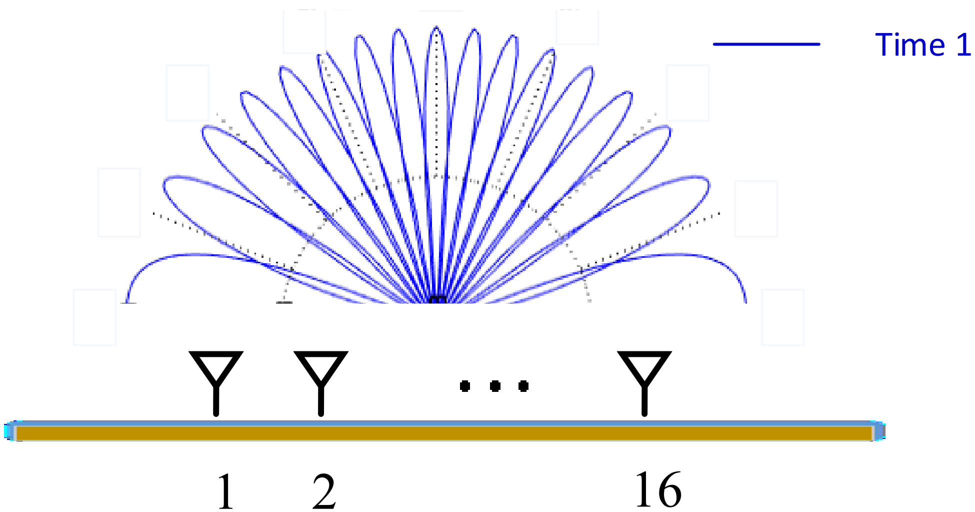
- **Beam-domain precoding** needs to be applied to user groups $\{UE_1, UE_2\}$ and $\{UE_3, UE_4\}$ to eliminate MU interference.
- One can directly serve UE_5 in the beam domain because of spatial division.



All-digital Architecture (5/5)

- With $Q = n_b$ TXRUs, n_b beams are used to cover the angular space of interest.
- No beam sweeping is needed. Beam width can be small for a large n_b.

- For example: $n_b = Q = 16$



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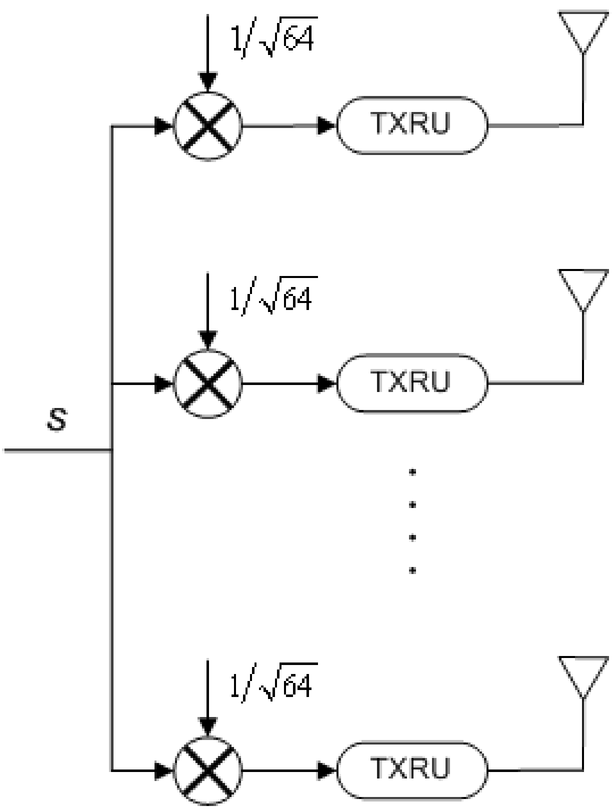
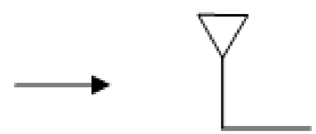
Antenna Gain from Beam-domain Processing (1/2)

- Downlink example: 64 antennas at gNB; single-antenna UE; $\mathrm{AoA} = 90^\circ$ (boresight) → beamforming weight vector $1/\sqrt{64} \times \mathbf{1}$





$$r = 64 \times \frac{s}{\sqrt{64}} + n = \sqrt{64}s + n$$

$$P_s = E\left[|s|^2\right], P_n = E\left[|n|^2\right]$$

$$\rightarrow SNR = \frac{64 P_s}{P_n}$$

$\rightarrow$ 18 dB antenna power gain

Antenna Gain from Beam-domain Processing (2/2)

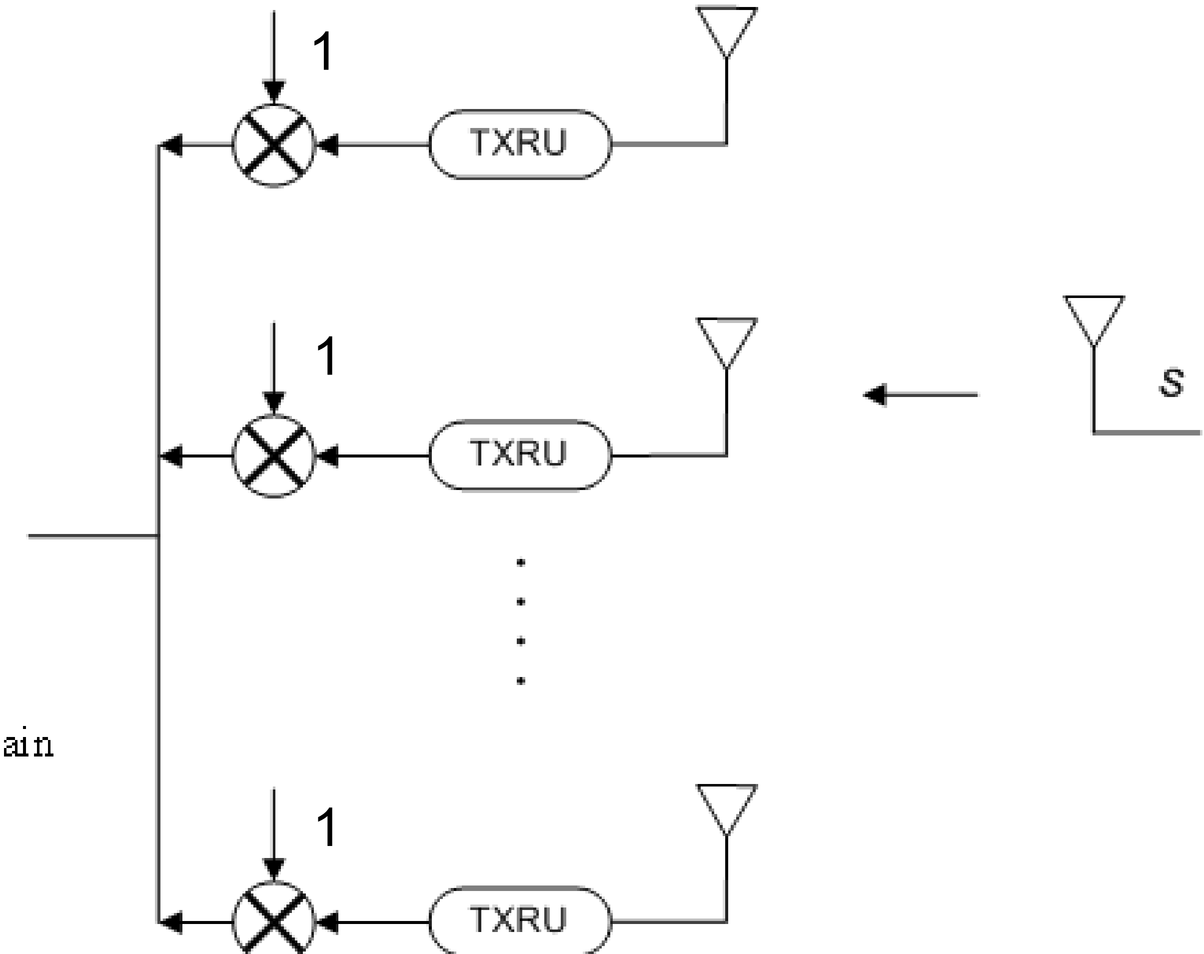
- Similarly for uplink

$$r = \sum_{i=1}^{64} s + n_i = 64s + \sum_{i=1}^{64} n_i$$

$$P_s = E\left[|s|^2\right], P_{n_i} = E\left[|n|^2\right]$$

$$\rightarrow SNR = \frac{64^2 P_s}{64 P_n} = \frac{64 P_s}{P_n}$$

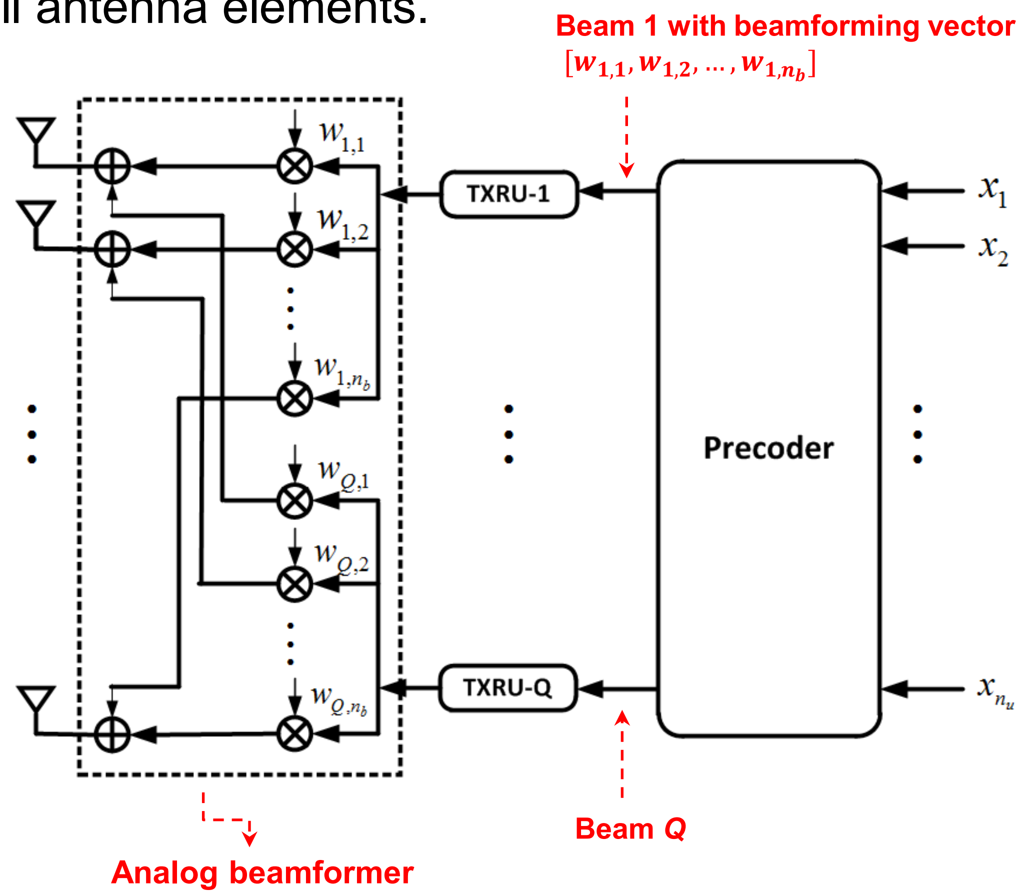
$$\rightarrow 18 \text{ dB antenna power gain}$$



Hybrid Architecture (1/6)

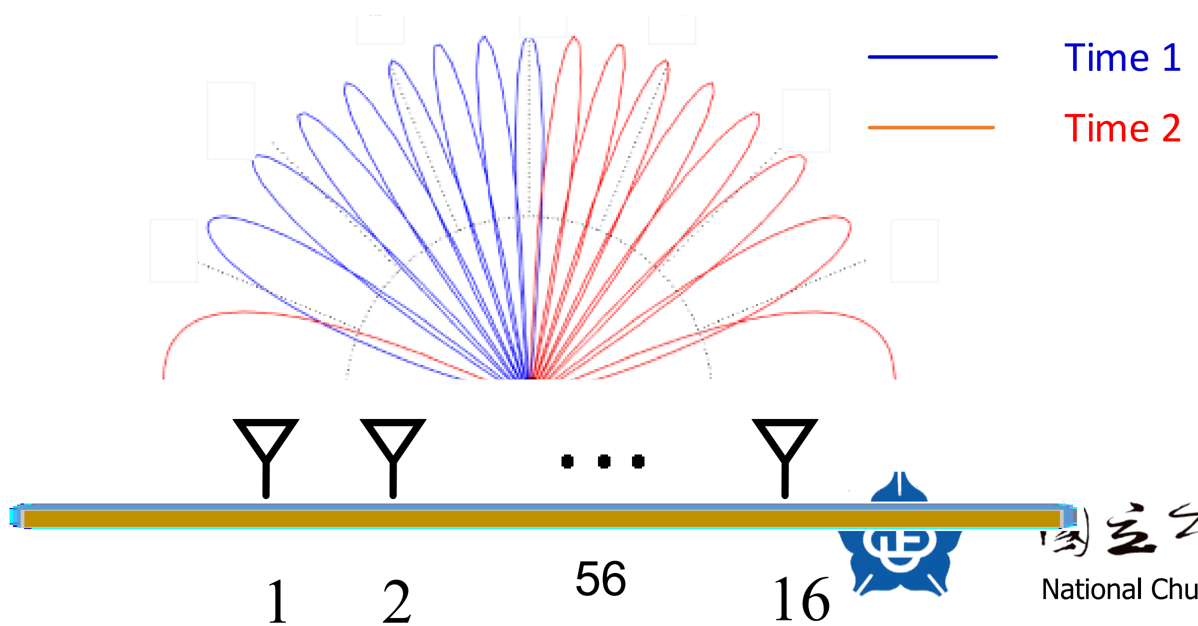
- $Q < n_b$: Less costly than all-digital architecture
- Case 1: fully connected Analog Beamformer
  - Each TXRU is connected to all antenna elements.

– Huge technical challenge due to the use of signal combiners
  - Beam-sweeping might be needed to have full angular coverage.
  - *Q* narrow beams are available for each beam sweeping.



Hybrid Architecture (2/6)

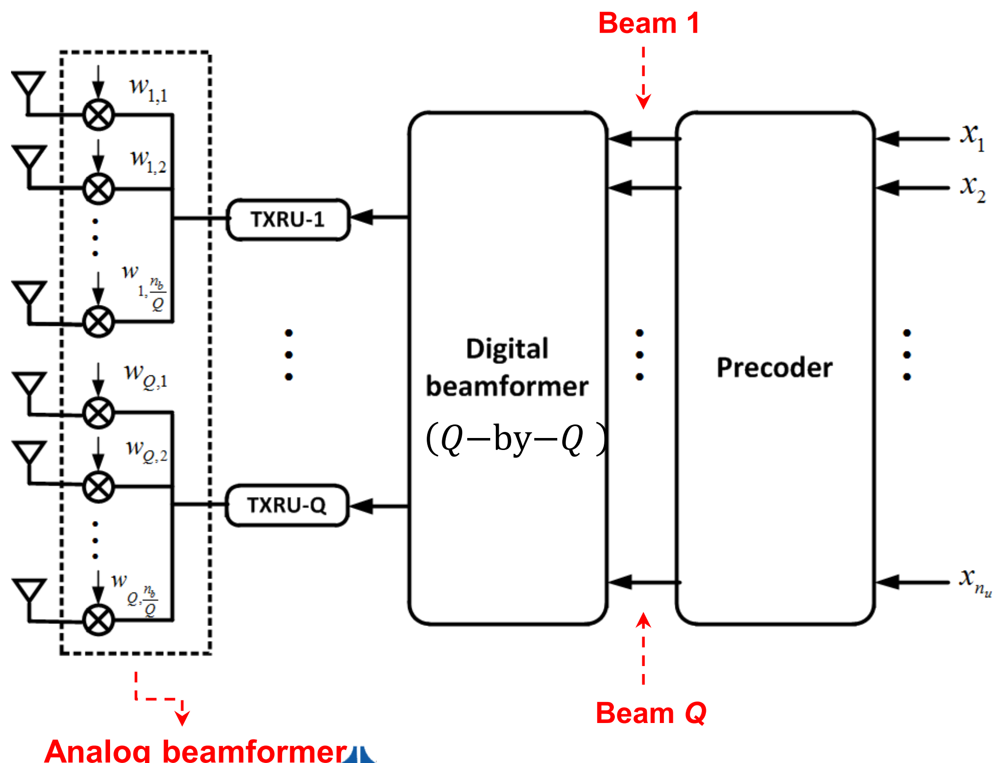
- Example of beam sweeping
  - Divide n_b beams into different subsets and use different subsets to serve users in time-division manner.
  - Beam sweeping induces excessive latency (scheduling) control signaling overhead.
  - Example: $n_b = 16$, $Q = 8$



Hybrid Architecture (3/6)

- Case 2: Partially connected (subarray) Analog Beamformer
  - Each antenna element is connected to one TXRU. And each TXRU is connected to a sub-array of antenna elements.
  - No signal combiner is needed.

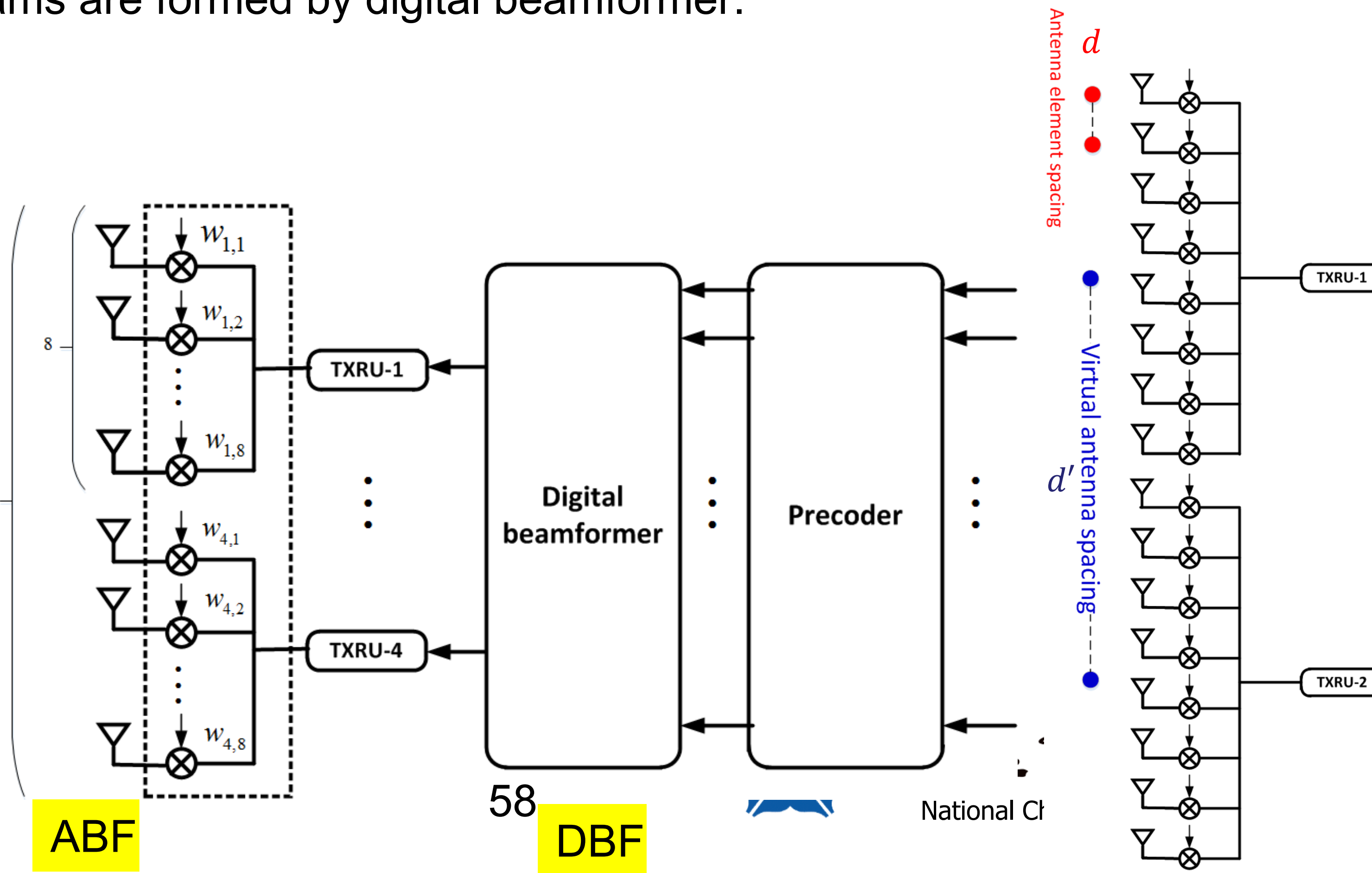
– Each subarray connected to a specific TXRU can be viewed as **an antenna** with specific antenna pattern. Narrow beam can still be formed if there is large enough number of antenna elements in a subarray.



Hybrid Architecture (4/6)

- Case 2a: Narrow beams are formed by use of digital beamformer.
  - Each subarray has the same wide (**broad**) beam pattern, which is formed by its analog beamformer (ABF).
  - **Narrow** beams are formed by digital beamformer.

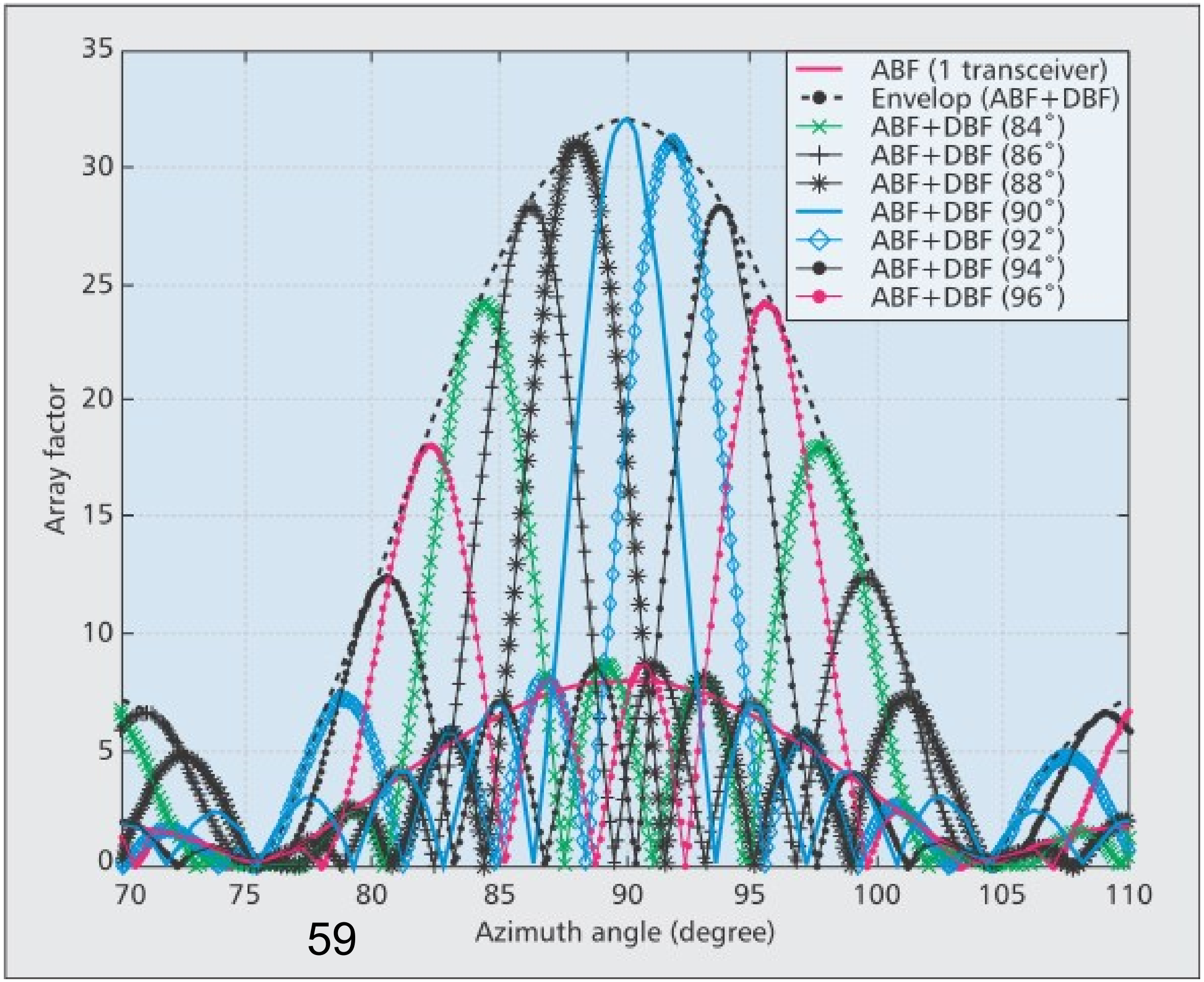
Example: $n_b = 32$, $Q = 4$; $d = \frac{\lambda}{2}$;
$d' > \frac{\lambda}{2}$
(subarray spacing)



Hybrid Architecture (5/6)

- Case 2a (cont.): May not be suitable for high-order MU-MIMO.

  – The same broad beam pattern (by ABF) for each subarray has max array gain at azimuth 90°.
  – The narrow beam can be formed by digital beamformer.



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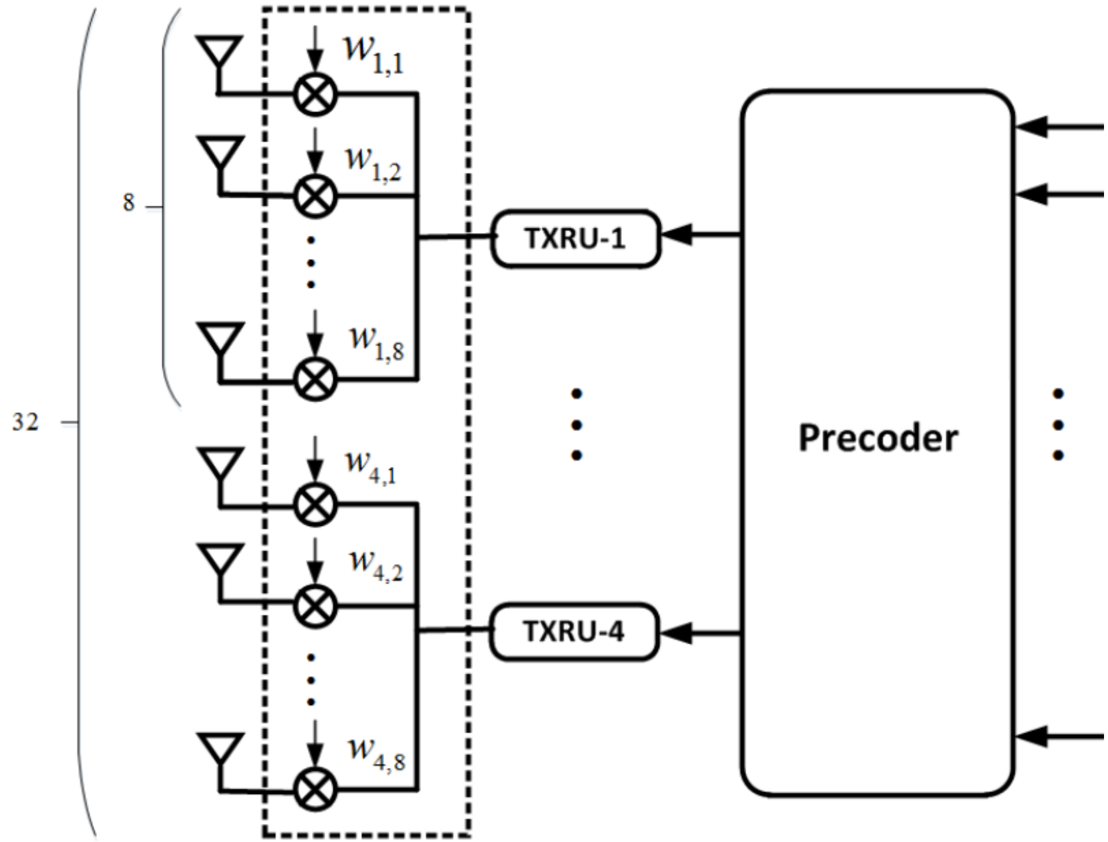
Hybrid Architecture (6/6)

- Case 2b:

Each subarray of Analog Beamformer has its own antenna pattern and all the subarrays are employed to cover (partial) angular space. Thus the digital beamformer is unnecessary.

Beam sweeping might be needed to have full angular space coverage.



Outline

Maximum Ratio Combining (MRC)

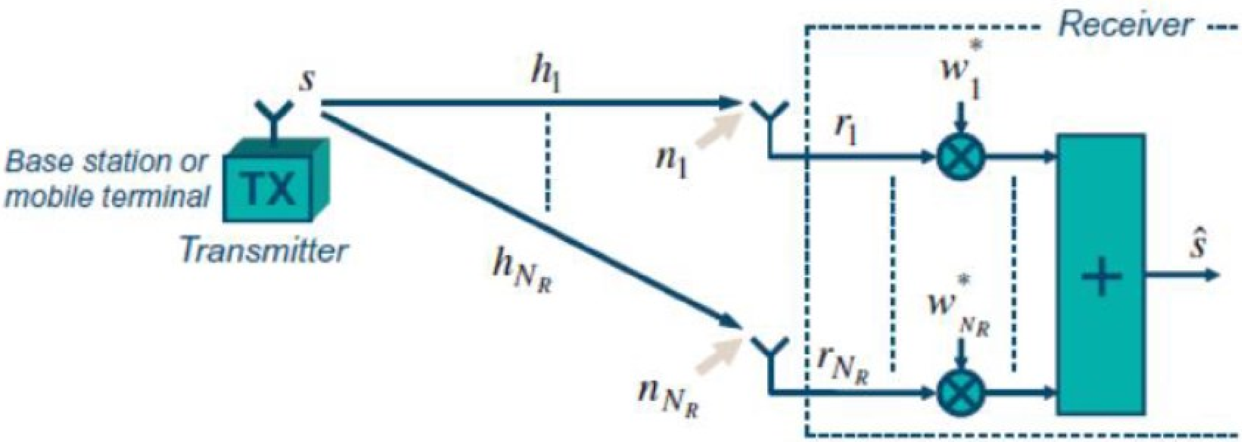
- The combined signal $\hat{s} = \sum_{k=1}^{N_R} w_k^* r_k = v + z$

$$v = \sum_{k=1}^{N_R} w_k^* h_k s \ ; \ \text{the signal part}$$

$$z = \sum_{k=1}^{N_R} w_k^* n_k \ ; \ \text{the noise part}$$

- The signal power

$$P_v = \mathrm{E}\left[|v|^2 \right] = P_s \cdot \left| \sum_{k=1}^{N_R} w_k^* h_k \right|^2, \ \text{where } P_s = \mathrm{E}\left[|s|^2 \right]$$



Maximum Ratio Combining (MRC)

- Assume that n_k are zero-mean and mutually independent

$$P_z = \mathrm{E}\left[|z|^2\right] = \sum_{k=1}^{M} |w_k|^2 P_{N_k}, \text{ where } P_{N_k} = \mathrm{E}\left[|n_k|^2\right]$$

- The output SNR: $\gamma = \dfrac{P_v}{P_z} = \dfrac{P_s \cdot \left|\sum_{k=1}^{N_R} w_k^* h_k\right|^2}{\sum_{k=1}^{N_R} |w_k|^2 P_{N_k}}$

- The Cauchy-Schwarz inequality

$$\left|\sum_{k=1}^{N_R} a_k^* b_k\right|^2 \leq \left(\sum_{k=1}^{N_R} |a_k|^2\right)\left(\sum_{k=1}^{N_R} |b_k|^2\right)$$

with equality if $b_k = Ca_k$ for all k, where C is a complex constant

Maximum Ratio Combining (MRC)

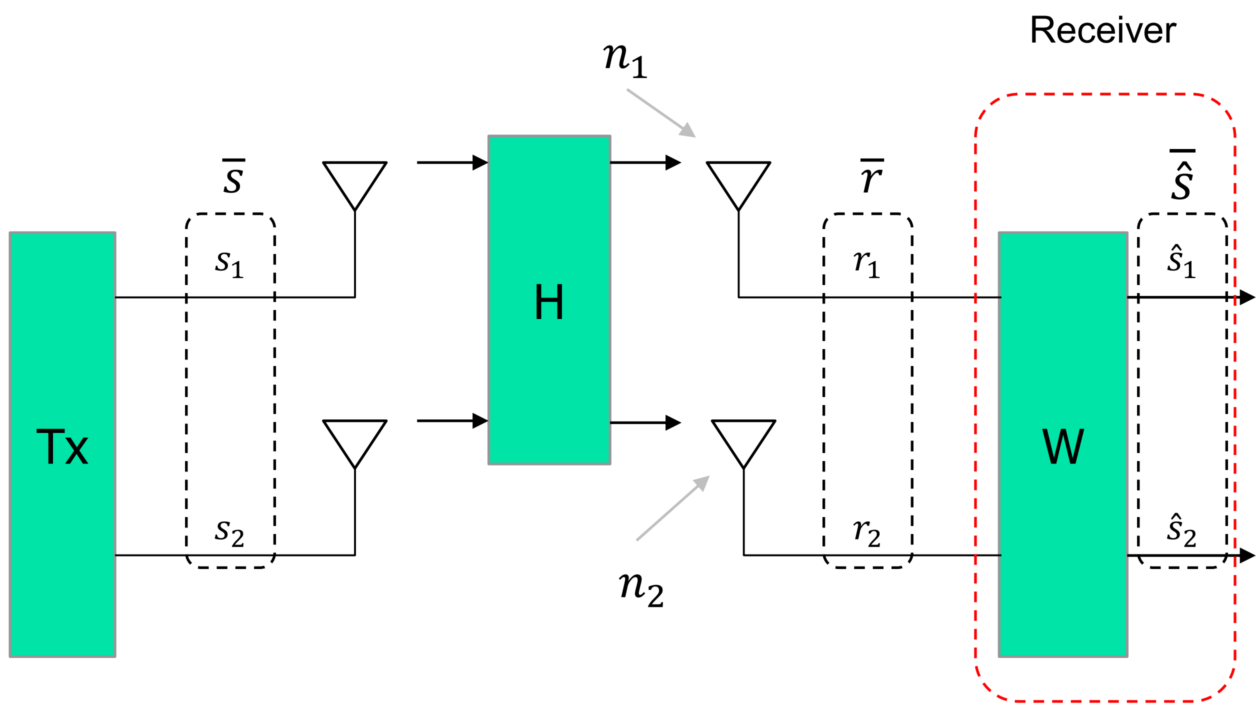
Let $a_k = w_k \sqrt{P_{N_k}}$, $b_k = \dfrac{h_k}{\sqrt{P_{N_k}}}$

$$\left| \sum_{k=1}^{N_R} w_k^* h_k \right|^2 \le \left(\sum_{k=1}^{N_R} |w_k|^2 P_{N_k} \right) \left(\sum_{k=1}^{N_R} \frac{|h_k|^2}{P_{N_k}} \right)$$

$$\gamma = \frac{P_s \sum_{k=1}^{N_R} |w_k^* h_k|^2}{\sum_{k=1}^{N_R} |w_k|^2 P_{N_k}} \le \frac{P_s \left(\sum_{k=1}^{N_R} |w_k|^2 P_{N_k} \right) \left(\sum_{k=1}^{N_R} \frac{|h_k|^2}{P_{N_k}} \right)}{\sum_{k=1}^{N_R} |w_k|^2 P_{N_k}} = \sum_{k=1}^{N_R} \frac{P_s |h_k|^2}{P_{N_k}}$$

i.e., $\overline{w}_{\mathrm{MRC}} = \overline{h}$. This says the maximum SNR is the sum of the SNR of respective receive branch.

Spatial Multiplexing



Zero-Forcing Receiver

- Zero-forcing receiver($\boldsymbol{H}^{-1}$ exists)

$$\bar{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \boldsymbol{H}\bar{s} + \bar{n}$$

$$\bar{\hat{s}}_{ZF} = \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = H^{-1}\bar{r} = \bar{s} + H^{-1}\bar{n}$$

  - For non-square channel matrix $H_{n_R \times n_T}, n_R > n_T$

$$\bar{\hat{s}}_{ZF} = \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = (H^H H)^{-1} H^H \bar{r} = (H^H H)^{-1} H^H (H\bar{s} + \bar{n})$$

$$= \bar{s} + (H^H H)^{-1} H^H \bar{n}\, ; (H^H H)^{-1}\ exists;\ H^H H : Gram\ matrix$$

  - Noise enhancement for ill-conditioned channels

MMSE(minimum mean-squared error) receiver

- In the MMSE detection algorithm, the expected value of the mean square error between the transmitted vector s and a linear combination of the received vector $\hat{\bar{s}} = W^H \bar{r}$ is minimized

$$W^H = \underset{L^H}{\operatorname{argmin}}\{E[\|\bar{s} - L^H \bar{r}\|^2]\}$$

- where w is an $n_R \times n_T$ matrix of MMSE coefficients

$$W = \left(H^H H + \frac{{\sigma_n}^2}{{\sigma_s}^2} I\right)^{-1} H^H$$

- See 補充教材MMSE推導

MMSE(minimum mean-squared error) receiver

$\bar{r} = H\bar{s} + \bar{n}$; $\hat{\bar{s}}_{MMSE} = \mathbf{K}\bar{r}$ is the MMSE estimator of $\bar{s}$, if

$$\mathbf{K} = \mathbf{R}_{\bar{s}\bar{r}}\mathbf{R}_{\bar{r}\bar{r}}^{-1},\quad \mathbf{R}_{\bar{s}\bar{r}} \doteq \mathrm{E}\left[\bar{s}\bar{r}^{\mathrm{H}}\right],\ \mathbf{R}_{\bar{r}\bar{r}} \doteq \mathrm{E}\left[\bar{r}\bar{r}^{\mathrm{H}}\right];$$

$$\mathbf{R}_{\bar{s}\bar{r}} \doteq \mathrm{E}\left[\bar{s}\bar{r}^{\mathrm{H}}\right] = \mathrm{E}\left[\bar{s}\left(H\bar{s} + \bar{n}\right)^{\mathrm{H}}\right] = \sigma_s^2 H^H$$

$$\mathbf{R}_{\bar{r}\bar{r}} \doteq \mathrm{E}\left[\bar{r}\bar{r}^{\mathrm{H}}\right] = \mathrm{E}\left[\left(H\bar{s} + \bar{n}\right)\left(H\bar{s} + \bar{n}\right)^{\mathrm{H}}\right] = \sigma_s^2 HH^H + \sigma_n^2 I$$

$$\rightarrow \mathbf{K} = \sigma_s^2 H^H\left(\sigma_s^2 HH^H + \sigma_n^2 I\right)^{-1} = \frac{\sigma_s^2}{\sigma_n^2}H^H\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H + I\right)^{-1}$$

$$= \left(\frac{\sigma_s^2}{\sigma_n^2}H^H H + I\right)^{-1}\frac{\sigma_s^2}{\sigma_n^2}H^H = \left(H^H H + \frac{\sigma_n^2}{\sigma_s^2}I\right)^{-1} H^H$$

$$\rightarrow \hat{\bar{s}}_{MMSE} = \mathbf{K}\bar{r} = \left(H^H H + \frac{\sigma_n^2}{\sigma_s^2}I\right)^{-1} H^H\bar{r};\quad \sigma_n^2 = 0 \rightarrow \hat{\bar{s}}_{MMSE} = \hat{\bar{s}}_{ZF}$$

MMSE(minimum mean-squared error) receiver

$$H^H\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H+I\right)=\left(\frac{\sigma_s^2}{\sigma_n^2}H^HH+I\right)H^H$$

$$H^H\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H+I\right)\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H+I\right)^{-1}=\left(\frac{\sigma_s^2}{\sigma_n^2}H^HH+I\right)H^H\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H+I\right)^{-1}$$

$$\left(\frac{\sigma_s^2}{\sigma_n^2}H^HH+I\right)^{-1}H^H=H^H\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H+I\right)^{-1}$$

$$\rightarrow\quad \frac{\sigma_s^2}{\sigma_n^2}H^H\left(\frac{\sigma_s^2}{\sigma_n^2}HH^H+I\right)^{-1}$$

$$=\left(\frac{\sigma_s^2}{\sigma_n^2}H^HH+I\right)^{-1}\frac{\sigma_s^2}{\sigma_n^2}H^H=\left(H^HH+\frac{\sigma_n^2}{\sigma_s^2}I\right)^{-1}H^H$$

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Thank you