5G NR實體層技術
Physical Layer Techniques for the 5G New Radio
上課教材

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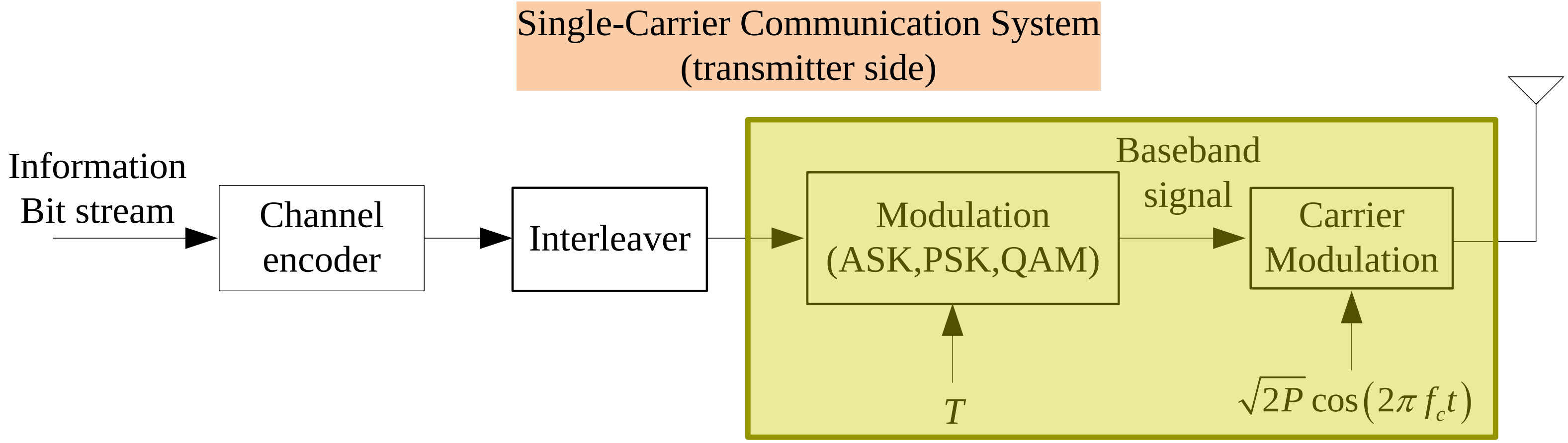
本上課教材分3大部分

- Part I： Cyclic Prefix Orthogonal Frequency Division Multiplexing (CP-OFDM)
- Part II：New Multi-Carrier Waveforms
  - Universal Filter Multi-Carrier Waveform
  - Filter Bank Multi-Carrier Waveform
  - Generalized Frequency Division Multiplexing
- Part III： Non-Orthogonal Multiple Access (NOMA) Techniques
  - Power-Domain Non-Orthogonal Multiple Access
  - Sparse Code Multiple Access

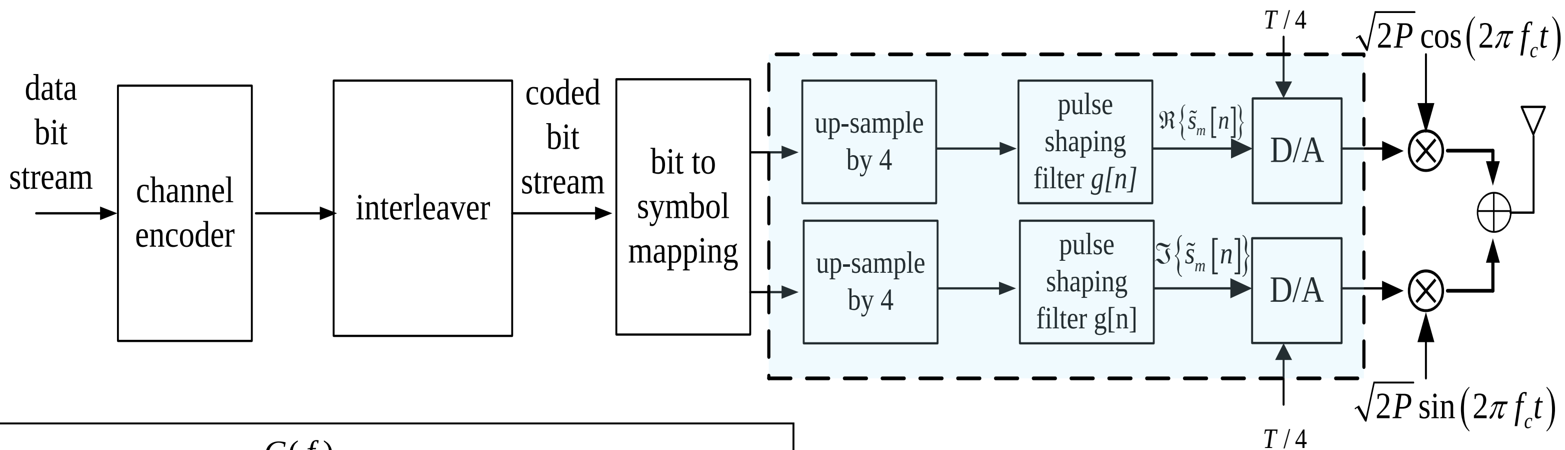
Part I：

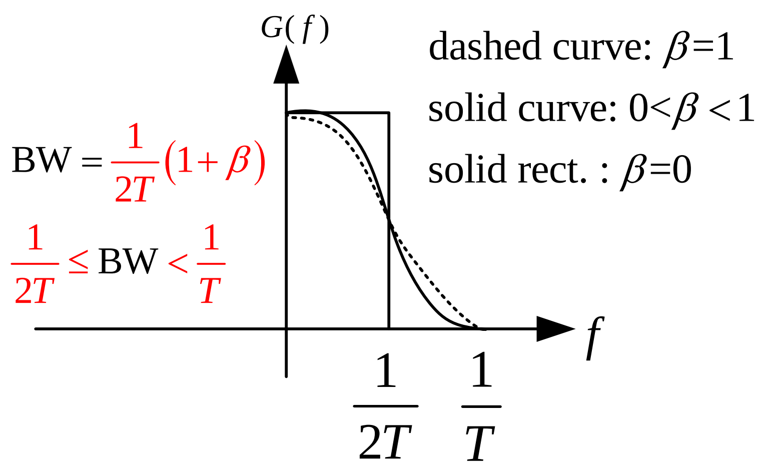
Cyclic Prefix Orthogonal Frequency Division Multiplexing (CP-OFDM)

Single-Carrier Communication System



Single Carrier Communication System (1/2)

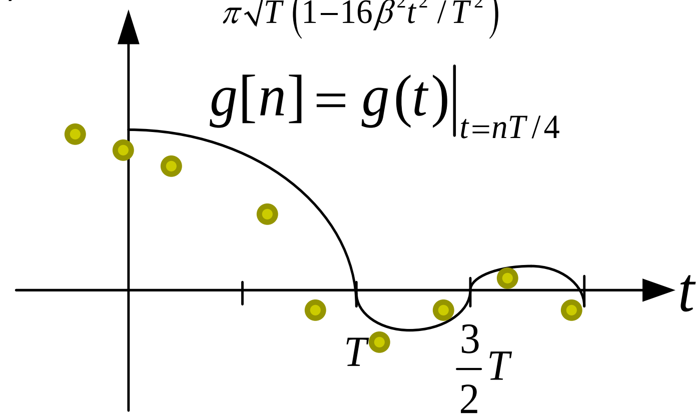




Spectrum of the root-raised-cosine function

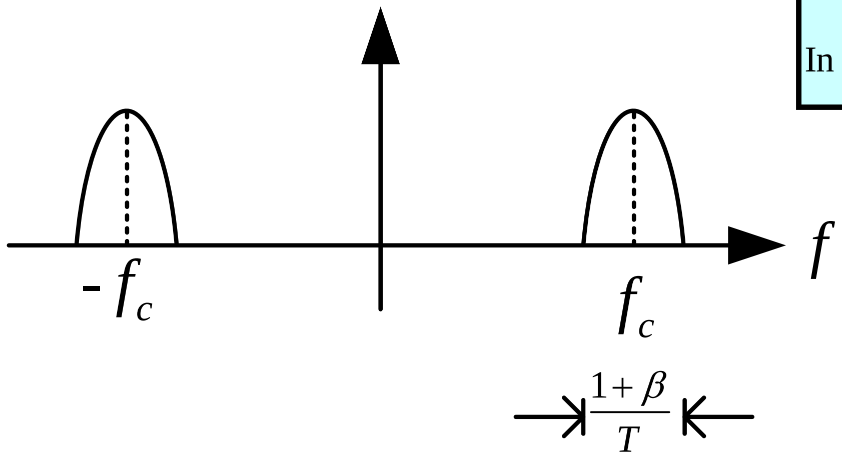
Baseband bandwidth $\frac{1+\beta}{2T}$

$$g(t) = 4\beta \frac{\cos\left[(1+\beta)\pi t / T\right] + \sin\left[(1-\beta)\pi t / T\right](4\beta t / T)^{-1}}{\pi\sqrt{T}\left(1-16\beta^2 t^2 / T^2\right)}$$



time domain of the root-raised-cosine function

Single Carrier Communication System (2/2)

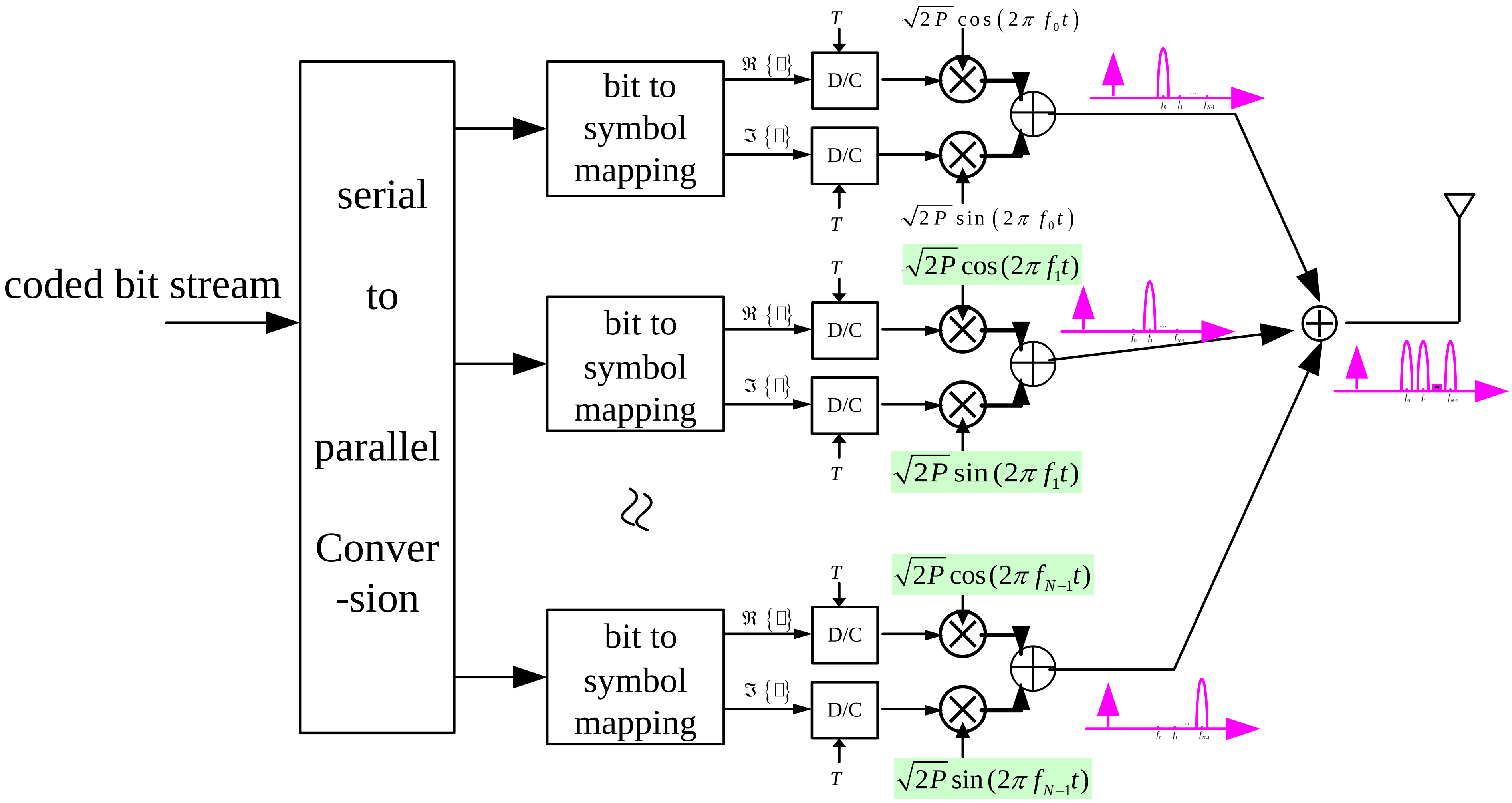


The RF bandwidth is $\frac{1+\beta}{T}, 0<\beta<1$.

In the following, assume $\beta=0$ and RF BW $\frac{1}{T}$.

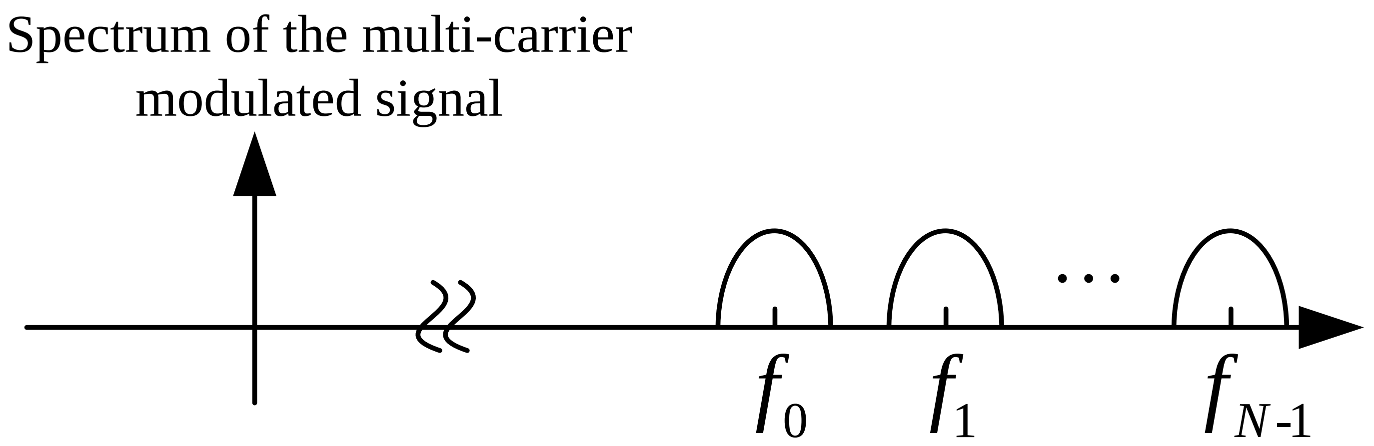
Modulation type	Bits per symbol	Symbols per sec/Hz	bps/Hz
BPSK	1	1	1
QPSK	2	1	2
16-QAM	4	1	4
64-QAM	6	1	6

Multi-Carrier Communication System (1/2)



Multi-Carrier Communication System (2/2)

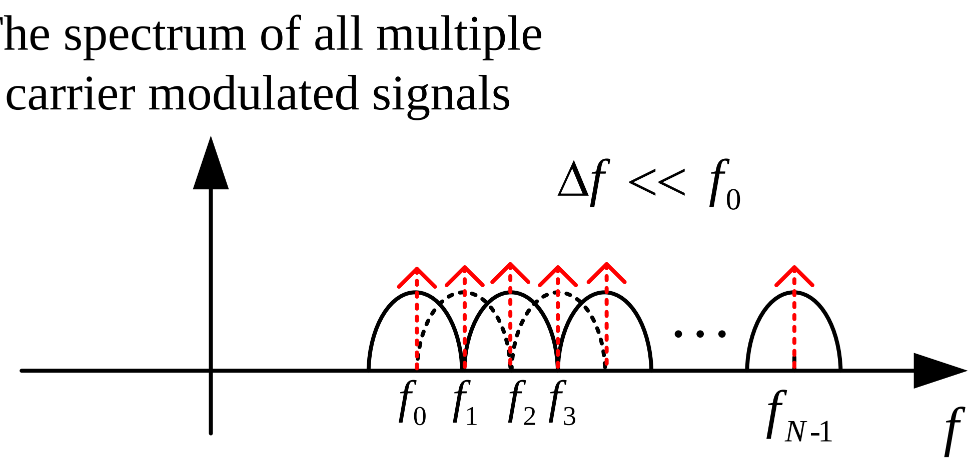
- If the $f_k, \forall k$, are far apart, the spectrum of the transmitted signal looks as follows.



- The carrier frequencies $f_k, \forall k$, are selected to avoid spectrum overlapping such that modulated signals associated with all carriers do not interfere with one another.

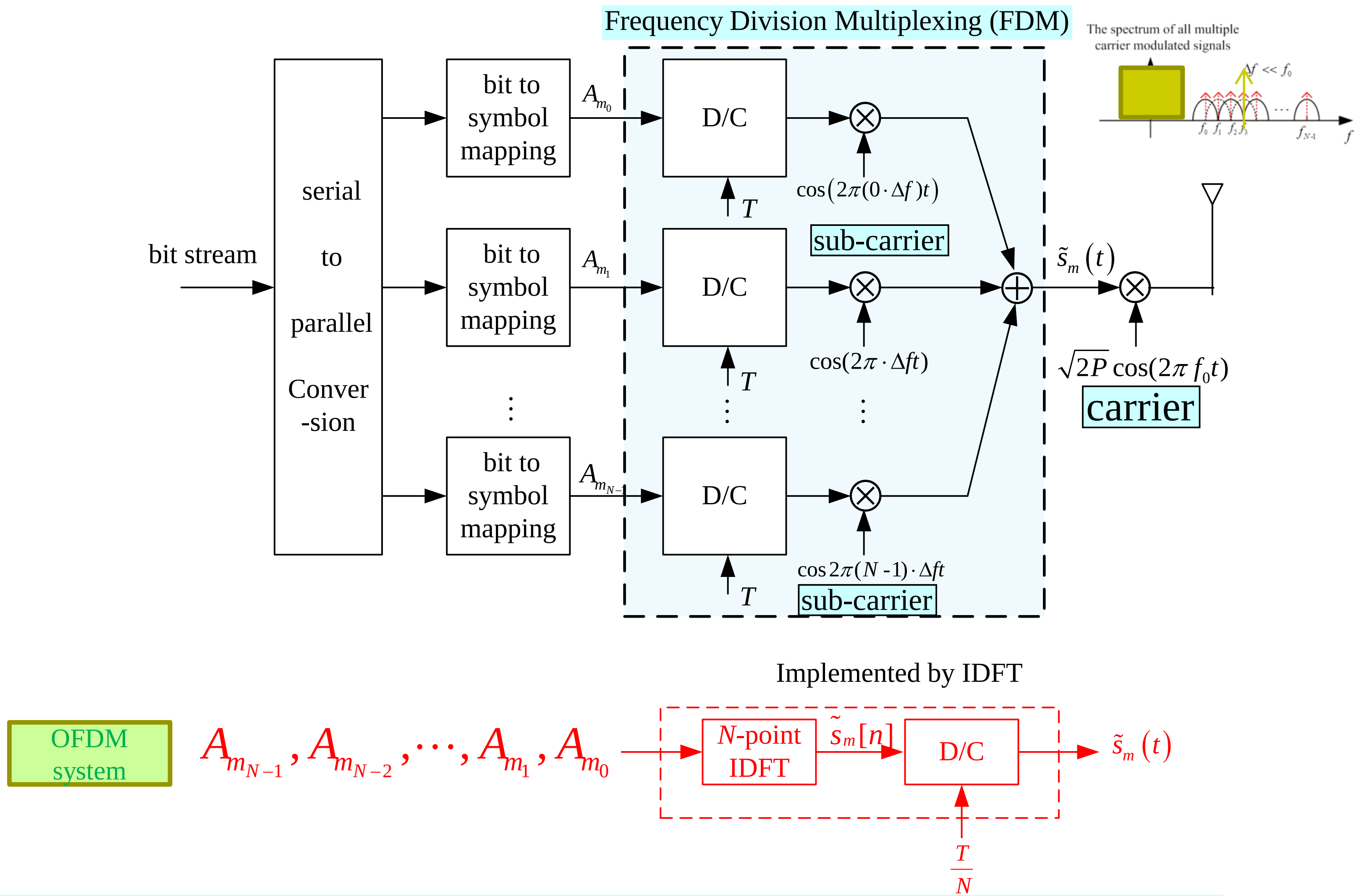
The OFDM System (1/3)

- However, if the carrier frequencies satisfy $f_k = f_0 + k\Delta f, \quad k = 0, \cdots, N-1$ where f_0 and $\Delta f = \frac{1}{T}$ are fixed values, the spectrum looks like



The spectra of all multiple sub-signals are overlapped.
It appears that the multiple sub-signals may interfere with one another.
However, the frequency components at frequency instants $f_k = f_0 + k\Delta, \forall k$,
do not interfere with one another.
Through precise frequency synchronization, the receiver can obtain through accurate sampling the frequency components at these frequency instants.
Hence, transmitting signal by this scheme requires accurate frequency synchronization.

The OFDM System (2/3)



The N sub-carriers $\{\cos(0\cdot 2\pi\Delta ft), \cos(1\cdot 2\pi\Delta ft), \cdots, \cos((N-1)\cdot 2\pi\Delta ft)\}$ are orthogonal.

The IDFT in OFDM plays digitally the role of FDM as in the multi-carier communication system

The OFDM System (3/3)

- The OFDM system is a structure of Orthogonal FDM of N parallel signal streams.

- Advantages of the OFDM system over the multi-carrier (MC)-system:
  - High spectral efficiency (two-fold)
  - Low-complexity (1-tapped) channel equalization
  - Only one RF chain (one mixer/power amplifier, one high-speed DAC)
  - Cheap and stable digital FFT to implement the Orthogonal FDM

Subcarrier Modulation Mapping (1/2)

- The encoded and interleaved binary serial input data shall be divided into groups of N_{BPSC} (1, 2, 4, *or* 6) bits and converted into complex numbers representing BPSK, QPSK, 16-QAM, or 64-QAM constellation points.
- The output values, *d*, are formed by multiplying the resulting ($I + jQ$) value by a normalization factor K_{MOD}, as follow

$$d = (I + jQ) \times K_{MOD}.$$

Table 81 - Modulation-dependent normalization factor K_{MOD} to make $E\{|d|^2\} = 1.$

Modulation	K_{MOD}
BPSK	1
QPSK	$1/\sqrt{2}$
16-QAM	$1/\sqrt{10}$
64-QAM	$1/\sqrt{42}$

Table 82 – BPSK encoding table

Input bit (b0)	I-out	Q-out
0	-1	0
1	1	0

Subcarrier Modulation Mapping (2/2)

Table 83 – QPSK encoding table b_0b_1

Input bit (b0)	I-out
0	-1
1	1

Input bit (b1)	Q-out
0	-1
1	1

Table 84 – 16-QAM encoding table $b_0b_1b_2b_3$

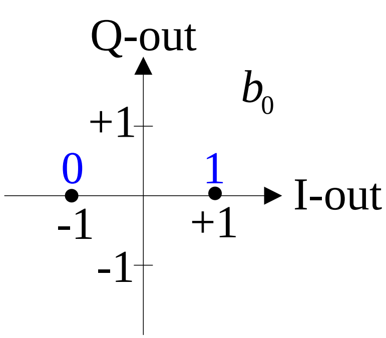
Input bits (b0 b1)	I-out
00	-3
01	-1
11	1
10	3

Input bits (b2 b3)	Q-out
00	-3
01	-1
11	1
10	3

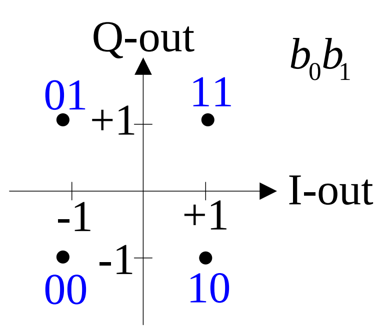
Table 85 – 64-QAM encoding table $b_0b_1b_2b_3b_4b_5$

Input bits (b0 b1 b2)	I-out
000	-7
001	-5
011	-3
010	-1
110	1
111	3
101	5
100	7

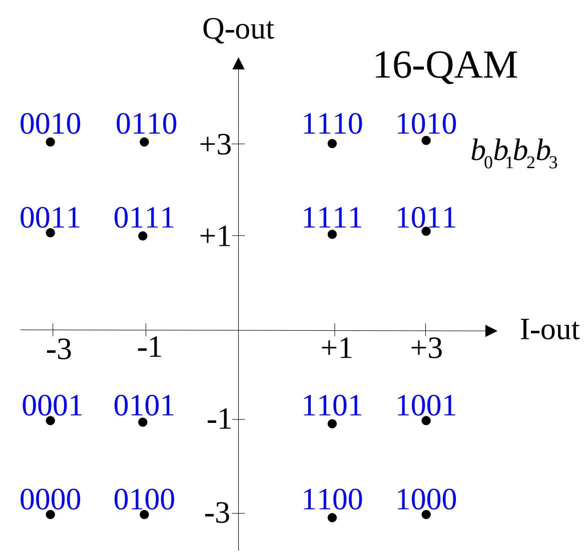
Input bits (b3 b4 b5)	Q-out
000	-7
001	-5
011	-3
010	-1
110	1
111	3
101	5
100	7



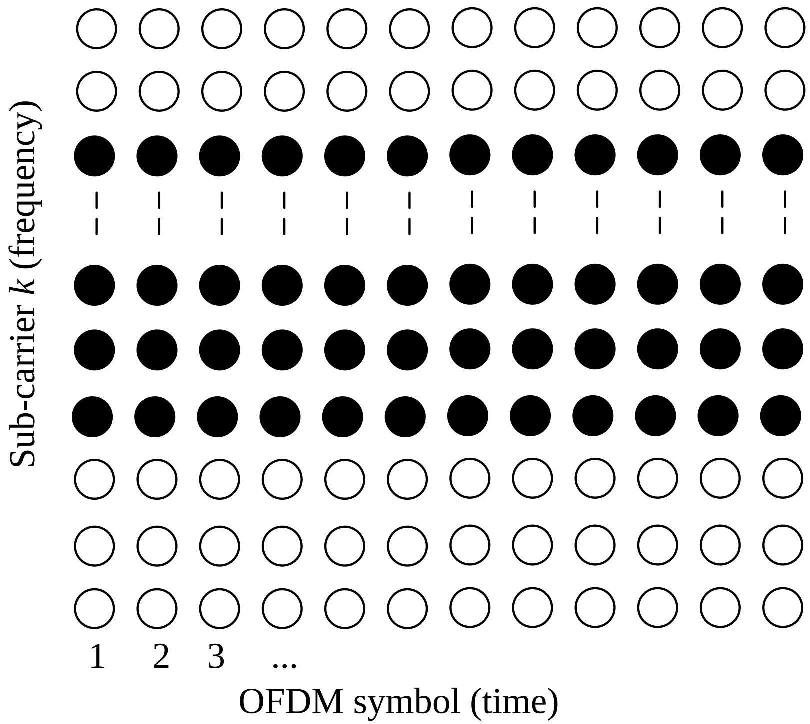
BPSK



QPSK



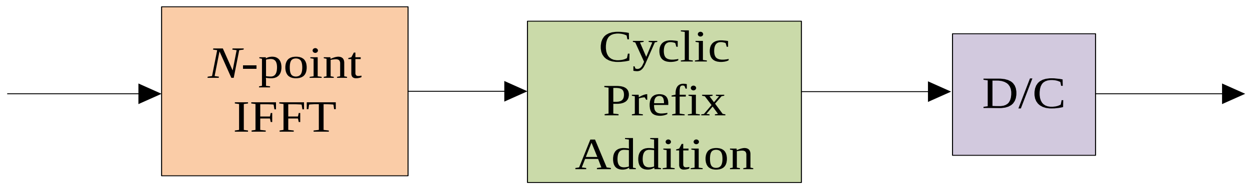
Time-Frequency Representation

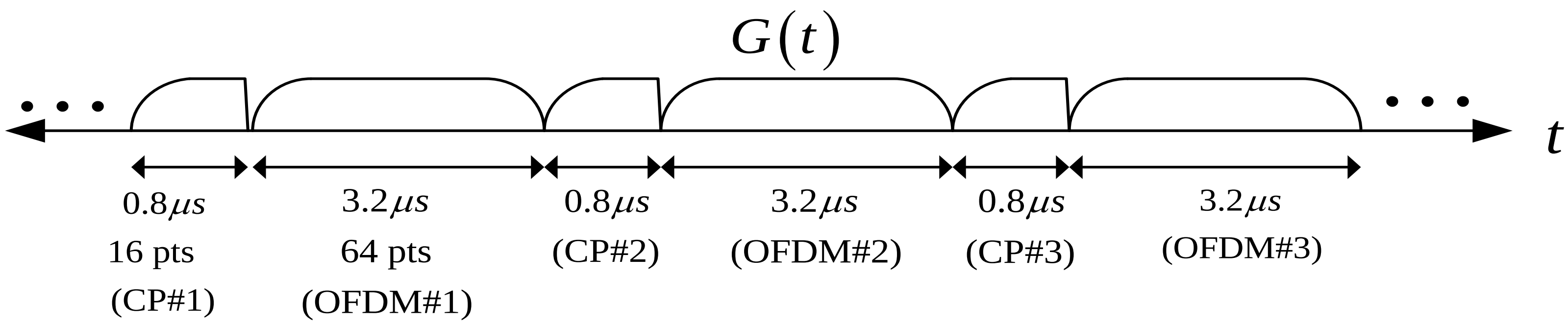


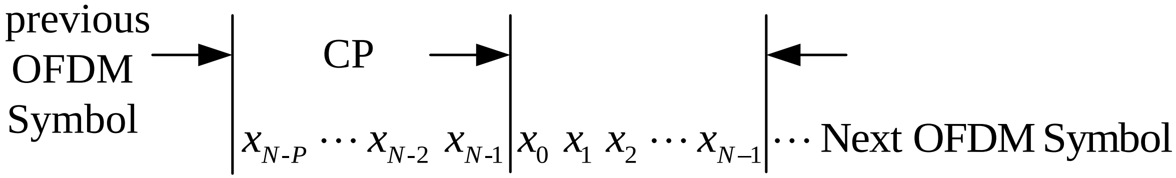
- Each black/white dot represents a sub-carrier symbol.
- One OFDM symbol is comprised of *N* (modulated) symbols.
- The *N* symbols are transmitted over a OFDM symbol duration of *T* seconds.

- The inter- subcarrier spacing is equal to *1/T* Hz.
- The sampling rate is *1/(NT)* Hz.

Cyclic Prefix for OFDM







One OFDM symbol may include:

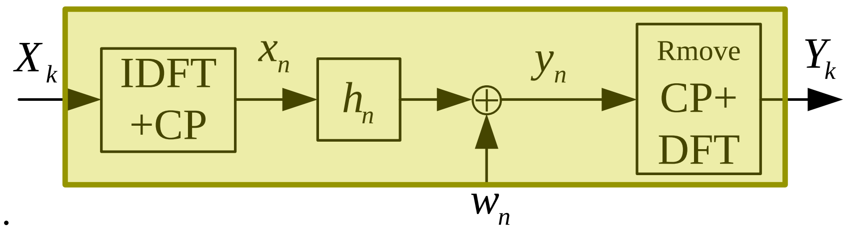
N symbols (data in frequency domain)

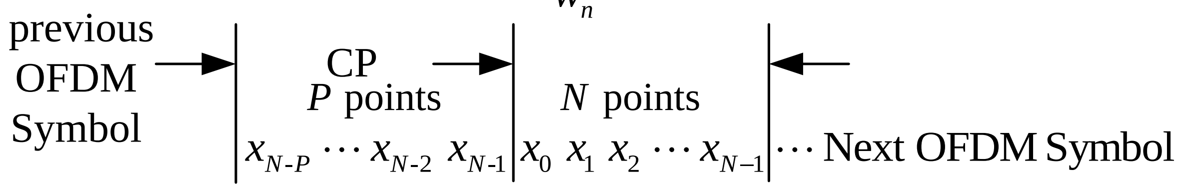
N samples (IFFT size in both time and frequency domains)

$N+P$ samples (IFFT size plus CP length in time domain)

Why is CP used ? (1/3)

- Consider the following system





$$y_n = \sum_{k=0}^{L-1} x_{n-k} h_k + w_n = h_0 x_n + h_1 x_{n-1} + \cdots + h_{L-1} x_{n-L+1} + w_n$$

$y[0]$	$y[1]$	$y[2]$	$y[3]$	$\cdots$	$y[N-1]$

$h_0 \times x_n$	[redacted]
$h_1 \times x_{n-1}$	[redacted]
$h_2 \times x_{n-2}$	$\cdots$ $x[N-4]$ $x[N-3]$ $x[N-2]$ $x[N-1]$ $x[0]$ $x[1]$ $\cdots$ $x[N-3]$ $x[N-2]$ $x[N-1]$ $\cdots$

$x[n]$	$x[0]$	$x[1]$	$x[2]$	$\cdots$	$x[N-1]$
$x[((n-1))_N]$	$x[N-1]$	$x[0]$	$x[1]$	$\cdots$	$x[N-2]$
$x[((n-2))_N]$	$x[N-2]$	$x[N-1]$	$\cdots$	$\cdots$	$x[N-3]$

Why is CP used ? (2/3)

$$
\underbrace{\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ \vdots \\ \vdots \\ y_{N-1} \end{bmatrix}}_{\mathbf{y}} {}_{N\times 1}
=
\underbrace{\begin{bmatrix} x_0 & x_{N-1} & x_{N-2} & \cdots & x_1 \\ x_1 & x_0 & x_{N-1} & \cdots & x_2 \\ \vdots & & \ddots & & \vdots \\ x_{N-1} & x_{N-2} & \cdots & \cdots & x_0 \end{bmatrix}}_{\mathbf{x}} {}_{N\times N}
\underbrace{\begin{bmatrix} h_0 \\ \vdots \\ h_{L-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\mathbf{h}} {}_{N\times 1}
+
\underbrace{\begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ \vdots \\ \vdots \\ w_{N-1} \end{bmatrix}}_{\mathbf{w}} {}_{N\times 1}
$$

circulant matrix

Define the DFT matrix by

$$
\mathbf{F} \triangleq \frac{1}{\sqrt{N}} \begin{bmatrix} w_N^{00} & \cdots & w_N^{0\cdot(N-1)} \\ \vdots & & \vdots \\ w_N^{(N-1)\cdot 0} & \cdots & w_N^{(N-1)\cdot(N-1)} \end{bmatrix}_{N\times N}, \text{ where } w_N = e^{-j2\pi\frac{nk}{N}}
$$

$$
\mathbf{F}\mathbf{F}^H = \mathbf{F}^H\mathbf{F} = \mathbf{I}
$$

Then,

$$
\underbrace{\mathbf{Fy}}_{\mathbf{Y}} = \mathbf{F}\underbrace{\mathbf{x}\mathbf{F}^{H}}_{\mathbf{X}} \underbrace{\mathbf{Fh}}_{\mathbf{H}} + \underbrace{\mathbf{Fw}}_{\mathbf{W}}
$$

Why is CP used ? (3/3)

- Note that

$$\underbrace{\begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N-1} \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} X_0 & 0 & \cdots & 0 \\ 0 & X_1 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & X_{N-1} \end{bmatrix}}_{\mathbf{X}} \underbrace{\begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_{N-1} \end{bmatrix}}_{\mathbf{H}} + \underbrace{\begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_{N-1} \end{bmatrix}}_{\mathbf{W}}$$

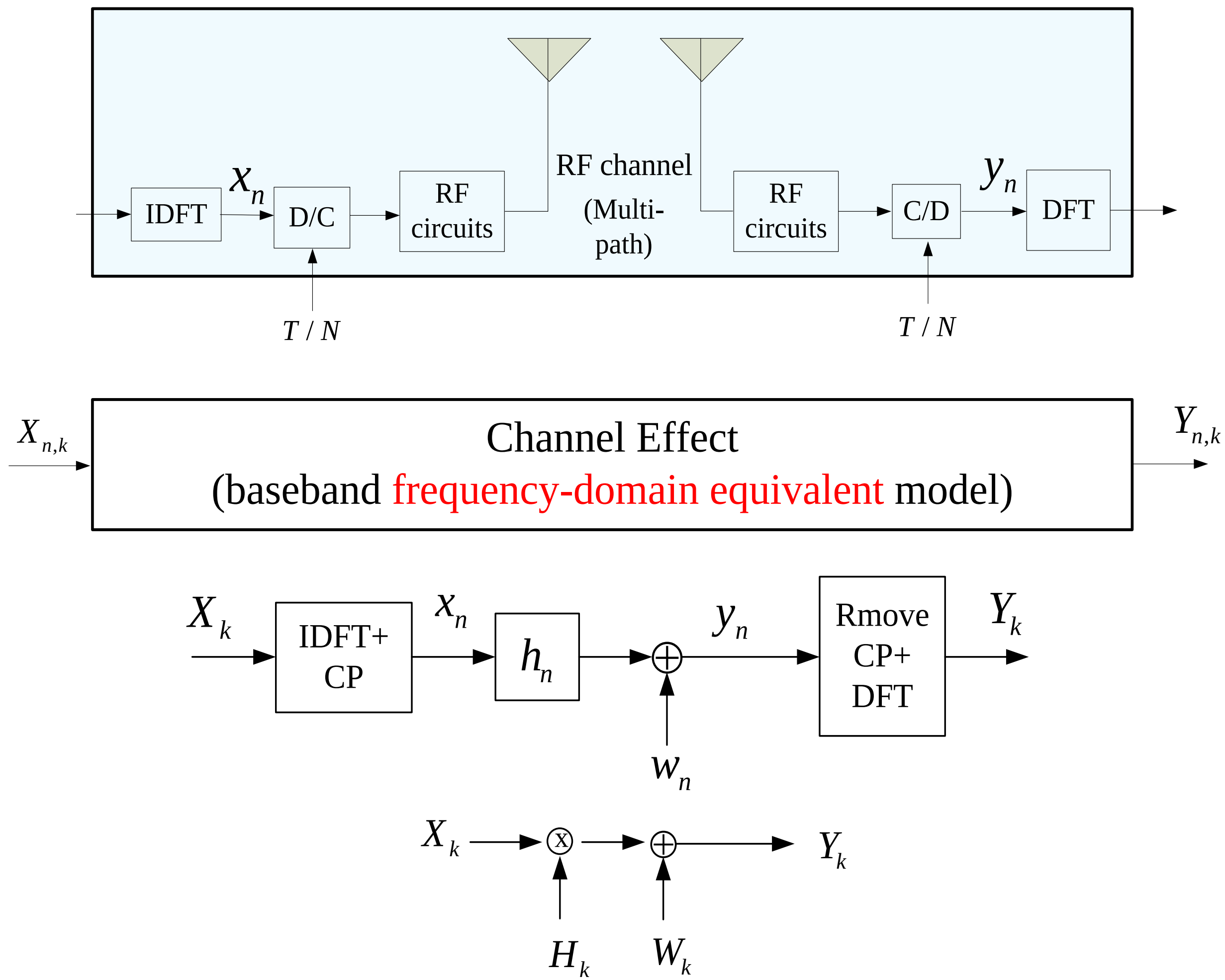
- It must be satisfied that $P \geq L$ to avoid ISI.
- It is essential to prove that $\mathbf{FxF^H}$ is diagonal.
- The above frequency-domain model can be written as

$$Y_k = X_k H_k + W_k, \quad k = 0,1,\cdots,N-1$$

k is the sub-carrier index.

- The frequency-selective channel now becomes frequency non-selective.

Complete Channel Effects

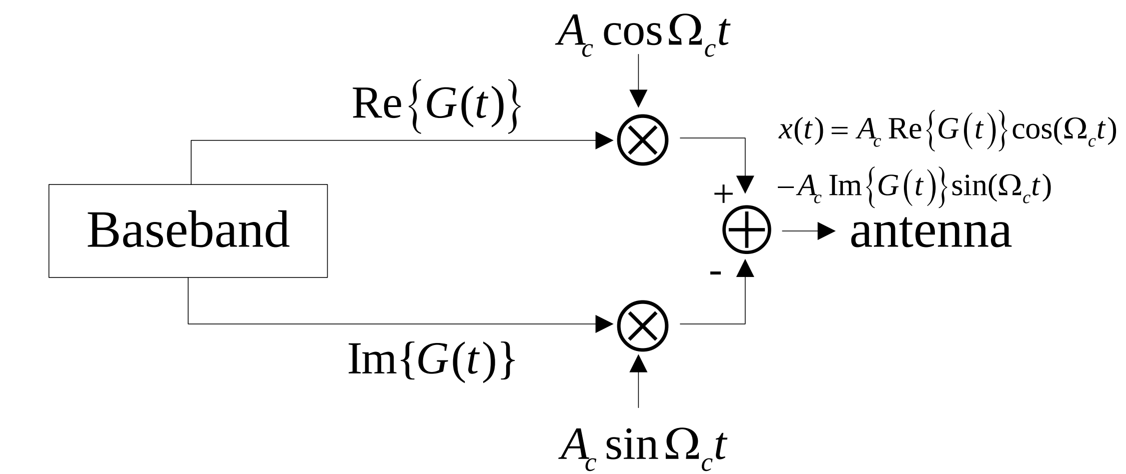


The frequency-selective channel now becomes frequency non-selective.

Cyclic Prefix vs Guard Time

Guard Time	Cyclic Prefix
Eliminates Inter-symbol Interference	Eliminates Inter-symbol Interference
Suffers from Inter-carrier Interference	Eliminates Inter-carrier Interference
Suffers from Intra-carrier Interference	Suffers from Intra-carrier Interference
Causes a reduction in data rate as a result of the increased OFDM symbol time	Causes a reduction in data rate as a result of the increased OFDM symbol time
Does not consume additional power associated with OFDM symbol time expansion due to the guard time	Necessitates additional power associated with OFDM symbol expansion due to the introduction of cyclic prefix

Pulse Shaping and Spectrum Mask



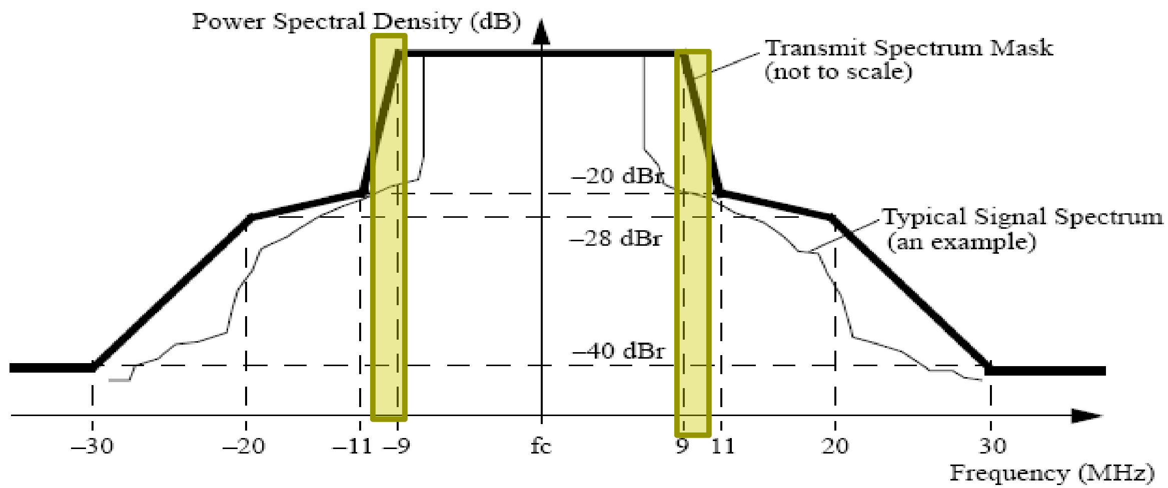
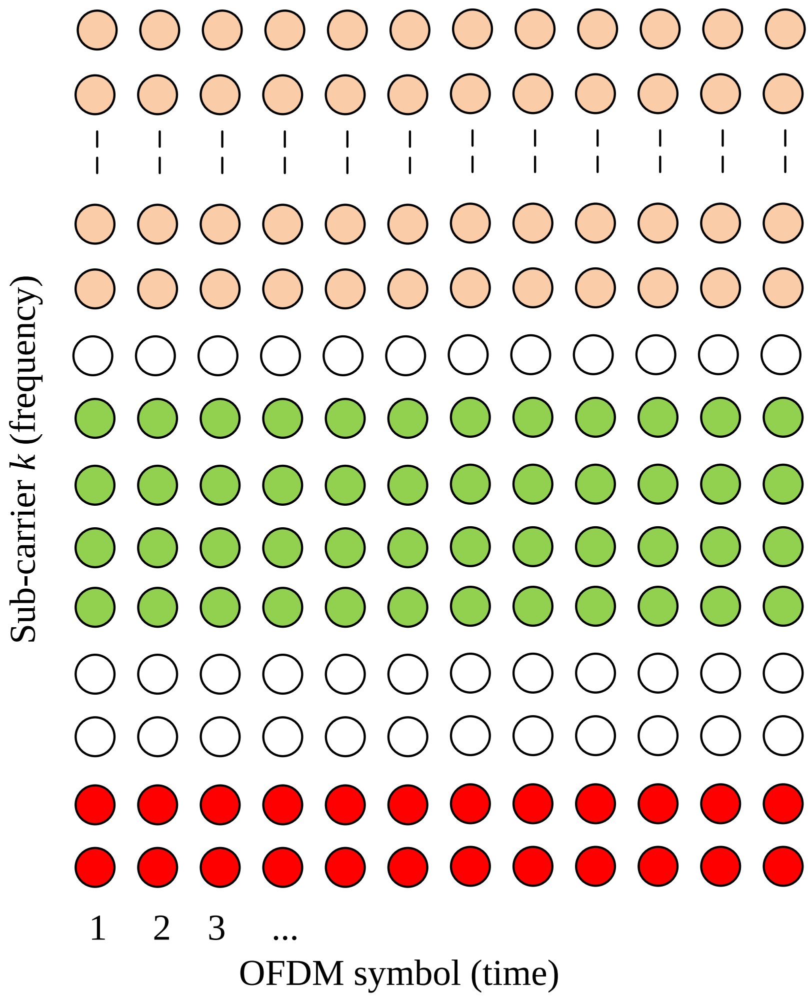


Figure 120—Transmit spectrum mask

Baseband bandwidth W:

$$\text{subcarrier } \#26 = 27 \times 312.5 \text{ KHz} = 8.4375 \text{ MHz or } 32 \times 312.5 \text{ KHz} = 10 \text{ MHz}$$

Orthogonal Frequency Division Multiple Access (OFDMA)



- Each user is allocated with a fixed number of sub-carriers

Physical Layer Parameters for LTE

Channel Bandwidth (MHz)	1.4	3	5	10	15	20
Frame Duration (ms)	10	10	10	10	10	10
Sub carrier spacing (Khz)	15	15	15	15	15	15
Sampling Frequency (Mhz)	1.92	3.84	7.68	15.36	23.04	30.72
FFT Size	128	256	512	1024	1536	2048
Occupied Subcarriers (including DC)	73	181	301	601	901	1201
Guard Subcarriers	55	75	211	423	635	847
Number of Resource Blocks	6	15	25	50	75	100
Occupied Channel Bandwith (Mhz)	1.095	2.715	4.515	9.015	13.515	18.015
DL Bandwidth Efficiency	78.2%	90%	90%	90%	90%	90%
OFDM Symbols for Subframe (for Short CP)	7	7	7	7	7	7
CP Length for Short CP (in us)	5.2 for the first symbol/4.69 for other symbols					

http://www.sharetechnote.com/html/Handbook_LTE_PhyParameter_DL_FDD.html

Advantages of CP-OFDM

- The OFDM spectrum is composed of overlapped narrow subcarriers. This makes efficient usage of frequency spectrum compared to traditional FDM method.
- The OFDM broadband channel is divided into smaller narrowband subchannels. This makes OFDM resistive to frequency selective fading. Moreover OFDM transmit/receive chain uses channel encoder/decoder and interleaver/deinterleaver which help in recovering lost OFDM symbols due to fading.
- OFDM makes use of cyclic prefix to eliminate ISI (Inter Symbol Interference) found in the multipath channel environment. Hence it is robust to multipath fading.
- Channel estimation and equalization has been carried out using known pattern (i.e. preamble) and embedded pilot carriers in a symbol. This is more simpler and efficient compare to channel equalization used in to SC (Single Carrier) system.
- Time offset estimation and correction algorithms are very easy due to correlation technique.
- It is possible to allocate bandwidth as per resource requirements. Hence OFDM is bandwidth scalable technique.

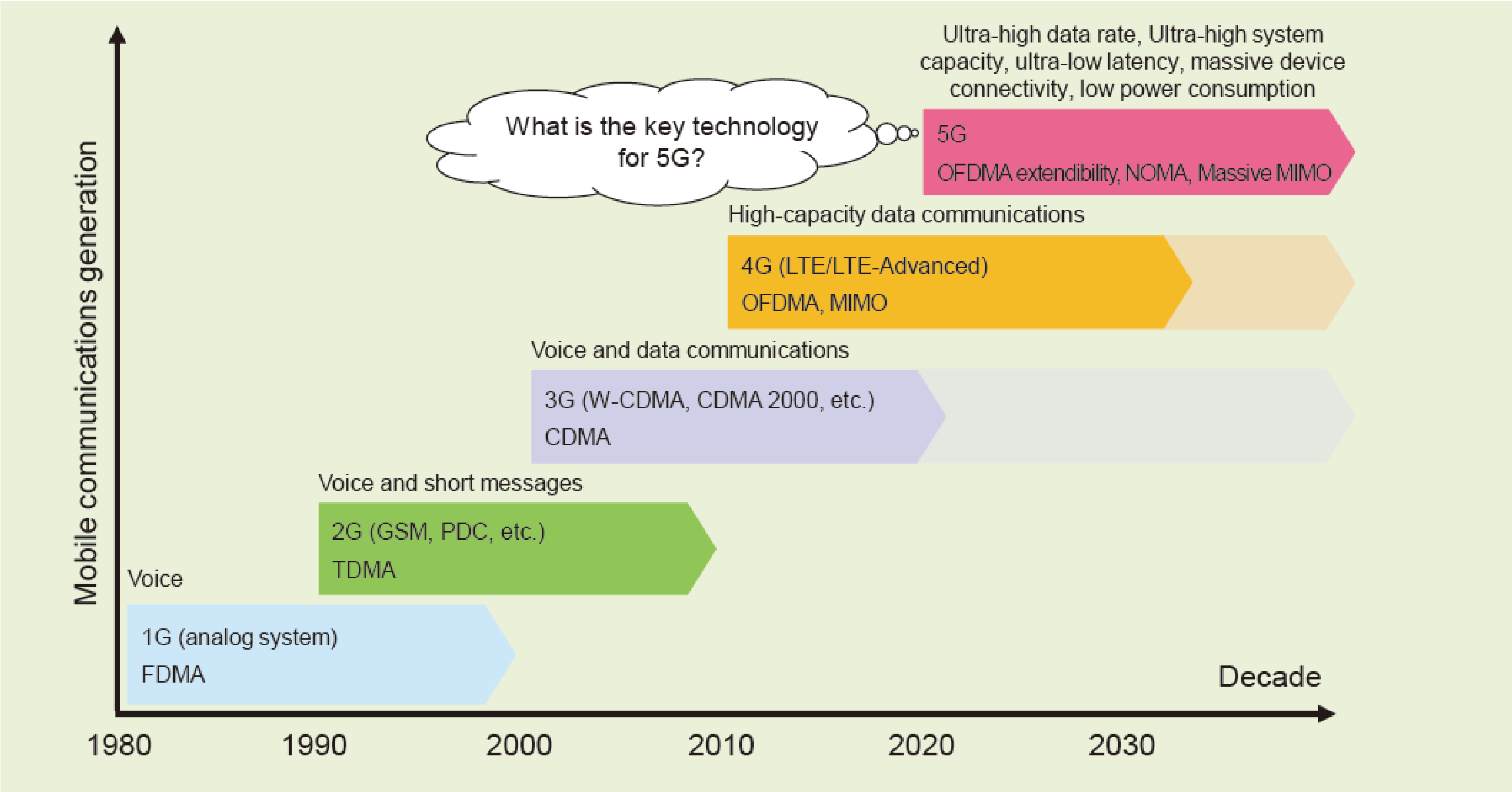
Disadvantages of CP-OFDM

- OFDM signal spectrum has higher peak to average power ratio (PAPR). Due to this, OFDM based transmission system requires radio frequency power amplifier (PA) with higher PAPR.
- It has higher carrier frequency offset due to different LOs (Local Oscillators) and DFT leakage. This requires complex frequency offset correction algorithms at the OFDM receiver.
- It is prone to Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI). This requires time offset and frequency offset correction algorithms.
- When OFDM signal travels through multiple paths, guard interval is required to avoid ISI errors due to timing offsets.

Part II：
New Multi-Carrier Waveforms

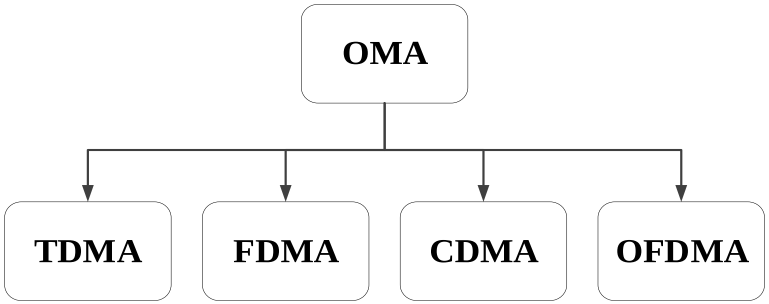
- Universal Filter Multi-Carrier (UFMC) Waveform
- Filter Bank Multi-Carrier (FBMC) Waveform
- Generalized Frequency Division Multiplexing (GFDM) Waveform

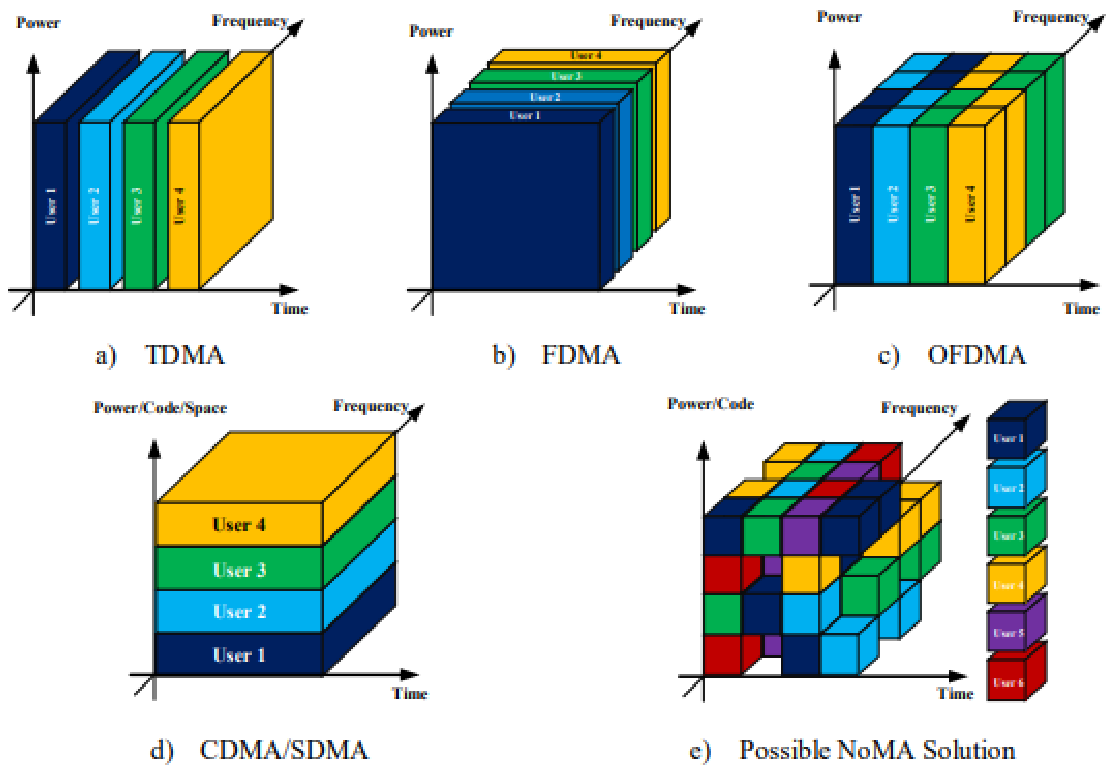
The 5G Communication



- **Enhanced Mobile Broadband (eMBB)**
- **Ultra-reliable and Low Latency Communications (URLLC)**
- **Massive Machine Type Communications (mMTC)**

Orthogonal Multiple Access (OMA)





Y. Chen, A. Bayesteh, Y. Wu, B. Ren, S. Kang, S. Sun, Q. Xiong, C. Qian, B. Yu, Z. Ding, S. Wang, S. Han, X. Hou, H. Lin, R. Visoz, and R. Razavi, “Towards the standardization of non-orthogonal multiple access for next generation wireless networks,” IEEE Commun. Mag., vol. 56, no. 3, pp. 19–27, Mar. 2018.

The Problem with CP-OFDM

- While windowing and filtering can indeed reduce the out-of-band (OOB) emissions of conventional OFDM, filter bank multicarrier modulation (FBMC) with offset quadrature amplitude modulation (OQAM) still performs much better, as shown in Fig. 3.1.

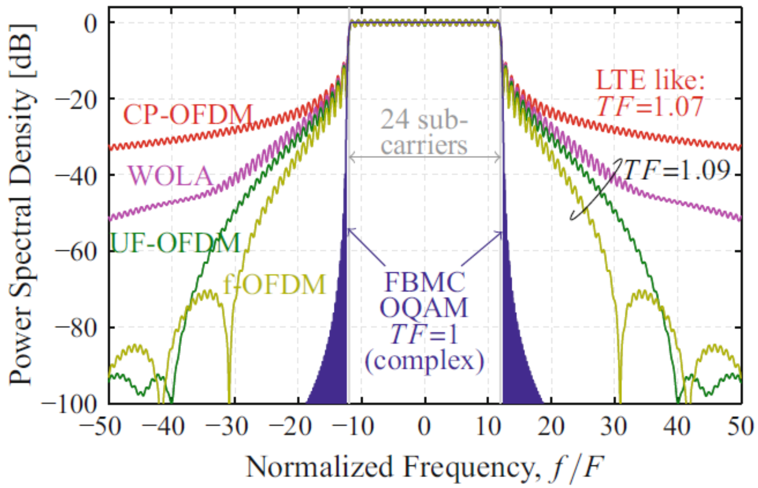


Fig. 3.1 FBMC has much better spectral properties compared with CP-OFDM. Windowing (WOLA) and filtering (UF-OFDM, f-OFDM) can improve the spectral properties of CP-OFDM. However, FBMC still performs much better and has the additional advantage of a maximum symbol density, $TF = 1$ (complex). ©2017 IEEE, [41]

R. Nissel, S. Schwarz, M. Rupp, "Filter bank multicarrier modulation schemes for future mobile communications," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 8, Aug. 2017.

New Waveforms

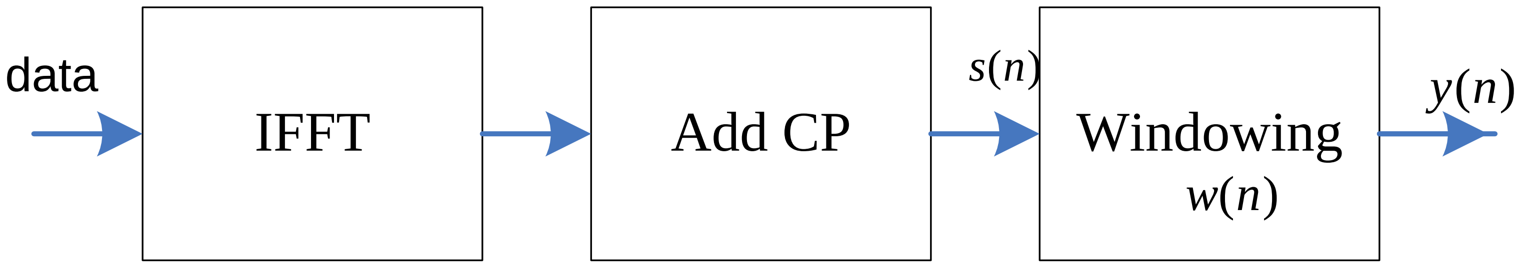
Table III *The summary and comparison of three new waveforms*

Waveforms \ Features	Key technologies	Advantages	Disadvantages
FBMC	multi-carrier filtering	(1) the flexible control of the degree of overlap between each sub- carrier (2) time-frequency efficiency improvement by about 10% in case of very short packets (3) low synchronization requirement	(1) large interference between sub-carriers (2) the long filter length, high complexity
UFMC	block-wise filtering	(1) the short filter length, Low complexity (2) time-frequency efficiency improvement by about 10% in any case (3) small interference between sub-carriers	(1) higher synchronization re- quirement than CP-OFDM
GFDM	Tx-filtering FFT-based equalization	(1) lower PARP (2) use of scattered spectrum resources (3) ultra-low out-of-band radiation	(1) receiver is rather complex

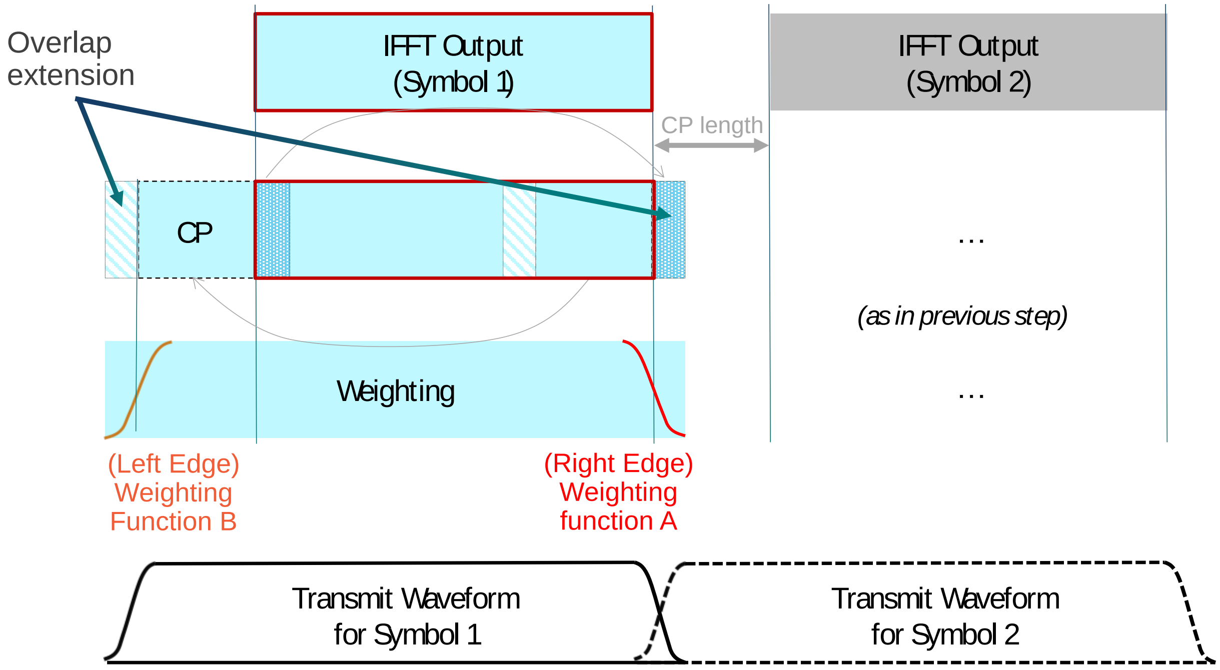
Y. Tao, L. Liu, S. Liu, and Z. Zhang, “A survey: several technologies of non-orthogonal transmission for 5G,” *China Communications*, vol. 12, no. 10, pp. 1-15, 2015.

The Universal Filter Multi-Carrier (UFMC) Waveform

CP-OFDM with Weighted Overlap and Add (WOLA) (1/2)

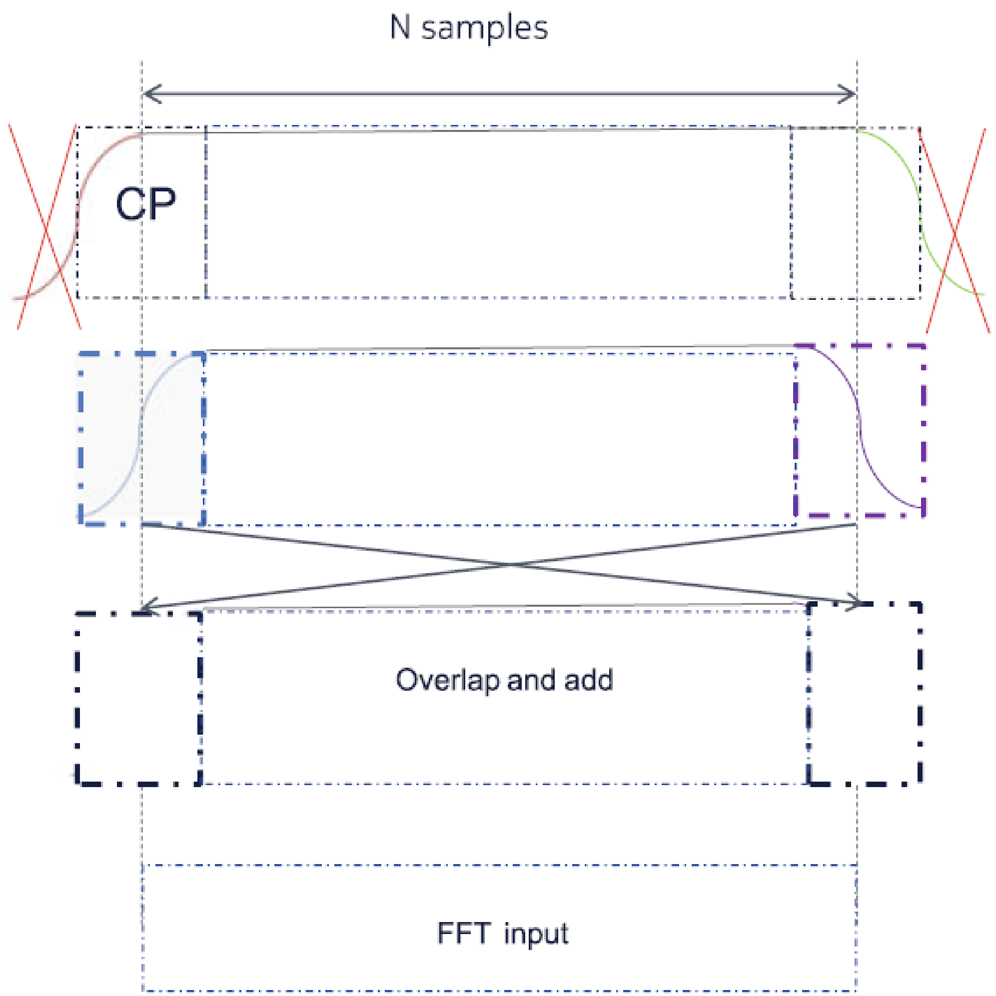


$$y(n) = s(n) \cdot w(n)$$



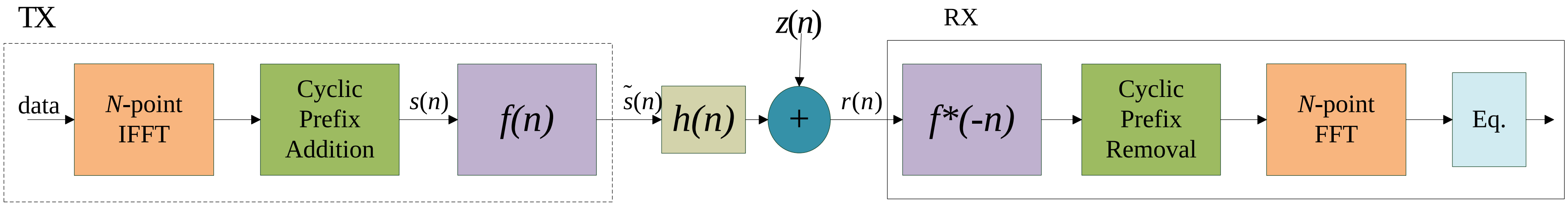
Qualcomm, R1-162199 Feasibility of Mixing Numerology in an OFDM System RAN1#84bis(2016)

CP-OFDM with Weighted Overlap and Add (WOLA) (2/2)



R. Zayani, Y. Medjahdi, H. Shaiek, and D. Roviras, “WOLA-OFDM: A Potential Candidate for Asynchronous 5G,” in *Proc. IEEE Globecom Workshops (GC Wkshps),* Feb. 2017, pp. 1-5.

Filtered-OFDM (1/3)



- Filtered OFDM (f-OFDM) is a 5G candidate waveform based on sub-band filtering by

$$f(n) = w(n)\,p_B(n),$$

where $p_B(n)$ is a sinc impulse response with bandwidth B in the frequency domain equal to the sub-band allocation size. Also $w(n)$ over duration $T_w = \frac{T_u}{2}$ is the windowing mask to have smooth transitions.

M. Vaezi, H. Vincent Poor, and Z. Ding, *Multiple Access Techniques for 5G Wireless Networks and Beyond. Springer*, 2018.

Filtered-OFDM (2/3)

- For practical implementation, the sine function is soft-truncated with different window functions:

1. Hanning window

$$w(t)=\begin{cases}0.5[1+\cos(2\pi|t|/T_w)], & |t|\le\frac{T_w}{2}\\ 0, & |t|>\frac{T_w}{2}\end{cases}.$$

2. Root-raised-cosine (RRC) window

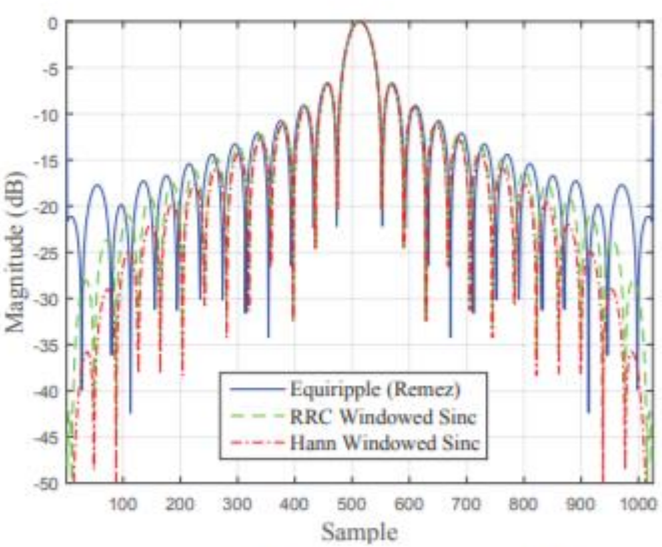


Figure 4. Impulse response of different filters.

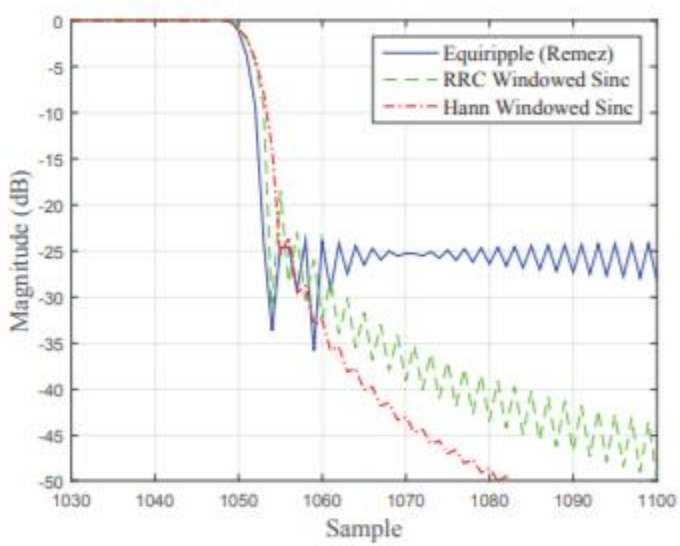
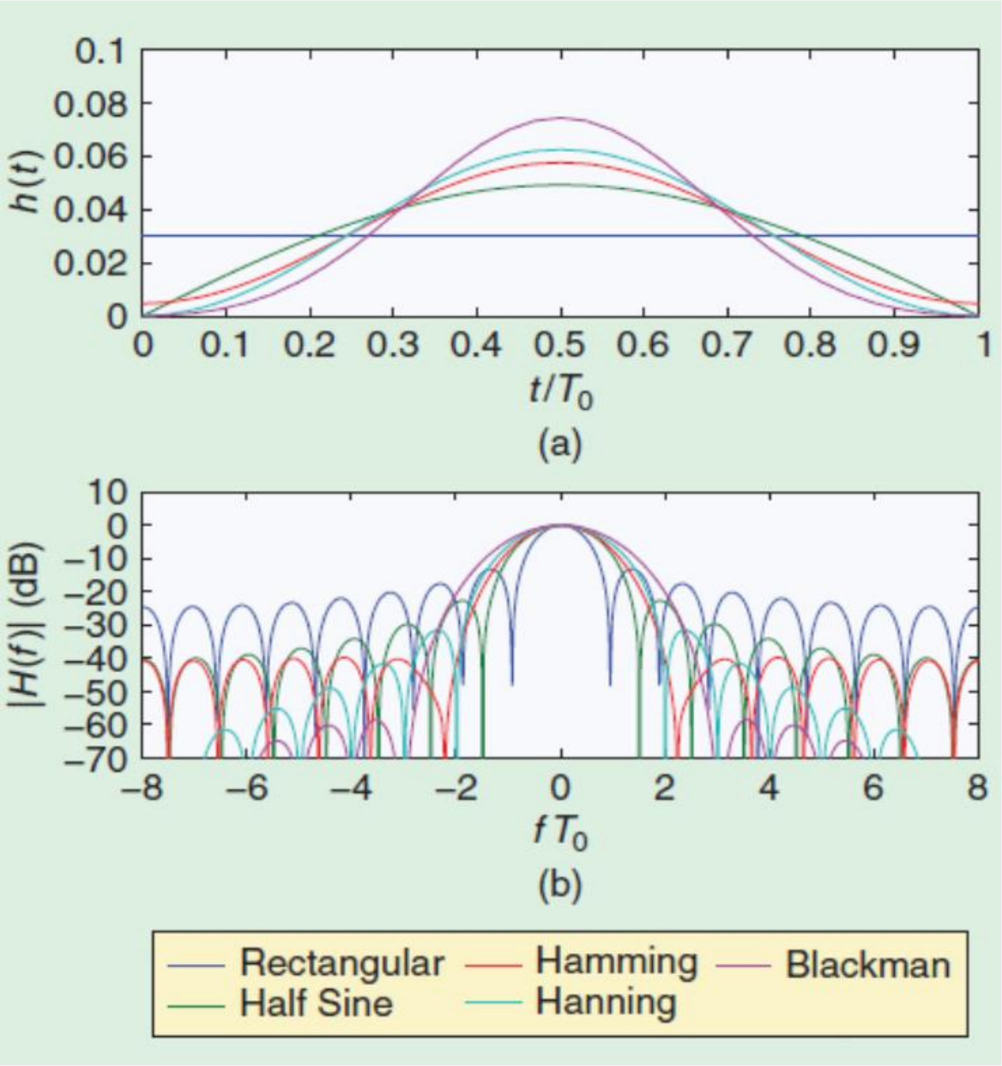


Figure 5. Frequency response of different filters.

X. Zhang, M. Jia, L. Chen, J. Ma, and J. Qiu, "Filtered-OFDM-enabler for flexible waveform in the 5th generation cellular networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Apr. 2015, pp. 1–6.

Filtered-OFDM (3/3)

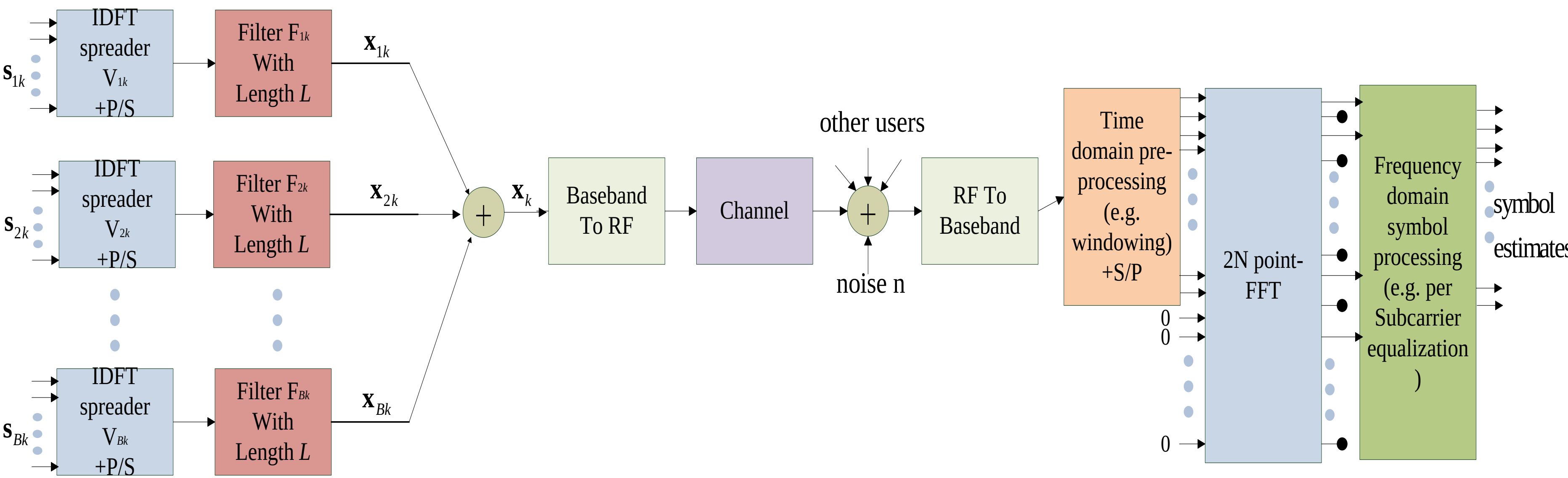


Hamming, Hanning, and Blackman windows offer lower side lobes at the cost of a wider main lobe.

B. Farhang-Boroujeny, "OFDM Versus Filter Bank Multicarrier," *Signal Processing Magazine*, *IEEE* , vol.28, no.3, pp.92-112, May 2011.

Universal Filtered Multi-Carrier (UFMC) (1/5)

- UF-OFDM is a 5G candidate waveform, also known as universal filtered-multi-carrier (UFMC), where blocks of subcarriers (sub-bands) are filtered.



F. Schaich, Thorsten Wild, Y. Chen , "Waveform contenders for 5G - suitability for short packet and low latency transmissions" in *Proc. IEEE Vehicular Technology Conference (VTC Spring)*, May. 2014, pp. 18-21.

Universal Filtered Multi-Carrier (UFMC) (2/5)

- IFFT symbols are generated in the same way as legacy CP-OFDM. Instead of CP, a guard interval (GI) filled with zeros is introduced between the IFFT symbols to prevent ISI due to transmit filter delay.
- Dolph–Chebychev filters are optimal in the sense that for a given side lobe level (SLL) the main lobe width is minimized. They are adjustable by the tuning parameter for the side lobe attenuation (SLA) as well as by the filter length N.

$$W(k)=\frac{\cos\left\{N\cos^{-1}\left[\beta\cos\left(\frac{\pi k}{N}\right)\right]\right\}}{\cosh\left[N\cosh^{-1}(\beta)\right]},\qquad k=0,1,2,\cdots,N-1$$

$$\beta=\cosh\left[\frac{1}{N}\cosh^{-1}\left(10^{\alpha}\right)\right]\ \text{with sidelobe in db} = \text{-}20\alpha$$

$$w_0(n)=\frac{1}{N}\sum_{k=0}^{N-1}W(k)e^{j2\pi kn/N},\ -N/2\le n\le N/2$$

$$w(n)=w_0(n-(N-1)/2),\ 0\le n\le N-1$$

https://en.wikipedia.org/wiki/Window_function#Dolph%E2%80%93Chebyshev_window

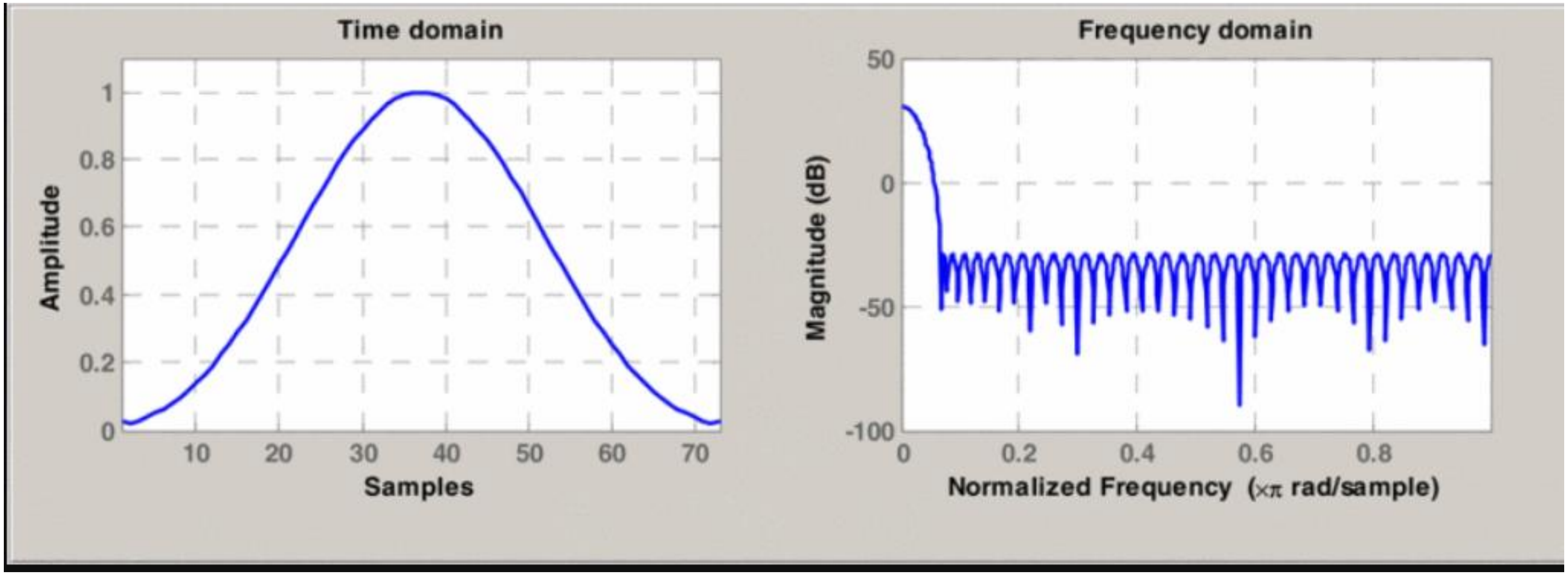
Universal Filtered Multi-Carrier (UFMC) (3/5)

- Dolph–Chebychev filters are optimal in the sense that for a given side lobe level (SLL) the main lobe width is minimized. They are adjustable by the tuning parameter for the side lobe attenuation (SLA) as well as by the filter length L.
- For example, on the one hand, in high ICI use cases with asynchronous transmission, it makes sense to use filters which are longer than the guard interval $L > N_{GI}$, at the price of higher vulnerability to delay spreads .
- On the other hand, in environments with high delay spread, a shorter filter length is used to protect against ISI. The SLA controls the trade-off between the main lobe width and the SLL.

M. Vaezi, H. Vincent Poor, and Z. Ding, *Multiple Access Techniques for 5G Wireless Networks and Beyond. Springer*, 2019.

Universal Filtered Multi-Carrier (UFMC) (4/5)

The side lobes of the Dolph-Chebyshev window transform are equal height, they are often called ``ripple in the stop-band'‘



Dolph-Chebyshev window (L=73,α_{SLA} =60 dB).

Y. Chen, F. Schaich and T. Wild, “Waveform contenders for 5G - suitability for short packet and low latency transmissions” in *Proc. IEEE Vehicular Technology Conference (VTC Spring)*, May. 2014, pp. 18-21.

Universal Filtered Multi-Carrier (UFMC) (5/5)

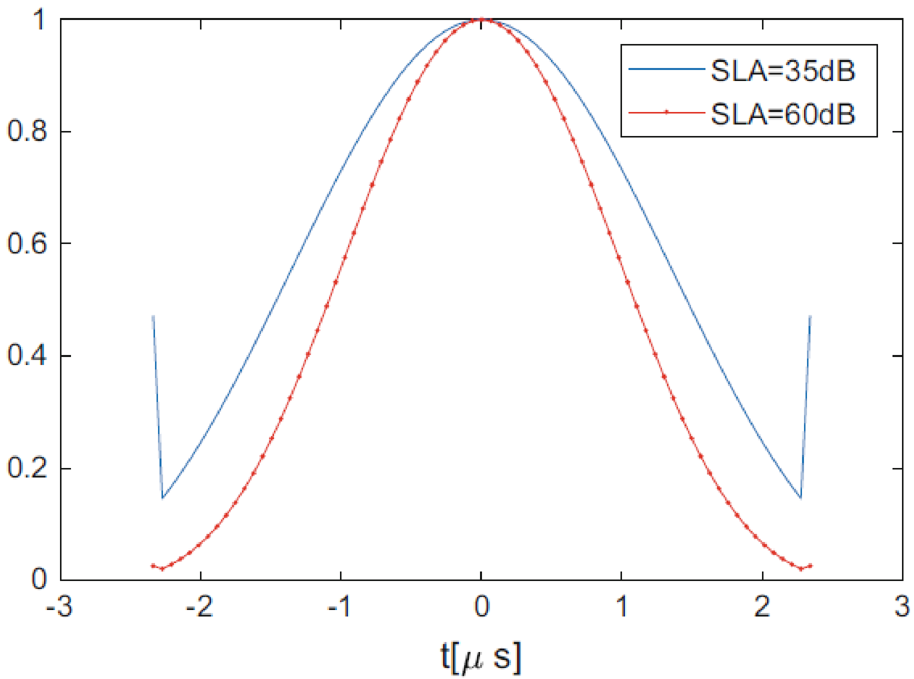


Fig. 2.11 Time domain impulse response for Dolph–Chebyshev filter with $L = 72$ and $SLA = 35, 60$ dB

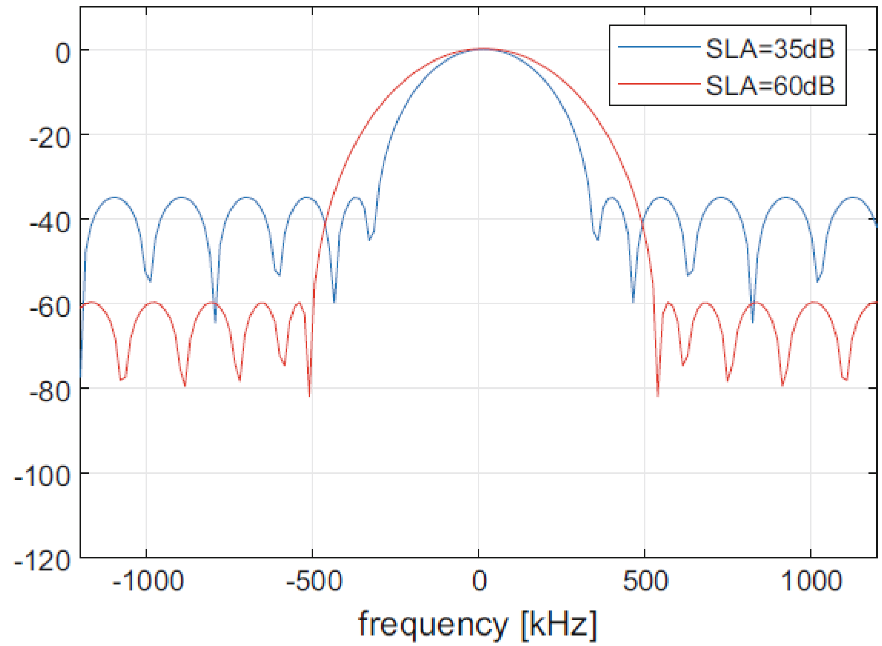
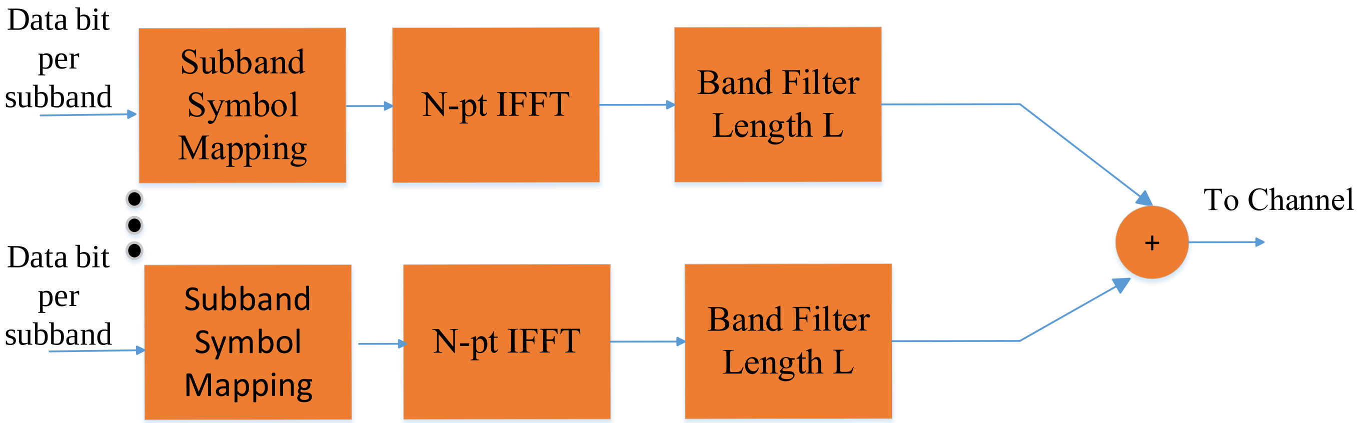
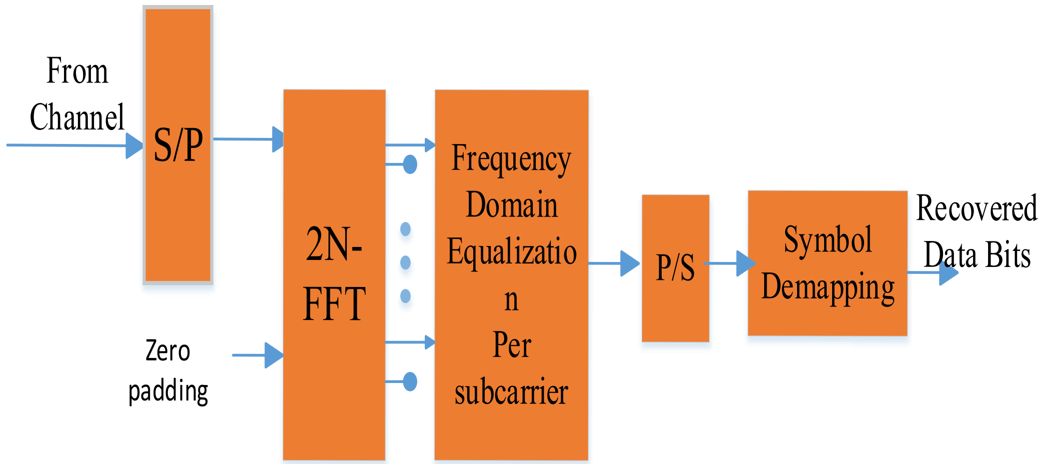


Fig. 2.12 Frequency domain response for Dolph–Chebyshev filter with $L = 72$ and $SLA = 35, 60$ dB

M. Vaezi, H. Vincent Poor, and Z. Ding, *Multiple Access Techniques for 5G Wireless Networks and Beyond. Springer*, 2019.

Matlab simulation (1/2)

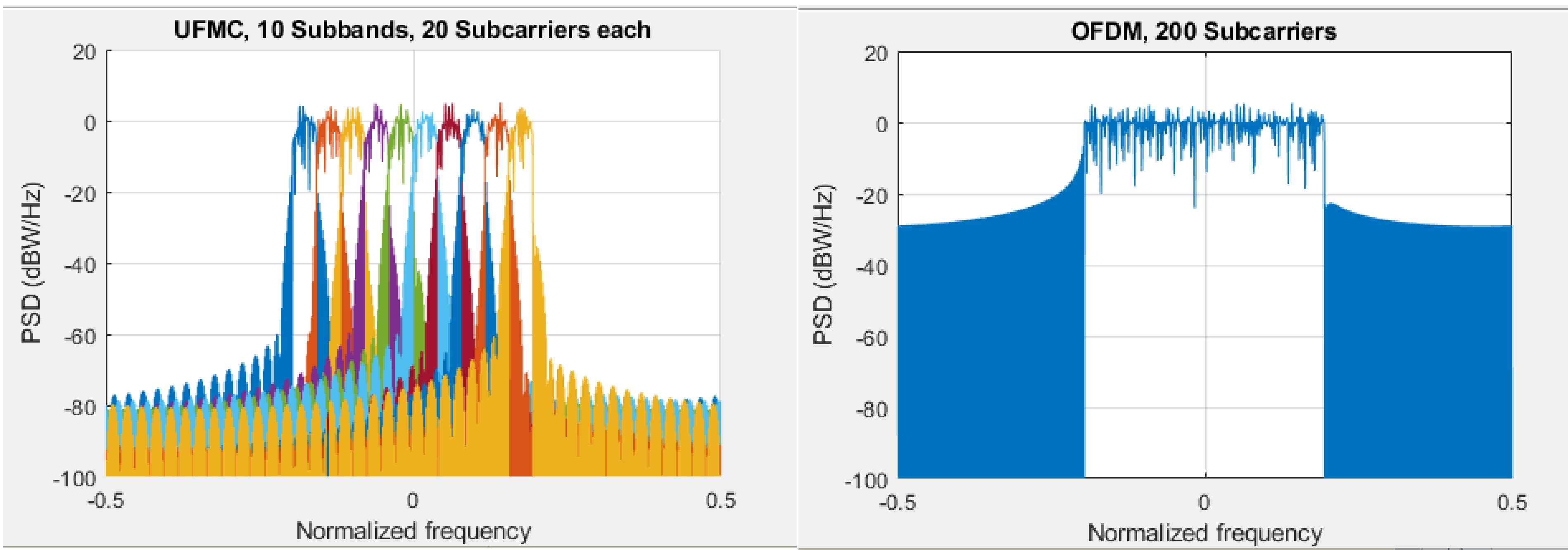




Simulation Parameter	Parameter Value
UFMC system	
The number of subbands	10
The number of subcarrier in one subband	20
Subband Offset	156
Filter length	43
sidelobe attenuation	40
Size of IFFT	512
OFDM system	
The number of subcarriers	200
Subband Offset	156
Size of IFFT	512

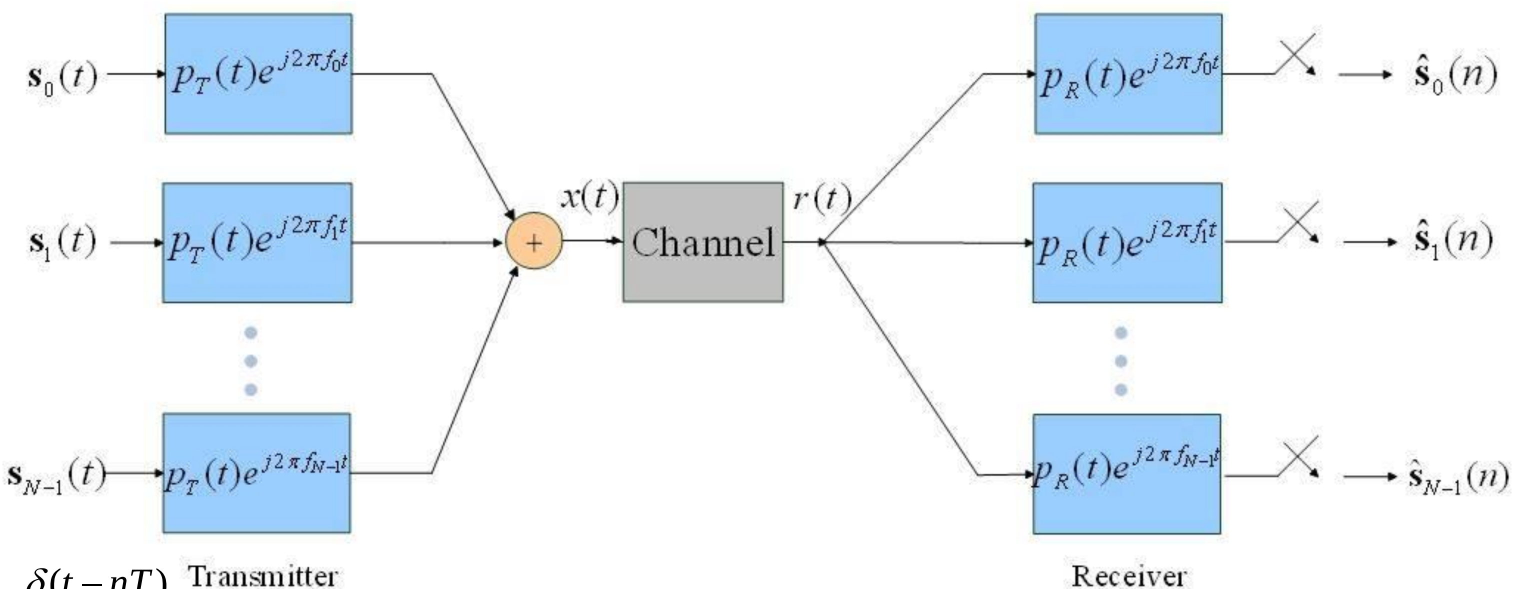
Matlab Simulation (2/2)

The PSD of UFMC and OFDM



The Filter Bank Multi-Carrier (FBMC) Waveform

FBMC System (1/3)



$$s_k(t) = \sum_n A_{k,n} \delta(t - nT)$$

- Mathematically, the transmitted signal, $x(t)$, of a multicarrier system in the time domain can be expressed as

$$x(t) = \sum_n \sum_{k=0}^{N-1} g_{k,n}(t) A_{k,n}$$

where $A_{k,n}$ denotes the transmitted symbol at subcarrier position k and time position n, and is chosen from a symbol alphabet, usually a QAM or a PAM signal constellation.

B. Farhang-Boroujeny, “OFDM versus filter bank multicarrier,” *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 92-112, May 2011.

FBMC System (2/3)

- The basis pulse $g_{k,n}(t)$ is defined by

$$g_{k,n}(t) = p(t - nT)e^{j2\pi kF(t-nT)}$$

and is, essentially, a time and frequency shifted version of prototype filter *p*(*t*), with *T* denoting the time spacing and *F* the frequency spacing (subcarrier spacing).

- After transmission over a channel, the received symbols are decoded by projecting the received signal, *r*(*t*), onto the basis pulses, $g_{k,n}(t)$, that is,

$$y_{k,n} = \langle r(t), g_{k,n}(t) \rangle = \int_{-\infty}^{\infty} r(t) g_{k,n}^{*}(t) \mathrm{d}t$$

FBMC System (3/3)

- Multicarrier systems are mainly characterized by prototype filter $p(t)$ as well as time spacing T and frequency spacing F, so that the ambiguity function,

$$A(\tau,\upsilon)=\int_{-\infty}^{\infty} p(t-\frac{\tau}{2})p^{*}(t+\frac{\tau}{2})e^{j2\pi\upsilon t}dt$$

captures the main properties of a multicarrier system in a compact way.

- The projection of the transmitted basis pulses $g_{k_1,n_1}(t)$ onto the received basis pulses $g_{k_2,n_2}(t)$ can then be expressed by the ambiguity function according to

$$\left\langle g_{k_1,n_1}(t),g_{k_2,n_2}(t)\right\rangle=\underbrace{e^{-j\pi TF(k_1+k_2)(n_1-n_2)}}_{\text{only a phase shift}}\underbrace{A(T(n_1-n_2),F(k_1-k_2))}_{\text{ambiguity function}}$$

M. Vaezi, H. Vincent Poor, and Z. Ding, *Multiple Access Techniques for 5G Wireless Networks and Beyond*. Springer, 2019.

Prototype Filter (1/3)

- There exist some fundamental limitations of multicarrier systems, as formulated by the Balian–Low theorem, which states that it is mathematically impossible that the following four desired properties are fulfilled at the same time [7]:

1. Maximum symbol density, $$\frac{1}{TF} = 1$$
2. Time-localization, $$\sigma_t = \sqrt{\int_{-\infty}^{\infty} (t-\bar{t})^2 \left|p(t)\right|^2 dt} < \infty$$
3. Frequency-localization, $$\sigma_f = \sqrt{\int_{-\infty}^{\infty} (f-\bar{f})^2 \left|P(f)\right|^2 df} < \infty$$
4. Orthogonality, $$\langle g_{l_1,k_1}(t), g_{l_2,k_2}(t) \rangle = \delta_{(l_1-l_2),(k_1-k_2)}$$ $$A(T(k_1-k_2), F(l_1-l_2)) = \delta_{(l_1-l_2),(k_1-k_2)}$$

R. Nissel, S. Schwarz, M. Rupp, "Filter bank multicarrier modulation schemes for future mobile communications," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 8, Aug. 2017.

Prototype Filter (2/3)

	Maximum symbol density	Time-localization	Frequency-localization	Orthogonality
OFDM (no CP)	yes	yes	no	yes
FBMC/QAM	no	yes	yes	yes
FBMC/OQAM	yes	yes	yes	Real only

R. Nissel, S. Schwarz, M. Rupp, "Filter bank multicarrier modulation schemes for future mobile communications," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 8, Aug. 2017.

Prototype Filter (3/3)

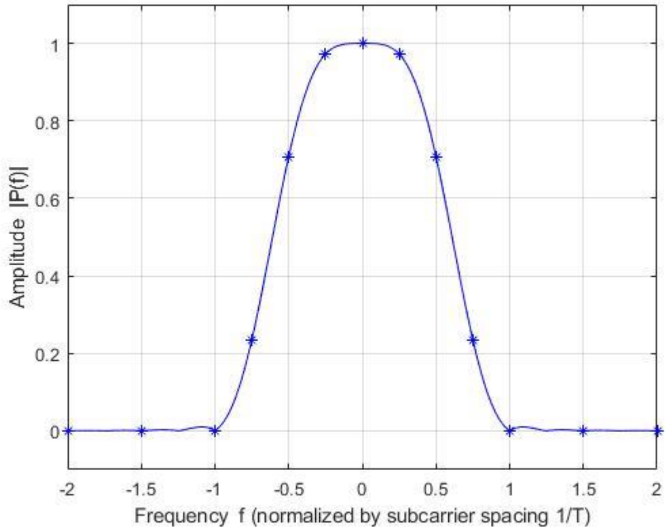
- A prominent filter is the PHYDYAS prototype filter [6]

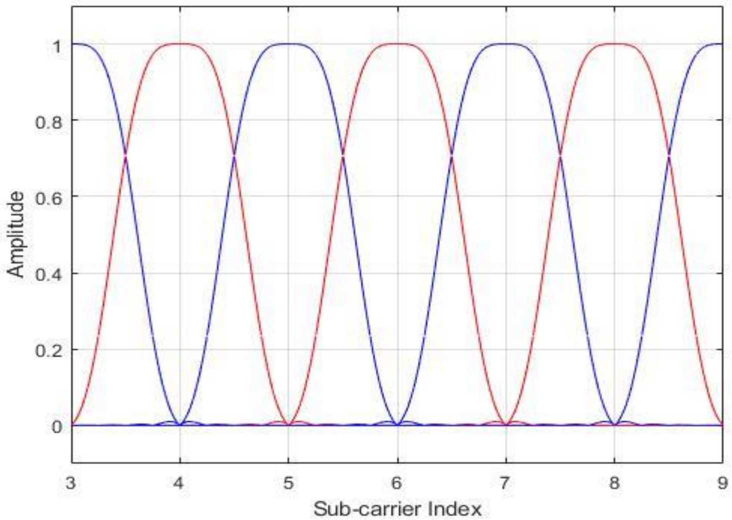
$$P(f)=\sum_{k=-(K-1)}^{K-1} H_k \frac{\sin(\pi(f-\frac{k}{NK})NK)}{NK\sin(\pi(f-\frac{k}{NK}))}$$

K	H_0	H_1	H_2	H_3	σ^2 (dB)
2	1	$\sqrt{2}/2$	-	-	-35
3	1	0.911438	0.411438	-	-44
4	1	0.971960	$\sqrt{2}/2$	0.235147	-65

Orthogonal : $T=T_0$; $F=2/T_0$ $\rightarrow TF=2$

Localization : $\sigma_t=0.2745T_0$; $\sigma_f=0.328/T_0$



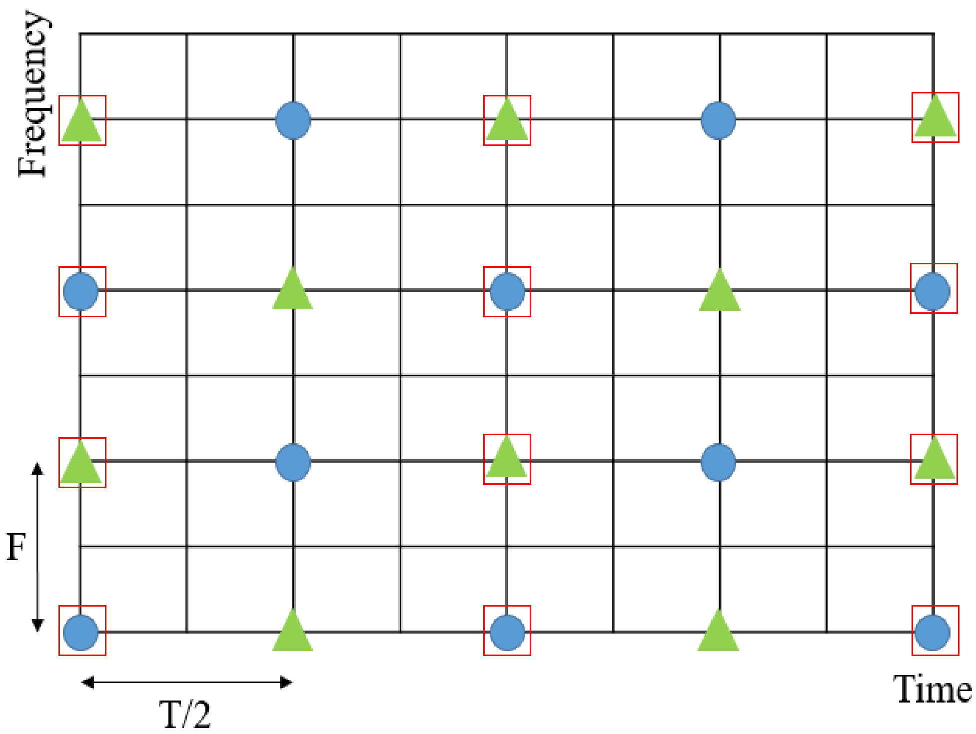


M. Bellanger et al., “FBMC physical layer: a primer,” June 2010. [Online] Available: http://www.ict-phydyas.org

Offset QAM (OQAM) (1/2)

- In FBMC systems, any kind of modulation can be used, whenever the sub-channels are separated.
- For example, if only the sub-channels with even (odd) index are exploited, there is no overlap and QAM modulation can be employed.
- However, if full speed is seeked, all the sub-channels must be exploited and a specific modulation is needed to cope with the frequency domain overlapping of the neighbouring sub-channels.
- Then, the strategy to reach full capacity is the following:
  - Double the symbol rate and, for each sub-channel, use alternatively the real and the imaginary part of the iFFT.
  - This way, the real and the imaginary part of a complex data symbol are not transmitted simultaneously as in OFDM, but the imaginary part is delayed by half the symbol duration.
- This is the so-called offset quadrature amplitude modulation (OQAM) and the term ‘offset’ reflects the time shift of half the inverse of the sub-channel spacing between the real part and the imaginary part of a complex symbol.

Offset QAM (OQAM) (2/2)



- Real part (blue circle)
- Imaginary part (green triangle)
- Complex QAM symbol (red square)

$F = 1/T$: subcarrier spacing

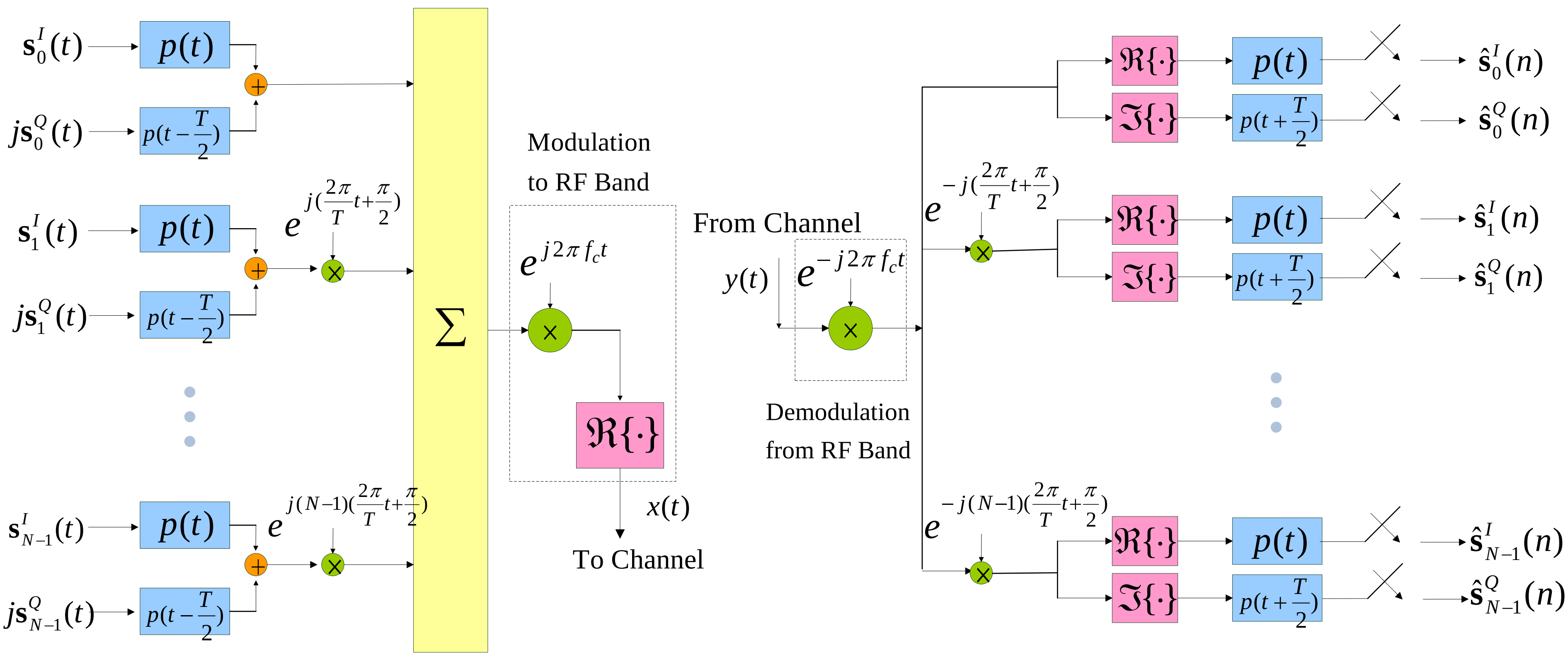
T: OFDM/QAM symbol duration

Symbol density

OFDM/QAM (without CP) : $1/TF = 1$

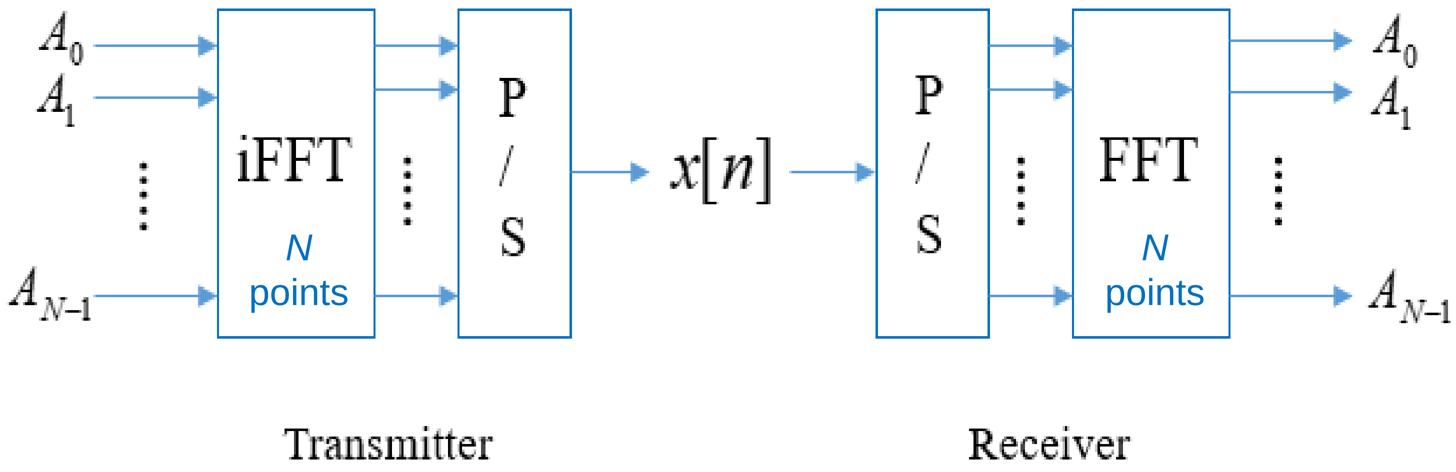
FBMC/OQAM : $\frac{1}{(T/2)F} = 2$ => 1 (complex)

FBMC/OQAM System



B. Farhang-Boroujeny, “OFDM versus filter bank multicarrier,” *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 92-112, May 2011.

Fast Fourier Transform Architecture (1/2)



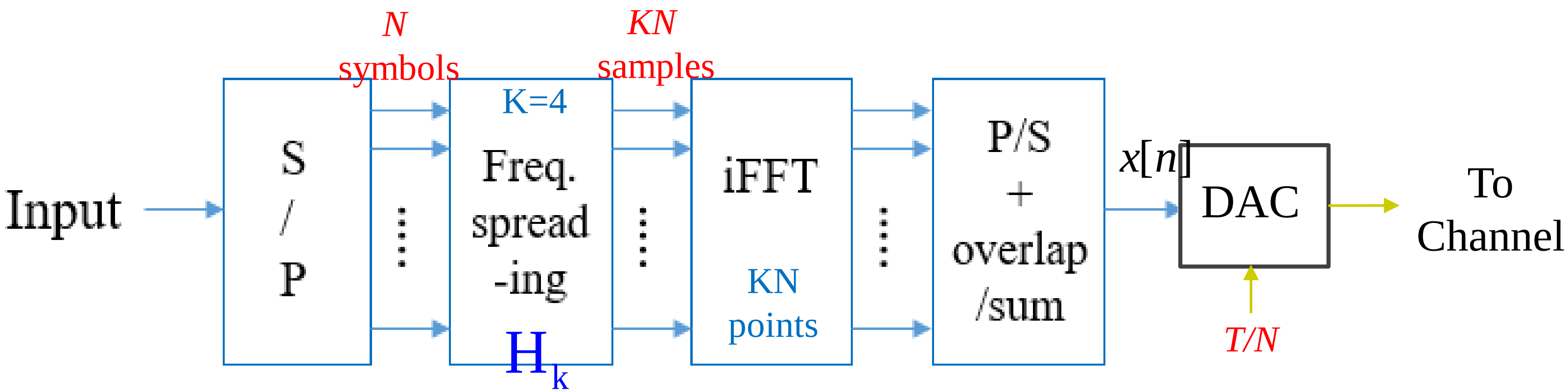
- In the presence of a channel with multipath propagation, due to the channel impulse response, the multicarrier symbols overlap at the receiver input and it is no more possible to demodulate with just the FFT, because inter-symbol interference has been introduced and the orthogonality property of the carriers has been lost.

Fast Fourier Transform Architecture (2/2)

1. Extend the symbol duration by a guard time exceeding the length of the channel impulse response and still demodulate with the same FFT. The scheme is called OFDM.

2. Keep the timing and the symbol duration as they are, but add some processing to the FFT. The scheme is called FBMC, because this additional processing and the FFT together constitute a bank of filters.

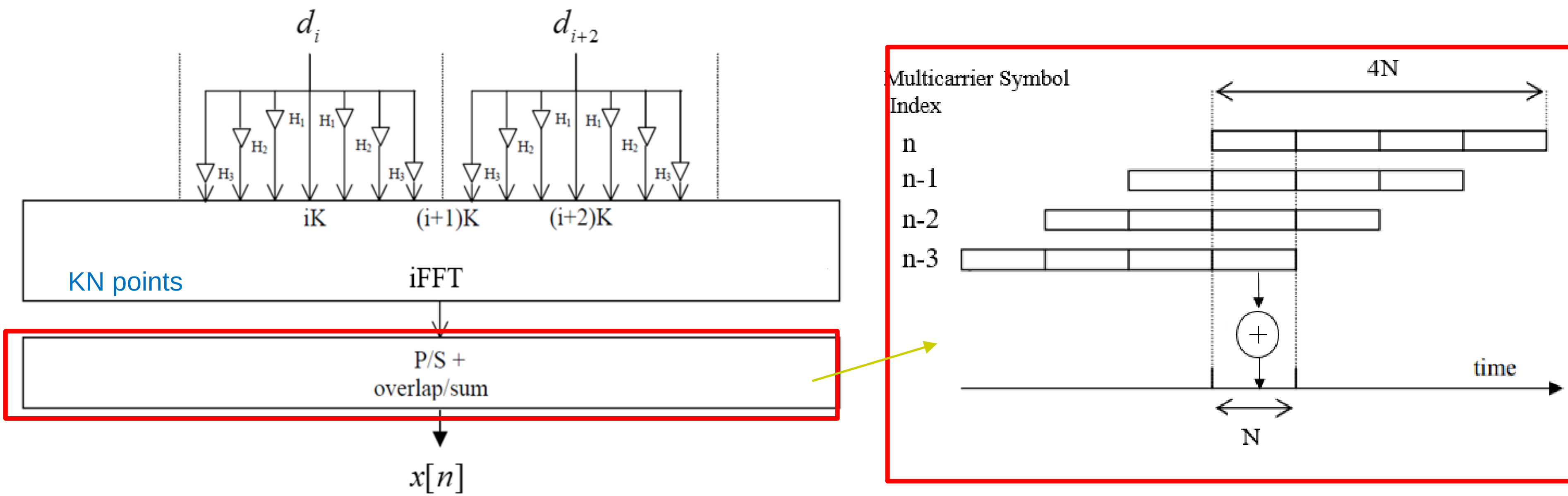
Extended FFT Method (1/3)



- The filter bank in the transmitter can be implemented as follows
- an iFFT of size *KN* is used, to generate all the necessary carriers,
- a particular data element after multiplication by the filter frequency coefficients, is fed to the *2K-1* inputs of the iFFT.
- Practically, the data element is spread over several iFFT inputs and the operation can be called "weighted frequency spreading".

M. Bellanger et al., "FBMC physical layer: a primer," June 2010. [Online] Available: http://www.ict-phydyas.org

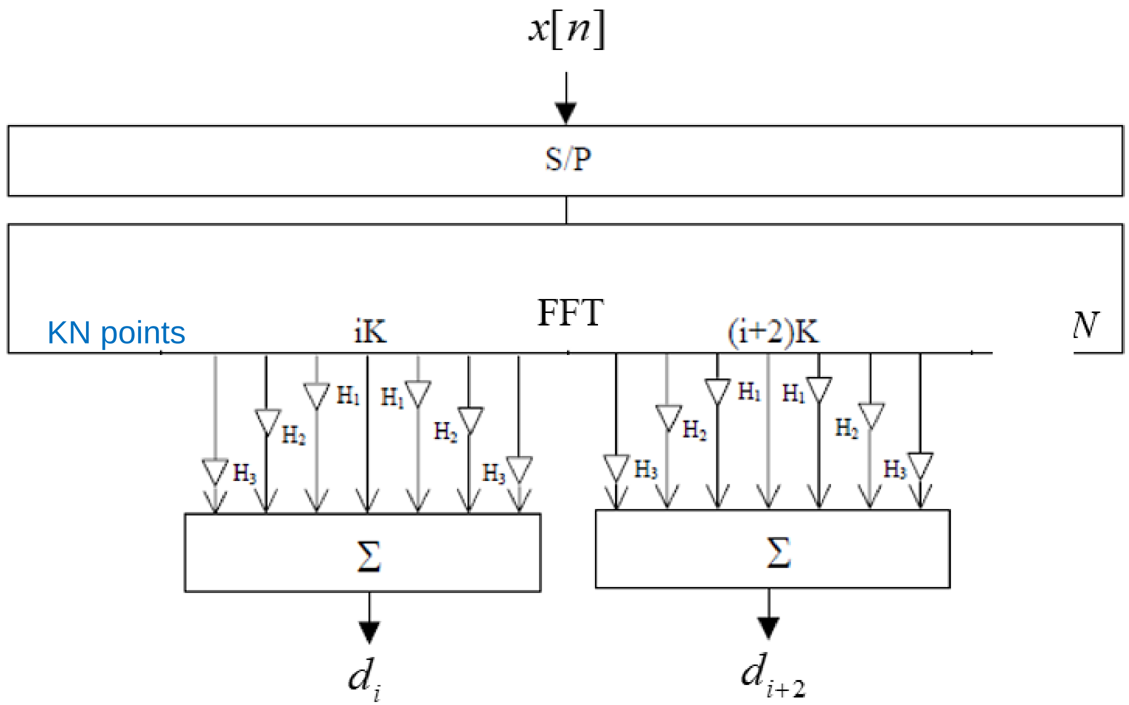
Extended FFT Method (2/3)



K	H_0	H_1	H_2	H_3	σ^2 (dB)
2	1	$\sqrt{2}/2$	-	-	-35
3	1	0.911438	0.411438	-	-44
4	1	0.971960	$\sqrt{2}/2$	0.235147	-65

M. Bellanger, “FS-FBMC: an alternative scheme for filter bank based multicarrier transmission,” in *Proc. The 5th Int. Symp. Commun., Control and Signal Processing (ISCCSP)*, May 2012, pp. 1-4.

Extended FFT Method (3/3)



- The implementation of the receiver is based on an extended FFT of size *KN*.
- At the output of the FFT, the data elements are recovered with the help of a weighted despreading operation.
- In fact, the data recovery rests on the following property of the frequency coefficients of the Nyquist filte $\frac{1}{K}\sum_{k=-K+1}^{K-1}\left|H_k\right|^2=1$

Matlab Simulation

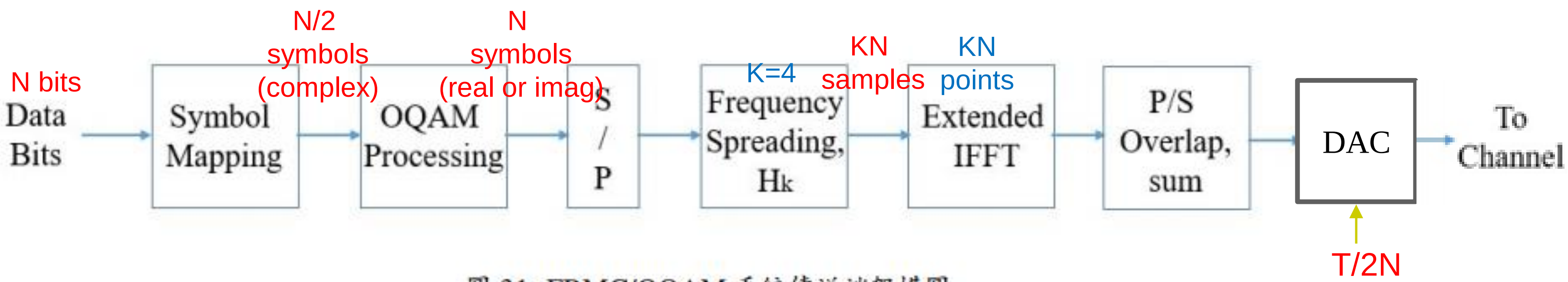


圖 31: FBMC/OQAM 系統傳送端架構圖

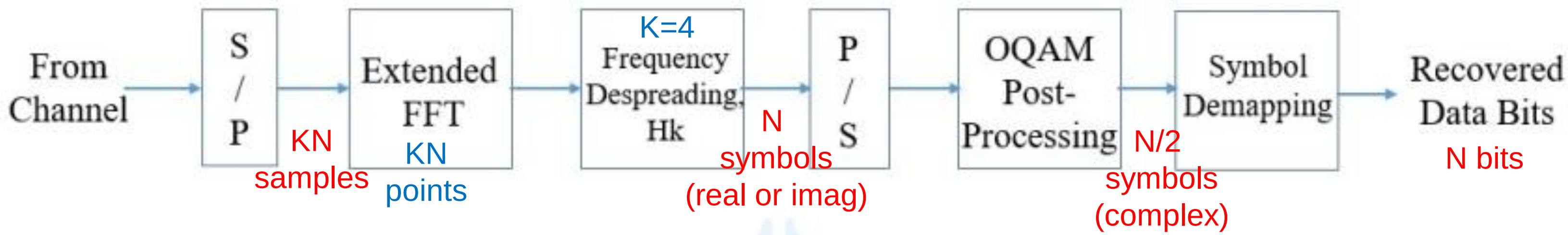
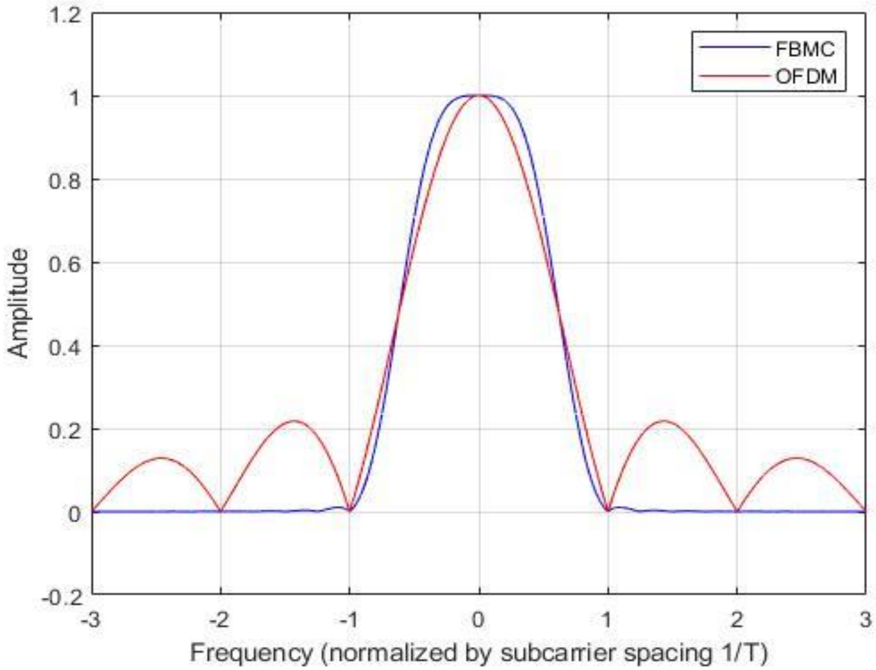


圖 34: FBMC/OQAM 系統接收端架構圖

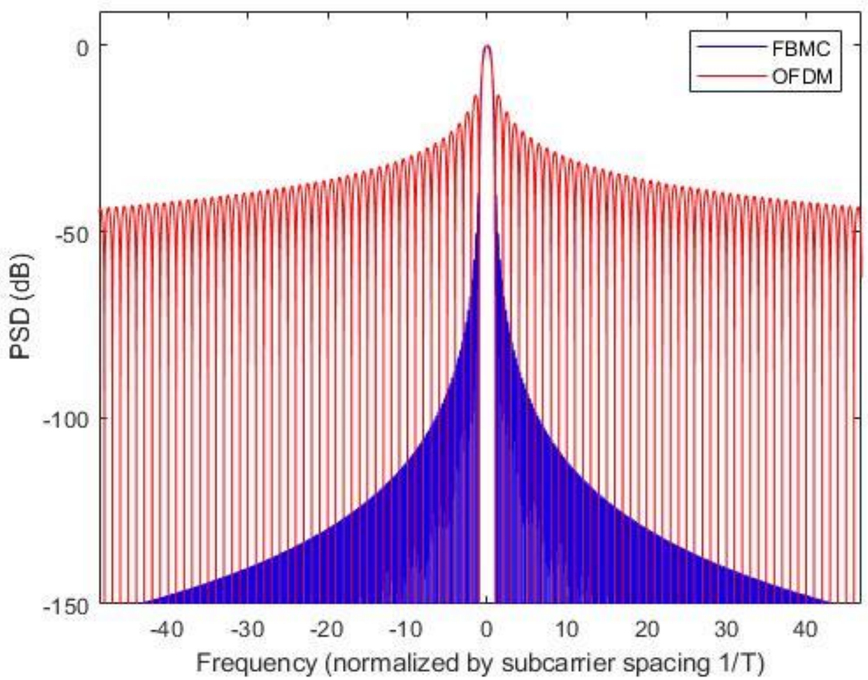
	OFDM (no CP)	FBMC
Number of subcarriers (N)	1024	1024
Number of data subcarriers	600	600
Guard bands on both sides	212	212
Constellation mapping	4-QAM	4-OQAM
Overlapping factor K	-	4
FFT size	1024 (N)	4096 (KN)

Mathworks. FBMC vs. OFDM Modulation - MATLAB & Simulink Example, June 2017. [Online] Available: https://ww2.mathworks.cn/help/comm/examples/fbmc-vs-ofdm-modulation.html?s_tid=srchtitle

Prototype Filter

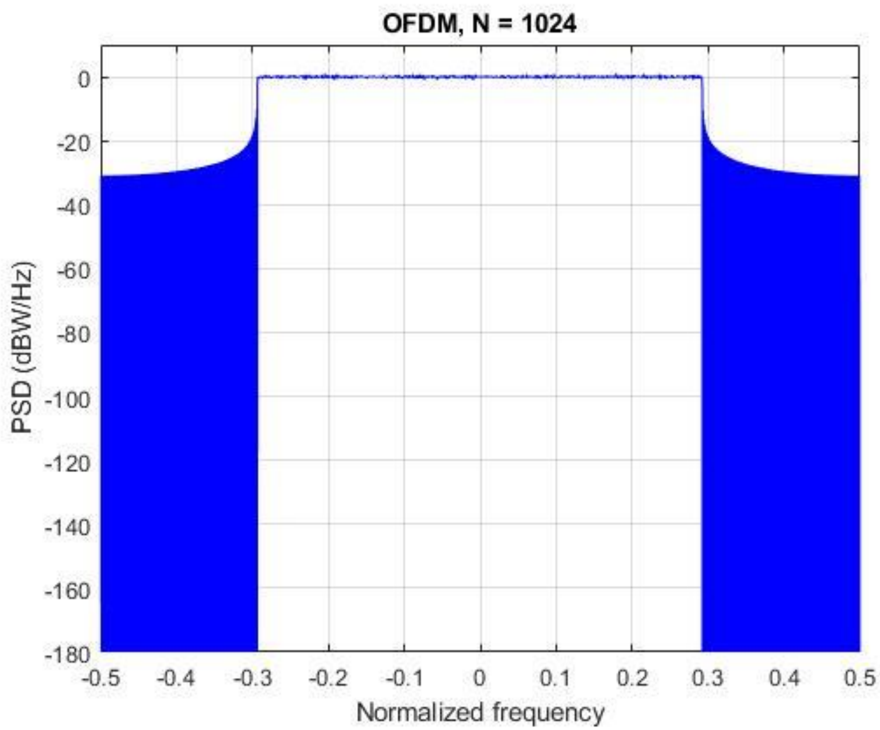


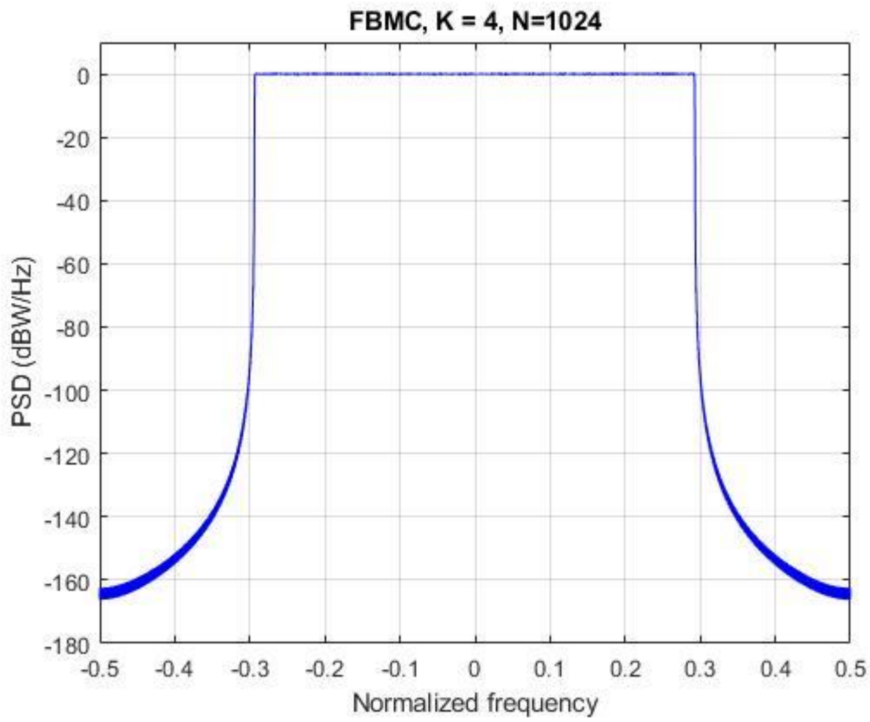
Frequency responses of the prototype filters of OFDM and FBMC (PHYDYAS filter, K=4).



Comparison of spectrum of OFDM and FBMC (PHYDYAS filter, K=4) for one subcarrier.

Power Spectral Density





Error Rate Analysis (1/2)

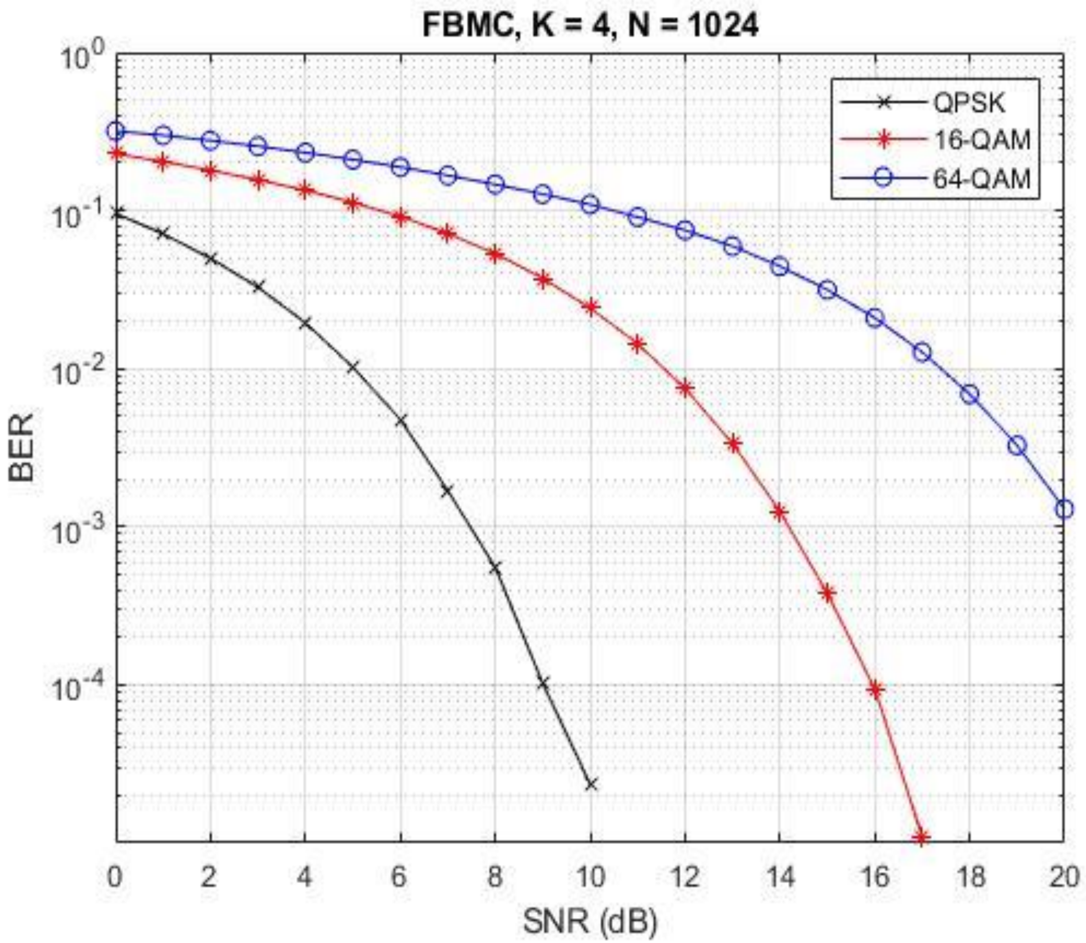


圖 35: FBMC/OQAM 系統在不同調變下的錯誤率比較 (AWGN channel)

Error Rate Analysis (2/2)

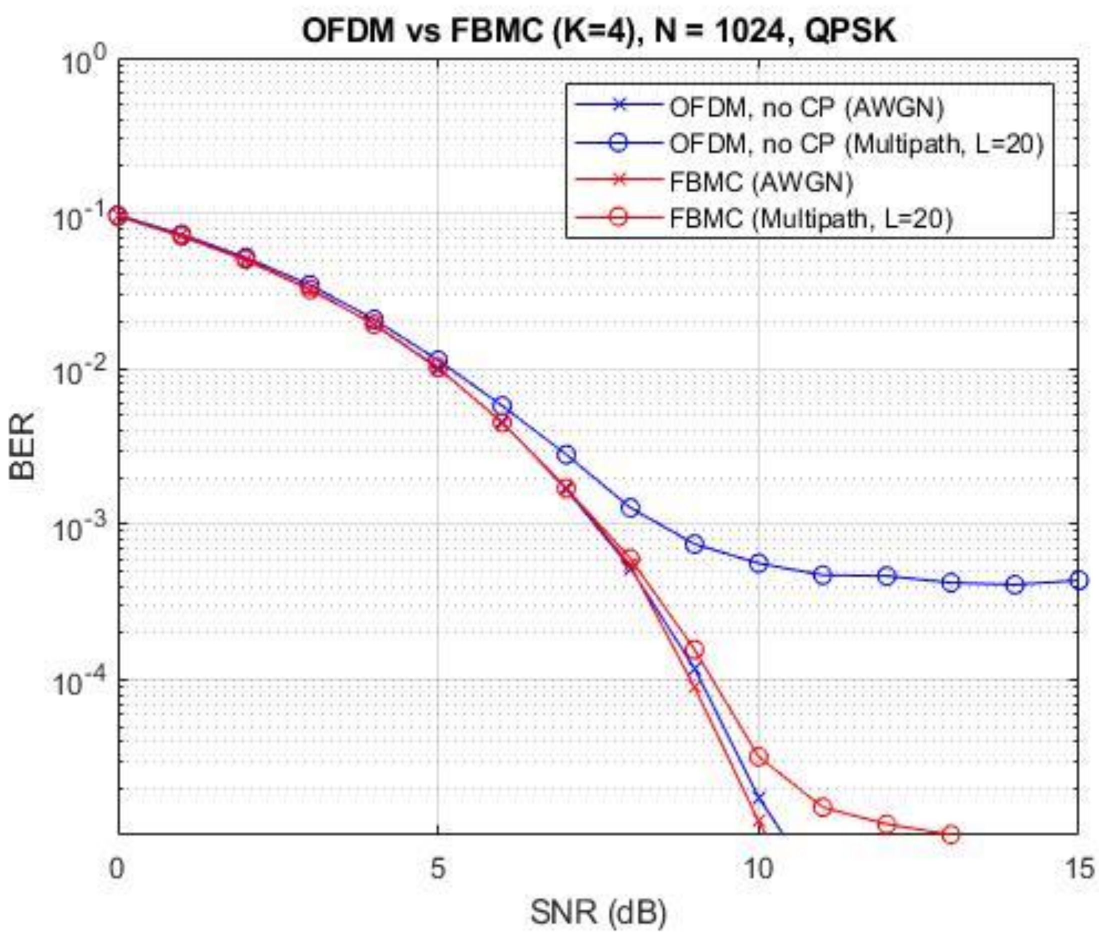


圖 36: 比較OFDM (未加CP) 及FBMC/OQAM 系統在多路徑通道效應下的差異

The Generalized Frequency Division Multiplexing (GFDM) Waveform

The GFDM System

- GFDM arranges the data symbols in a time-frequency grid, consisting of M sub-symbols and K subcarriers, and applies a circular prototype filter for each subcarrier[13].

- The total number of symbols follows as $N=KM$.

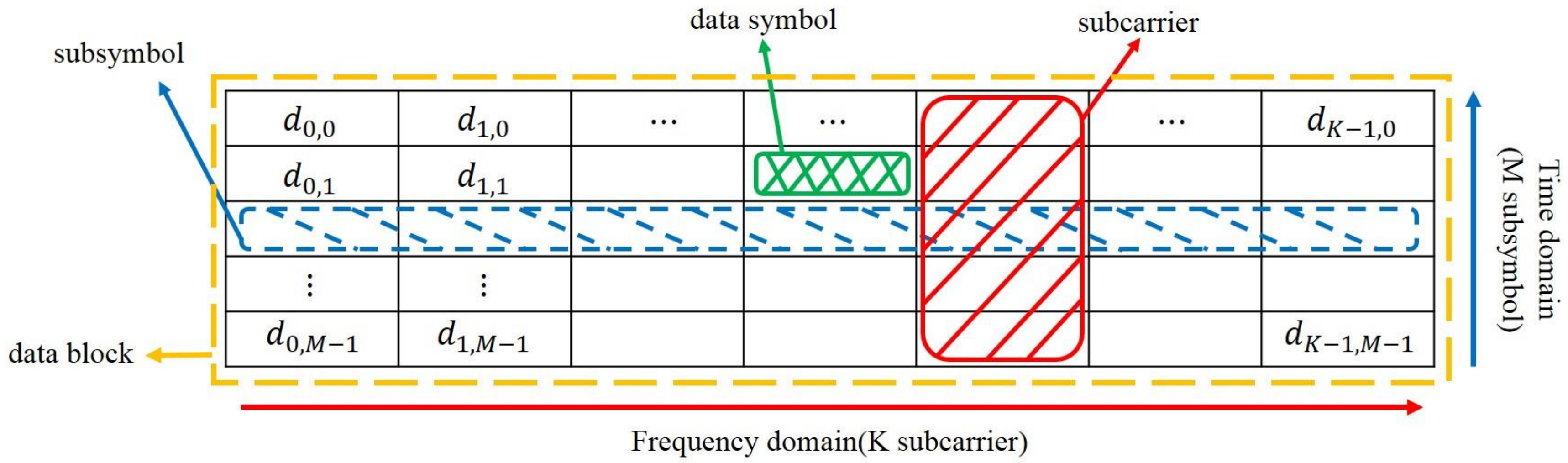
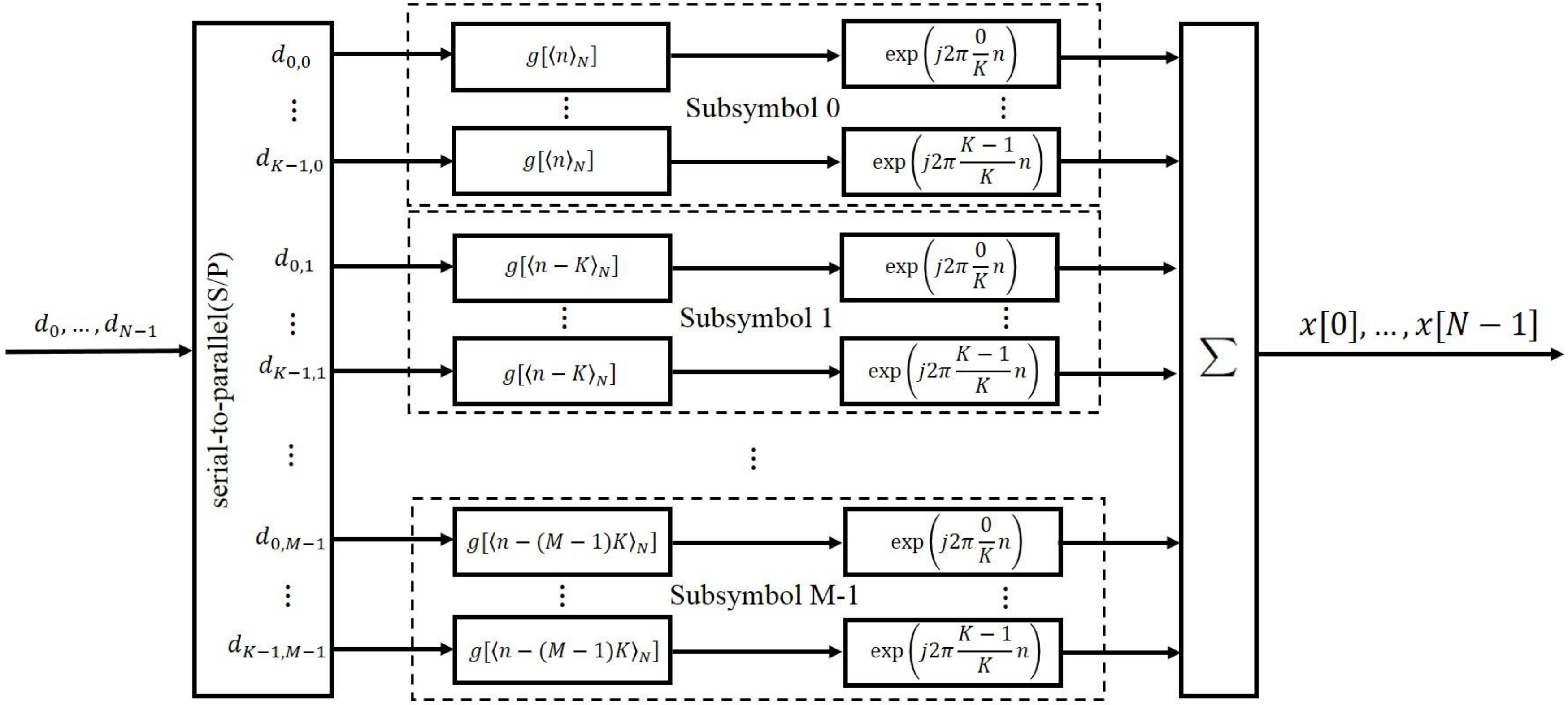


Fig 1: Overview of block structure and terminology

[13] G.Fettweis, M. Krondorf, and S. Bittner, “GFDM – generalized frequency division multiplexing ,” in *Proc. 69th IEEE Vehicular Technology Conference (VTC), Barcelona*, Spain, April. 2009, pp.1-4.

GFDM Modulator (1/4)



GFDM Modulator (2/4)

Each $d_{k,m}$ is transmitted with the corresponding pulse shape[14]

$$g_{k,m}[n] = g\left[\langle n - mK \rangle_N\right] \cdot \exp\left(-j2\pi \frac{k}{K} n\right)$$

With $n = 0,1,\ldots,N-1$ denoting the sampling index.

Each $g_{k,m}[n]$ is a time and frequency shifted version of a prototype filter $g[n]$,the filter on different sub-symbols as

$$g\left[\langle n - mK \rangle_N\right] = \delta\left[\langle n - mK \rangle_N\right] \otimes g[n]$$

where $\otimes$ is the circular convolution.

The data symbols as

$$\mathbf{d} = \left(d_{0,0}, d_{1,0}, \ldots, d_{K-1,0}, d_{0,1}, \ldots, d_{K-1,1}, \ldots, d_{0,M-1}, \ldots, d_{K-1,M-1}\right)^T$$

[14] N. Michailow, M. Matthé, I. Gaspar, A. Caldevilla,L. Mendes, A. Festag, and G. Fettweis, "Generalized frequency division multiplexing for 5th generation cellular networks ," *IEEE Trans. On Commun.*, vol. 62, no. 9, pp. 1-17, Sep. 2014.

GFDM Modulator (3/4)

Explain from a series of modulations in Fig 2, the GFDM transmit samples $\mathbf{x}=\left(x[n]\right)^T$ are obtained by superposition of all transmit symbols

$$x[n]=\sum_{k=0}^{K-1}\sum_{m=0}^{M-1} g\left[\langle n-mK\rangle_N\right]\cdot\exp\left(-j2\pi\frac{k}{K}n\right)d_{k,m} \qquad (1)$$
$$=\sum_{k=0}^{K-1}\sum_{m=0}^{M-1} g_{k,m}[n]\,d_{k,m}$$

Collecting the filter samples in a vector $\mathbf{g}_{k,m}=\left(g_{k,m}[n]\right)^T$ allows to formulate (1) as

$$\mathbf{x}=\left[\mathbf{g}_{0,0},\mathbf{g}_{1,0},\cdots,\mathbf{g}_{K-1,0},\mathbf{g}_{0,1},\cdots,\mathbf{g}_{K-1,M-1}\right]_{N\times N}\begin{bmatrix} d_{0,0}\\ d_{1,0}\\ \vdots\\ d_{K-1,0}\\ d_{0,1}\\ \vdots\\ d_{K-1,M-1}\end{bmatrix}_{N\times 1}$$

GFDM Modulator (4/4)

And allows to formulate (1) as

$$\mathbf{x} = \mathbf{A}\mathbf{d}$$

Where $\mathbf{A}$ is a $KM \times KM$ transmitter matrix [15].

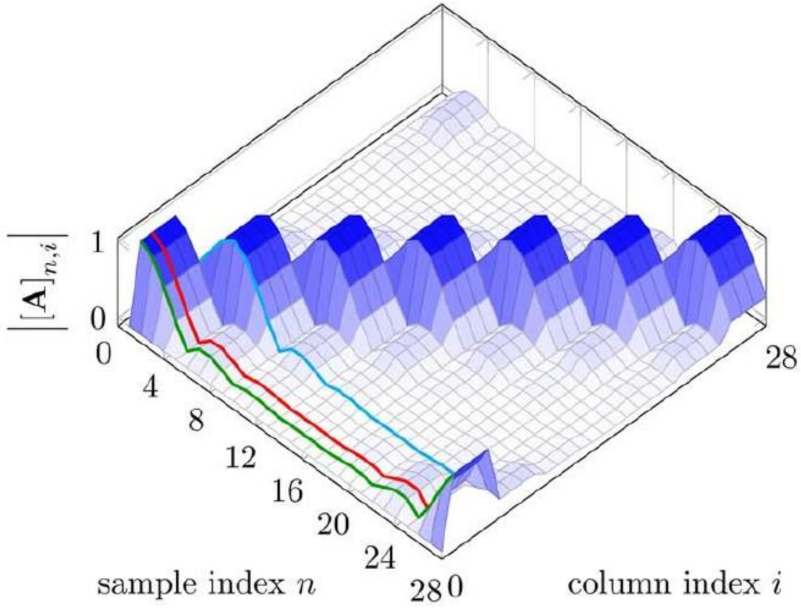
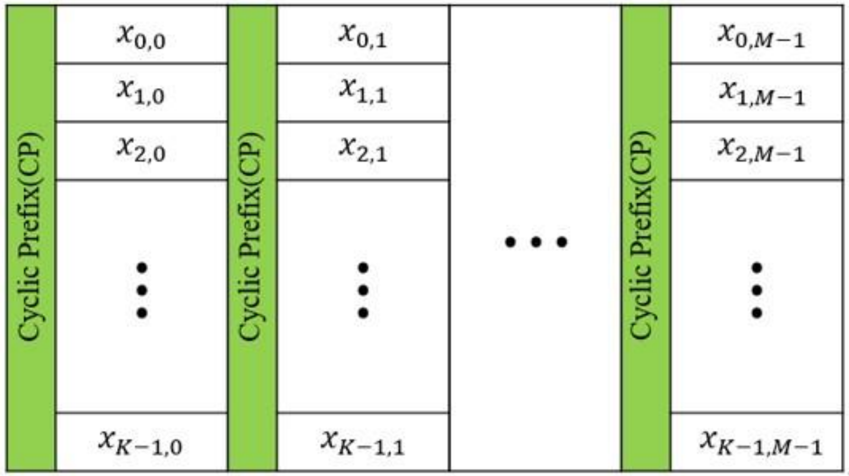


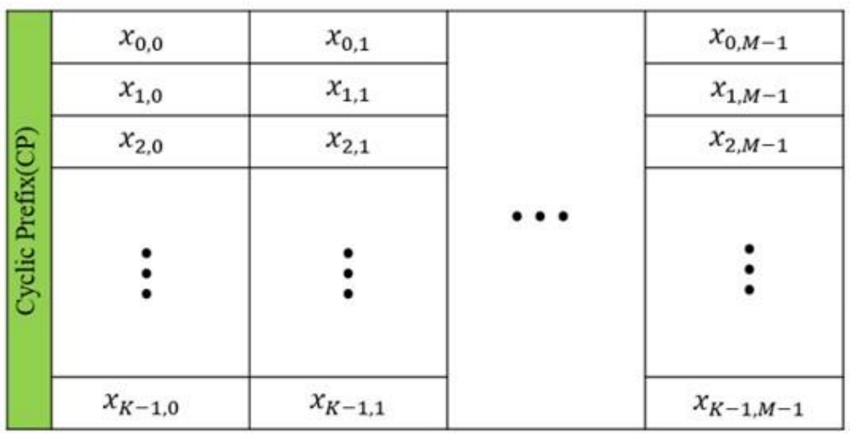
Fig 3:Illustration of GFDMtransmitter matrix for N=28, K=4, M=7 and RC filter roll off factor = 0.4.

[15] N. Michailow, S. Krone, M. Lentmaier, G. Fettweis, "Bit Error Rate Performance of Generalized Frequency Division Multiplexing," in *IEEE Vehicular Technology Conference*, Sep. 2012, pp. 1-5.

Cyclic Prefix



(a) OFDM 訊號



(b) GFDM 訊號

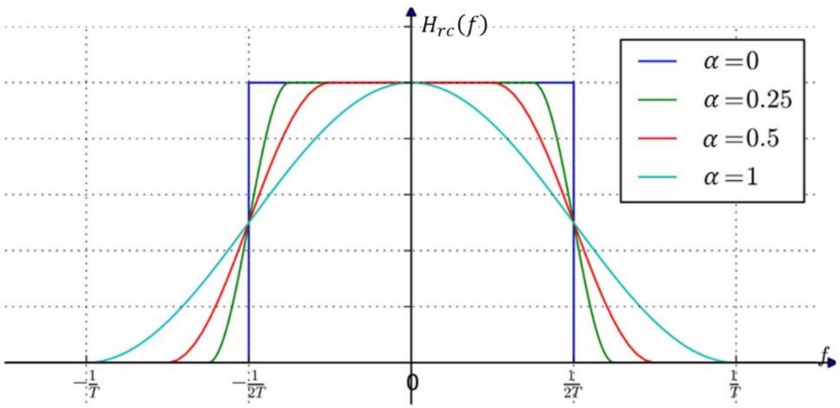
[16] B. Farhang-Boroujeny, and H. Moradi, “Derivation of GFDM based on OFDM principles,” in *Proc. IEEE International Conference Communications(ICC)*, Jun. 2015, pp. 2680-2685.

Prototype Filter

Raised-cosine filter

$$H_{rc}(f)=\begin{cases}1, & |f|\le\frac{1-\alpha}{2T}\\ \frac{1}{2}\left[1+\cos\left(\frac{\pi T}{\alpha}\left[|f|-\frac{1-\alpha}{2T}\right]\right)\right], & \frac{1-\alpha}{2T}<|f|\le\frac{1+\alpha}{2T}\\ 0, & \text{otherwise}\end{cases}$$

$$h_{rc}(t)=\begin{cases}\frac{\pi}{4T}\operatorname{sinc}\left(\frac{1}{2\alpha}\right), & t=\pm\frac{T}{2\alpha}\\ \frac{1}{T}\operatorname{sinc}\left(\frac{t}{T}\right)\frac{\cos\left(\frac{\pi\alpha t}{T}\right)}{1-\left(\frac{2\alpha t}{T}\right)^2}, & \text{otherwise}\end{cases}$$



Square-root-raised-cosine filter

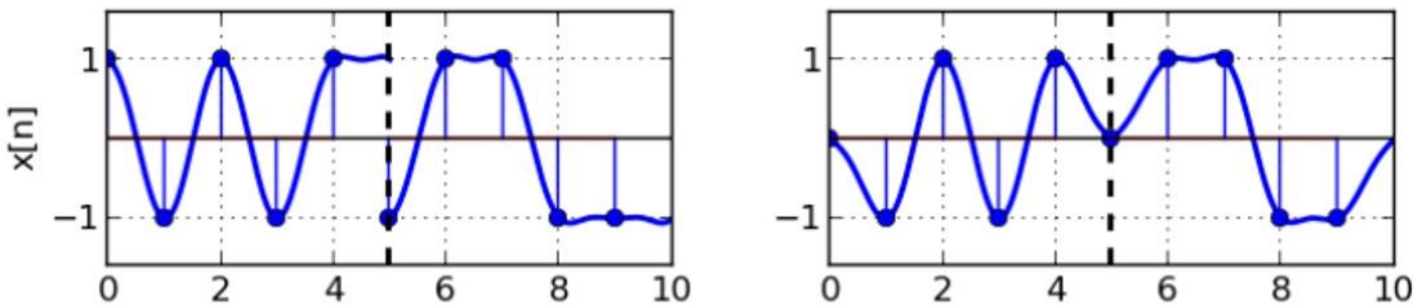
$$h_{srrc}(t)=\begin{cases}\frac{1}{T}\left(1+\alpha\left(\frac{4}{\pi}-1\right)\right), & t=0\\ \frac{\alpha}{T\sqrt{2}}\left[\left(1+\frac{2}{\pi}\right)\sin\left(\frac{\pi}{4\alpha}\right)+\left(1-\frac{2}{\pi}\right)\cos\left(\frac{\pi}{4\alpha}\right)\right], & t=\pm\frac{T}{4\alpha}\\ \frac{1}{T}\frac{\sin\left[\pi\frac{t}{T}(1-\alpha)\right]+4\alpha\frac{t}{T}\cos\left[\pi\frac{t}{T}(1+\alpha)\right]}{\pi\frac{t}{T}\left[1-\left(4\alpha\frac{t}{T}\right)^2\right]}, & \text{otherwise}\end{cases}$$

where T is the symbol period, and α is roll-off factor.

Guard Symbol Insertion

For the spectrum of the GFDM signal not only the pulse shaping but also the transition between subsequent blocks is important, since an abrupt change of the signal between two blocks creates a high OOB radiation.

In order to achieve more smooth transitions, a guard symbol can be inserted into each block, which means that $d_{k,0} = 0$ for all subcarriers [20].



(a) No guard symbol (b) One guard symbol

[20] M. Matthé, N. Michailow, I. Gaspar, and G. Fettweis, “Influence of pulse shaping on bit error rate performance and out of band radiation of generalized frequency division multiplexing,” in *IEEE International Conference Communications Workshops (ICC*), June. 2014, pp. 43-48

GFDM Receiver

The overall transceiver equation can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

Introducing as the received signal after channel equalization.

$$\mathbf{z} = \mathbf{H}^{-1}\mathbf{H}\mathbf{x} + \mathbf{H}^{-1}\mathbf{w}$$

$$= \mathbf{A}\mathbf{d} + \mathbf{H}^{-1}\mathbf{w}$$

Linear demodulation of the signal can be expressed as

$$\hat{\mathbf{d}} = \mathbf{B}\mathbf{z}$$

Where $\mathbf{B}$ is a $KM \times KM$ receiver matrix.

[14] N. Michailow, M. Matthé, I. Gaspar, A. Caldevilla,L. Mendes, A. Festag, and G. Fettweis, "Generalized frequency division multiplexing for 5th generation cellular networks ," *IEEE Trans. On Commun.*, vol. 62, no. 9, pp. 1-17, Sep. 2014.

Several Receivers

The matched filter (MF) receivermaximizes the signal-to-noise ratio (SNR) per subcarrier, but with the effect of introducing self-interference when a non-orthogonal transmit pulse is applied.

$$\mathbf{B}_{MF} = \mathbf{A}^{H}$$

The zero-forcing (ZF) receiver on the contrary completely removes any self-interference at the cost of enhancing the noise.

$$\mathbf{B}_{ZF} = \mathbf{A}^{-1}$$

The linear minimum mean square error (MMSE) receiver

$$\mathbf{B}_{MMSE} = \left(\mathbf{R}_{w}^{2} + \mathbf{A}^{H}\mathbf{H}^{H}\mathbf{H}\mathbf{A}\right)^{-1}\mathbf{A}^{H}\mathbf{H}^{H}$$

makes a trade-off between self-interference and noise enhancement.

[14] N. Michailow, M. Matthé, I. Gaspar, A. Caldevilla,L. Mendes, A. Festag, and G. Fettweis, “Generalized frequency division multiplexing for 5th generation cellular networks ,” *IEEE Trans. On Commun.*, vol. 62, no. 9, pp. 1-17, Sep. 2014.