教育部下世代尖端無線技術聯盟計畫

5G大型陣列天線基頻模組課程

Part I: 理論部分

Wireless Communications Laboratory, YZU

大綱

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理論一

5G Massive MIMO Baseband Processing (Channel, ABF, Precoding, Hybrid-BF)

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Outline

- 1. Introduction
- 2. Channel Model for mmWave Massive MIMO
- 3. Digital Precoding
  - Single User DP
  - Multiuser DP
- 4. Analog Beamforming (ABF)
  - Beam Steering
  - Beam Training (Beam Alignment)
- 5. Hybrid Precoding
  - Single User HP
  - Multiuser HP
- 6. Conclusions

1. Introduction

- The fundamentals of precoding (beamforming) are the regardless of carrier frequency.
  - Signal processing in mmWave massive MIMO systems needs to be subject to a practical constraints.
- For the traditional analog beamforming with small RF chains:
  - Analog circuit with phase shifter (PS) network
  - Constant amplitude constraints
  - Suffering the performance loss (Interference Problems)
- For the digital precoding with large antennas and RF chains:
  - High cost and energy consumption (large number of RF chains)

Introduction

  - Controlling both phase and amplitude to cancel interferences and achieve the optimal performance.
- For the hybrid analog and digital precoding:
  - Small RF chains with small digital precoder to cancel interferences.
  - Large analog beamformer with large PS to increase the antenna array gain.
  - Significantly reducing the RF chains without obvious performance loss.

2. Channel Model for mmWave Massive MIMO

- High free-space path loss is a characteristic of mmWave propagation.
  - Limitted spatial selectivity or scattering.
- Large tightly packed antenna arrays are characteristics of mmWave transceivers.
  - High levels of antenna correlation.
- Characteristics in mmWave channels.
  - Sparse scattering channel
  - Adopting a narrowband clustered channel representation, based on the extended Saleh-Valenzuela model.

Channel Model for mmWave Massive MIMO

- Using the clustered channel model, the discrete-time channel matrix H can be expressed by a sum of the L propagation paths, i.e.,

$$\mathbf{H} = \sqrt{\frac{N_{\mathrm{t}}N_{\mathrm{r}}}{L}} \sum_{l=1}^{L} \alpha_l \Lambda_{\mathrm{r}}(\phi_l^{\mathrm{r}}, \theta_l^{\mathrm{r}}) \Lambda_{\mathrm{t}}(\phi_l^{\mathrm{t}}, \theta_l^{\mathrm{t}}) \mathbf{a}_{\mathrm{r}}(\phi_l^{\mathrm{r}}, \theta_l^{\mathrm{r}}) \mathbf{a}_{\mathrm{t}}^{H}(\phi_l^{\mathrm{t}}, \theta_l^{\mathrm{t}})$$

where

α_l: complex gain of the l_{th} path

$\phi_l^r(\theta_l^r)$: AZ/EL angles of AoAs

$\phi_l^t(\theta_l^t)$: AZ/EL angles of AoDs

$\Lambda_r(\phi_l^r, \phi_l^r)$: RX antenna element gain at l_{th} AoA, $\Lambda_r = 1$ for simplicity

$\Lambda_t(\phi_l^t, \phi_l^t)$: TX antenna element gain at l_{th} AoD, $\Lambda_t = 1$ for simplicity

Channel Model for mmWave Massive MIMO

- For the uniform linear array (ULA) with N elements, the normalized array response vector:

$$\mathbf{a}_{\mathrm{ULA}}(\phi) = \frac{1}{\sqrt{N}}\left[1, e^{j\frac{2\pi}{\lambda}d\sin(\phi)}, \ldots, e^{j(N-1)\frac{2\pi}{\lambda}d\sin(\phi)}\right]^T$$

- For the uniform planar array (UPA) with W_1 and W_2 elements (W_1 W_2 $=N$), the normalized array response vector:

$$\mathbf{a}_{\mathrm{UPA}}(\phi,\theta) = \frac{1}{\sqrt{N}}\left[1, \ldots, e^{j\frac{2\pi}{\lambda}d(x\sin(\phi)\sin(\theta)+y\cos(\theta))}, \ldots, e^{j\frac{2\pi}{\lambda}d((W_1-1)\sin(\phi)\sin(\theta)+(W_2-1)\cos(\theta))}\right]^T$$

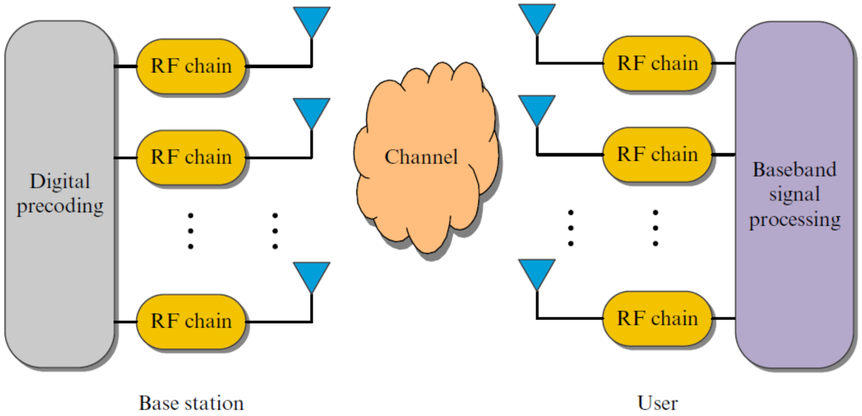
where $0 \le x \le W_1 - 1$ (horizontal) and $0 \le y \le W_2 - 1$ (vertical)

3. Digital Precoding (DP)

- DP can control "phases and amplitudes" of original signals to cancel interferences.
- Two categories of DP:
  - Linear precoding: TX signals with the linear combination of the original signals.
  - Nonlinear precoding: TX signals with the nonlinear processing.
- Two system used by DP techniques:
  - Single-user precoding system: Matched Filter (MF) and Zero-Forcing (ZF) precoding.
  - Multiuser precoding system: Block diagonalization (BD) precoding.

3.1 Single-User Digital Precoding

- Architecture of DP for single-user mmWave massive MIMO system:



Architecture of digital precoding for single-user mmWave massive MIMO system.

- Consider N_t TX antennas, N_r RX antennas, N_r data streams ($N_r < N_t$), the TX signal with $N_t \times N_r$ DP D matrix:

$$\mathbf{x} = \mathbf{D}\mathbf{s}$$

where

$\mathbf{s}: N_r \times 1$ original signal vector with $E(\mathbf{s}\mathbf{s}^H) = \frac{1}{N_r}\mathbf{I}_{Nr}$

$\mathbf{D}:$ precoding matrix to meet total TX power constraint, $\|\mathbf{D}_F\|^2 = tr(\mathbf{D}\mathbf{D}^H) = N_r$

Single-User Digital Precoding

- Under the narrowband system, the RX signal vector $\mathbf{y}(N_r \times 1)$:

$$\mathbf{y} = \sqrt{\rho}\mathbf{HDs} + \mathbf{n},$$

where

$\mathbf{H}: N_r \times N_t$ channel matrix with normalized power $E(\|\mathbf{H}\|_F^2) = N_t N_r$

ρ : The averaged RX power

$\mathbf{n}$: AWGN noise vector, i.i.d. noise element $CN(0, \sigma_n^2)$

$\mathbf{H}$: Assume H known at BS to enable precoding

Single-User Digital Precoding

- The simplest linear digital precoding: MF precoding

$$\mathbf{D} = \sqrt{\frac{N_{\mathrm{r}}}{\operatorname{tr}\left(\mathbf{F}\mathbf{F}^{H}\right)}}\mathbf{F},$$
$$\mathbf{F} = \mathbf{H}^{H}.$$

  - MF can maximize the SNR at user side.
  - MF involves severe interferences among different data streams.

- The well-known linear digital precoding: ZF precoding

$$\mathbf{D} = \sqrt{\frac{N_{\mathrm{r}}}{\operatorname{tr}\left(\mathbf{F}\mathbf{F}^{H}\right)}}\mathbf{F},$$
$$\mathbf{F} = \mathbf{H}^{H}\left(\mathbf{H}\mathbf{H}^{H}\right)^{-1}$$

Single-User Digital Precoding

- ZF can entirely eliminate the interferences among different data streams.
- **D** required to satisfy the total TX power constraint. ZF precoding may enhance the power of noise and lead performance loss.

- The Wiener filter (WF) precoding: WF precoding (or MMSE precoding)

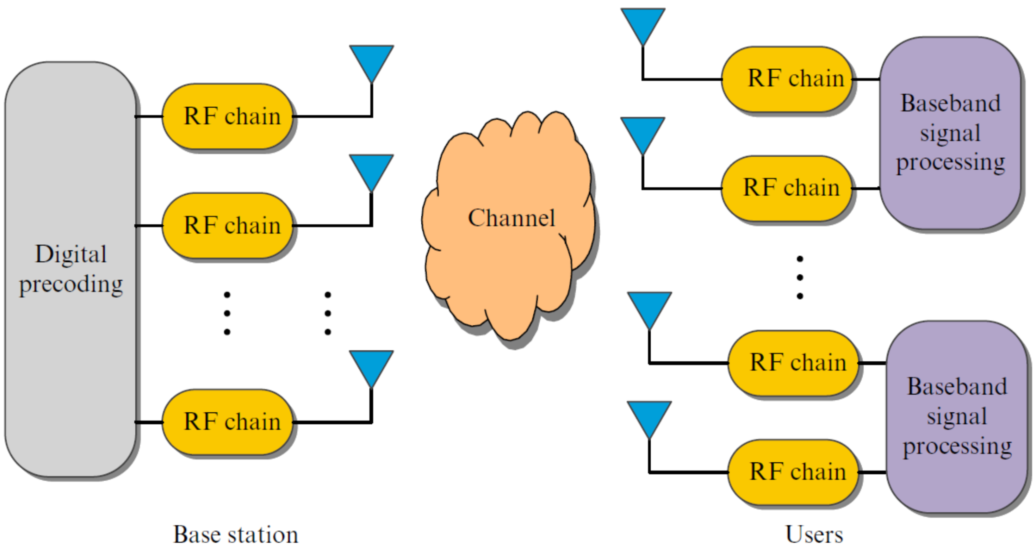
$$\mathbf{D} = \sqrt{\frac{N_{\mathrm{r}}}{\operatorname{tr}\left(\mathbf{F}\mathbf{F}^{H}\right)}}\mathbf{F},$$

$$\mathbf{F} = \mathbf{H}^{H}\left(\mathbf{H}\mathbf{H}^{H} + \frac{\sigma_n^2 N_{\mathrm{r}}}{\rho}\mathbf{I}\right)^{-1}$$

∴ WF precoding can make a better trade-off between the RX SNR and interferences.

3.2 Multiuser Digital Precoding

- Architecture of digital precoding for multiuser mmWave massive MIMO Systems.



Architecture of digital precoding for multiuser mmWave massive MIMO systems.

- Consider BS with N_{BS} antennas and RF chains.
- Communicate with U MSs, each MS with N_{MS} antennas.
- Total data streams is $N_{\mathrm{MS}}U$ ($N_{\mathrm{MS}}U \leq N_{\mathrm{BS}}$)

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Multiuser Digital Precoding

- For downlink communication, BS employs U digital precoders, i.e., $\mathbf{D}=[\mathbf{D}_1\ \mathbf{D}_2\ \ldots\ \mathbf{D}_U]$, where $\mathbf{D}_u$ is the $N_{BS}\times N_{MS}$ precoder for the uth user.
  - $\mathbf{D}_U$ satisfying the total TX power constraint $\|\mathbf{D}_u\|_F = N_{MS}$
- Considering the narrowband block-fading channel, the RX signal vector $\mathbf{r}_u$ of the uth MS:

$$\mathbf{r}_u = \mathbf{H}_u \sum_{n=1}^{U} \mathbf{D}_n \mathbf{s}_n + \mathbf{n}_u$$

where

$\mathbf{s}_n$: $N_{MS\times 1}$ original signal vector with normalized power

$\mathbf{H}_u$: $N_{MS}\times N_{BS}$ mmWave massive MIMO matrix between BS and the uth MS

$\mathbf{n}_u$: iid AWGN noise $CN(0,\sigma_n^{\ 2})$

Multiuser Digital Precoding

- The terms $\mathbf{H}_u\mathbf{D}_n\mathbf{s}_n \;\; for\; n \neq u$ are interferences to the uth MS.
- Design the BD precoding $\mathbf{D}_n$ to satisfy $\mathbf{H}_u\mathbf{D}_n = \mathbf{0}$. (Nulling)
- $\mathbf{D}_n$ is designed to lie in the null space of $\bar{\mathbf{H}}_u$,

$$\bar{\mathbf{H}}_u = \left[\mathbf{H}_1^H \cdots \mathbf{H}_{u-1}^H \; \mathbf{H}_{u+1}^H \cdots \mathbf{H}_U^H\right]^H$$

$$= \bar{\mathbf{U}}_u \bar{\mathbf{\Lambda}}_u \bar{\mathbf{V}}_u^H = \bar{\mathbf{U}}_u \bar{\mathbf{\Lambda}}_u \left[\bar{\mathbf{V}}_u^{nonzero}, \bar{\mathbf{V}}_u^{zero}\right]^H$$

where

$\bar{\mathbf{V}}_u^{nonzero}$: The right singular vectors corresponding to nonzero singular values of $\bar{\mathbf{H}}_u$

$\bar{\mathbf{V}}_u^{zero}$: The right singular vectors corresponding to zero singular values of $\bar{\mathbf{H}}_u$

- The digital precoder $\mathbf{D}_u$ of the uth MS is (first N_{MS} columns of $\bar{\mathbf{V}}_u^{zero}$)

$$\mathbf{D}_u = \bar{\mathbf{V}}_u^{zero}(:, 1 : N_{\mathrm{MS}})$$

Multiuser Digital Precoding

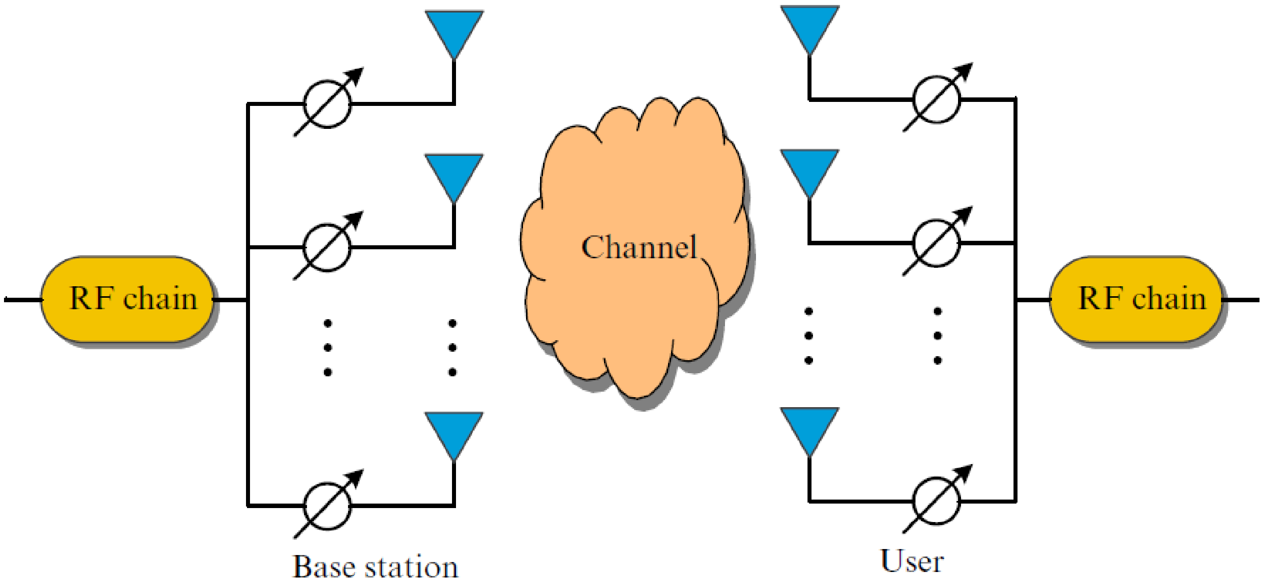
- Note: The optimal DPC and the near-optimal Tomlinson-Harashima (TH) precoding involve high computational complexity.

4. Analog Beamforming

- Analog beamforming is developed in point-to-point mmWave systems with large antenna arrays.
  - Only one RF chain
  - TX a single data stream
  - Control the phases of original signals to achieve the maximum antenna array gain and effective SNR.
- The widely used analog beamforming scheme:
  - Beam steering
  - To obtain the best analog beamforming vectors:
    - Beam training schemes

4.1 Beam Steering

- Architecture of analog beamforming for single-user mmWave massive MIMO Systems.



Architecture of analog beamforming for single-user mmWave massive MIMO systems.

  - N_t TX antennas, N_r RX antennas, only one RF chain to TX one data stream.

Beam Steering

- Define $\mathbf{f}_{Nt\times 1}$ as BS anlaog beamforming vector, $\mathbf{w}_{Nr\times 1}$ as user analog combining vector.
- Design $\mathbf{f}$ and $\mathbf{w}$ to maximize the effective SNR:

$$(\mathbf{w}^{\mathrm{opt}}, \mathbf{f}^{\mathrm{opt}}) = \arg\max \left|\mathbf{w}^{H}\mathbf{H}\mathbf{f}\right|^{2}$$

$$\text{s.t.} \quad w_i = \sqrt{N_{\mathrm{r}}^{-1}} e^{j\varphi_i}, \quad \forall i,$$

$$f_l = \sqrt{N_{\mathrm{t}}^{-1}} e^{j\phi_l}, \quad \forall l.$$

- For the optimal solution (unconstraint):

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{H} \qquad (\text{SVD of } \mathbf{H})$$

$$\mathbf{w}^{opt} = \mathbf{U}(:,1) \quad (\text{Max. eig.value, eig.vector})$$

$$\mathbf{f}^{opt} = \mathbf{V}(:,1) \quad (\text{No amplitude constriant})$$

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Beam Steering

- Design the practical solutions $\mathbf{f}$ and $\mathbf{w}$ satisfying the amplitude constraint to be close to the optimal unconstraint solutions $\mathbf{f}^{opt}$ and $\mathbf{w}^{opt}$.
  - Each right singular vector of **H** with $L=o(N_t)$ converges in chordal distance to an array response vector $\mathbf{a}_t(\phi_\ell^t,\theta_\ell^t)$.
  - Each left singular vector of **H** with $L=o(N_r)$ converges in chordal distance to an array response vector $\mathbf{a}_r(\phi_\ell^r,\theta_\ell^r)$.
  - Singular values converge to $N_t N_r |\alpha_\ell|^2 / L$.
- In other words, we can select

$$\mathbf{f}=\mathbf{a}_t(\phi_{k^*}^t,\theta_{k^*}^t) \text{ and } \mathbf{w}=\mathbf{a}_r(\phi_{k^*}^r,\theta_{k^*}^r)$$

where

$k^* = \arg\max_\ell |\alpha_\ell|^2$, to steer the beam in the strongest direction.

(For large N_t and N_r, it can achieve near-optimal performance)

4.2 Beam Training

- For the above beam steering, we need to know the perfect CSI
  - Impractical in the realistic systems due to only one RF chain
  - It observes a noisy version of the effective channel of smaller size.
- For the no full CSI, using the subspace sampling for beam training to find the $\mathbf{f}$ and $\mathbf{w}$.
  - BS and User collaborate to search the best beamformer and combiner pair from the predefined codebooks during the beam training.

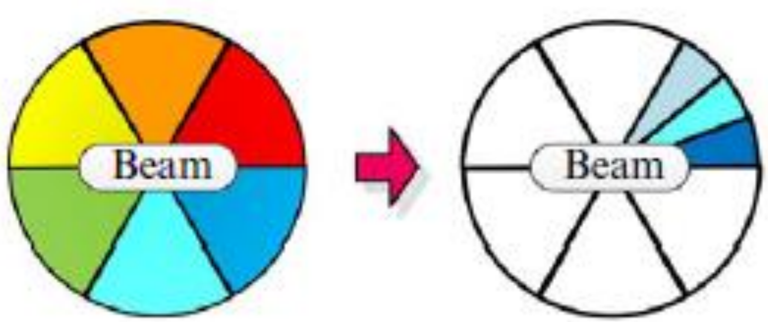
Beam Training

- The different codebook sizes are designed based on the beam steering scheme:

$$\mathbf{f} \in \mathcal{F} = \left\{ \mathbf{a}_\mathrm{t}\left(\overline{\phi}_1^\mathrm{t}, \overline{\theta}_1^\mathrm{t}\right), \mathbf{a}_\mathrm{t}\left(\overline{\phi}_2^\mathrm{t}, \overline{\theta}_2^\mathrm{t}\right), \ldots, \mathbf{a}_\mathrm{t}\left(\overline{\phi}_{|\mathcal{F}|}^\mathrm{t}, \overline{\theta}_{|\mathcal{F}|}^\mathrm{t}\right) \right\}$$

$$\mathbf{w} \in \mathcal{W} = \left\{ \mathbf{a}_\mathrm{r}\left(\overline{\phi}_1^\mathrm{r}, \overline{\theta}_1^\mathrm{r}\right), \mathbf{a}_\mathrm{t}\left(\overline{\phi}_2^\mathrm{r}, \overline{\theta}_2^\mathrm{r}\right), \ldots, \mathbf{a}_\mathrm{t}\left(\overline{\phi}_{|\mathcal{W}|}^\mathrm{r}, \overline{\theta}_{|\mathcal{W}|}^\mathrm{r}\right) \right\}$$

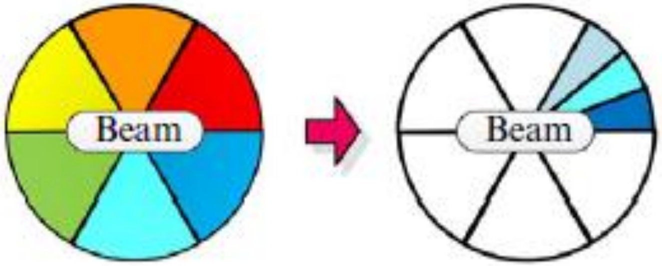
  - It can uniformly cover the whole range of AoDs/AoAs.



(A) Multilevel codebook

Beam Training

- For the optimal beam training scheme:
  - It is to **exhaustively search** all possible $|F|\ |W|$ pairs of beamforming and combining based on the maximized SNR criterion.
  - It cannot be affordable due to very large $|F|$ and $|W|$.
- For the hierarchical beam training scheme: (reducing the overhead of the exhaustive search)
  - (1) Construct a series of codebooks $F_1\ F_2 \cdots F_K\ \ (W_1 W_2 \cdots W_K)$ with the increasing resolution.

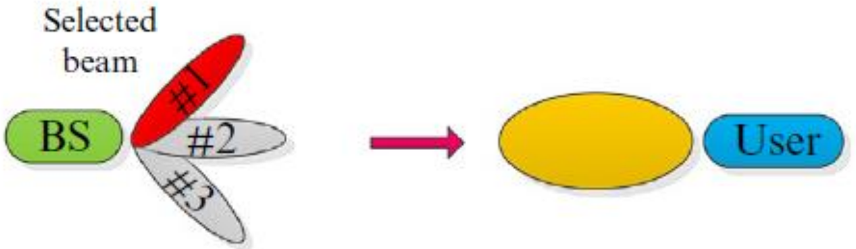


(A) Multilevel codebook

Beam Training

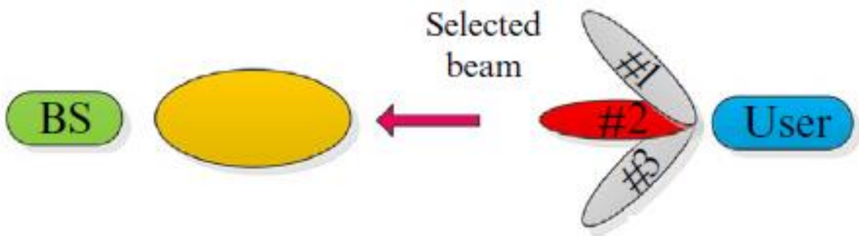
- (2) At the first level (lowest resolution codebook F_1), beam sweep at BS side (MS only RX and find the index of selected beamforming)

(Sending training data)



(B) Beam sweep at BS side

- (3) Swap their roles and beam sweep at user side to find the best beamforming index.



(Sending training data)

(C) beam sweep at user side

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Beam Training

- (4) Feedback the index of the selected beamforming vector to each other.

(D) feedback phase

- (5) Repeat the above procedure with a higher resolution codebook within the chosen beam until the last level (highest resolution codebook, F_K)

5. Hybrid Precoding

- Hybrid (analog and digital) Precoding used in mmWave massive MIMO systems
  - Step1: a small-size digital precoder used to cancel interferences.
  - Step2: a large-size analog beamformer used to increase antenna array gain.
- Two architectures of hybrid beamforming:
  - (1) Fully connected architecture: each RF chain connected to all BS antennas via PSs.
  - (2) Subconnected architecture: each RF chain connected to only a subset of BS antennas.

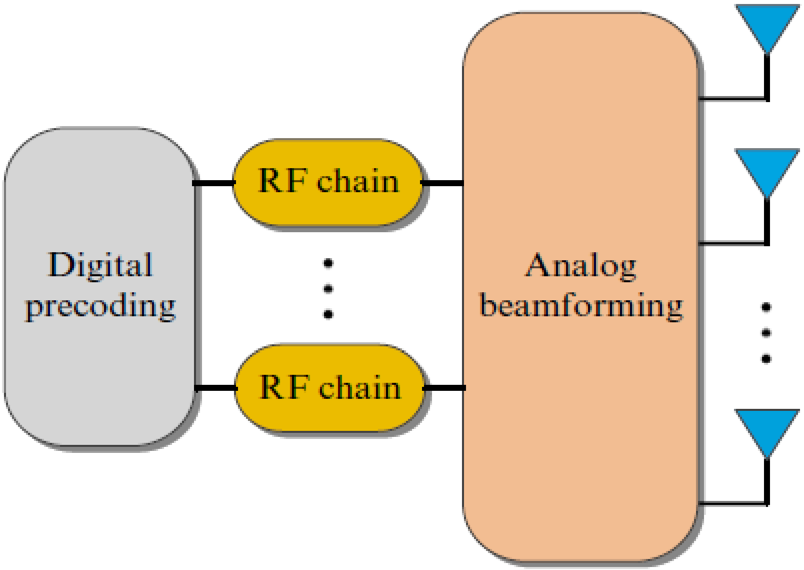
Hybrid Precoding

- Two systems of hybrid precoding:
  - (1) Single-user hybrid precoding
  - (2) Multiuser hybrid precoding
- Two hybrid beamforming schemes of single-user system:
  - (1) Spatially sparse hybrid precoding (fully connected architecture)
  - (2) Successive interference cancellation (SIC)-based hybrid precoding (subconnected architecture)
- Two-stage hybrid precoding scheme is used for multiuser system.

5.1 Single-User Hybrid Precoding

<System Model>

- Hardware architecture of hybrid precoding for single-user mmWave massive MIMO systems



Hardware architecture of hybrid precoding for single-user mmWave massive MIMO systems.

Single-User Hybrid Precoding

- N_t *TX antennas,* TX N_s *data streams, a* user with N_r *antennas*
- N_t^{RF} *RF chains to* TX *multistream,* $N_s \le N_t^{RF} \le N_t$
- *BS applies an* $N_t^{RF} \times N_s$ *digital precoder* D using N_t^{RF} RF *chains* *and an* $N_t \times N_t^{RF}$ analog beamformer A using analog phase shifters(PSs)

- The transmitted signal:

$$\mathbf{x} = \mathbf{ADs}$$

where $\mathbf{s} : N_s \times 1$ *original signal vector with normalized power* $E(\mathbf{ss}^H) = \frac{1}{N_s}\mathbf{I}_{N_s}$

- Consider a simple narrowband system:
  - Coherence bandwidth is usually very large at mmWave , e.q. the order of 100MHz.

Single-User Hybrid Precoding

- The received signal $\mathbf{y}_{N_r \times 1}$:

$$\mathbf{y} = \sqrt{\rho}\mathbf{HADs} + \mathbf{n}.$$

where

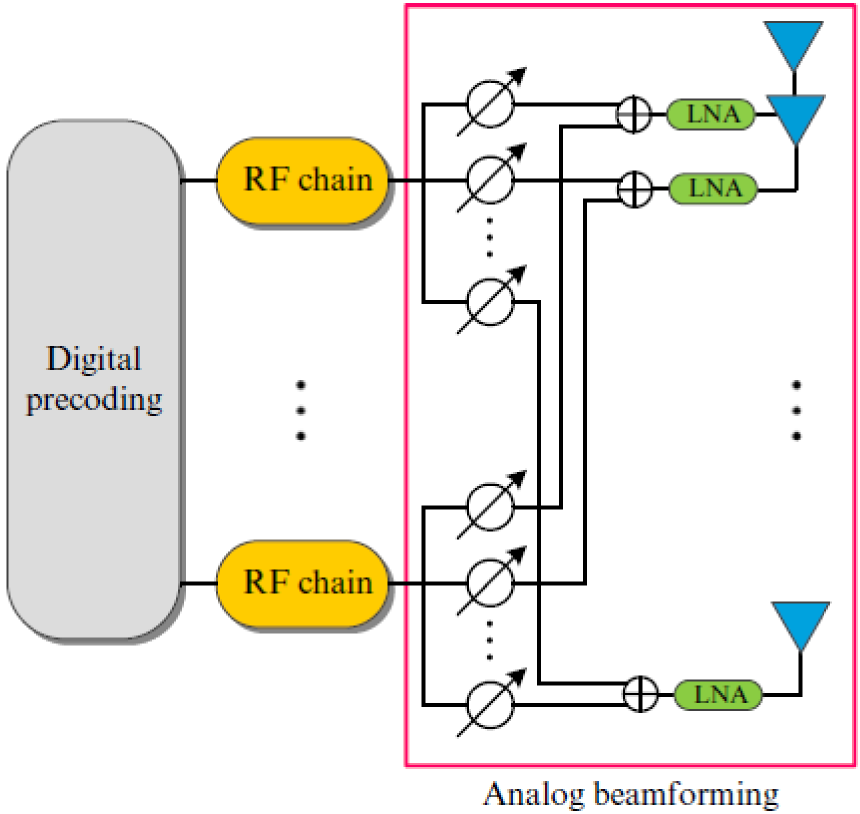
$\mathbf{H}$: *known at BS* and user

CSI: obtained at RX via training and shared with TX via limited feedback

<Spatially Sparse Hybrid Precoding (full connected)>

- Fully connected architecture of hybrid precoding for single-user mmWave massive MIMO system.

Single-User Hybrid Precoding



Fully connected architecture of hybrid precoding for single-user mmWave massive MIMO system.

Single-User Hybrid Precoding

- All elements of analog beamformer **A** have the same amplitude $\frac{1}{\sqrt{N_t}}$ but the different phase shifters.
- Total TX power constraint via the normalizing **D** to satisfy $\|\mathbf{AD}\|_F^2 = N_s$

- Design (**A**,**D**) to maximize the sum rate R(**A**,**D**) achieved by Gaussian signaling over mmWave channel:

$$R(\mathbf{A},\mathbf{D}) = \log_2\left(\left|\mathbf{I} + \frac{\rho}{N_s \sigma_n^2}\mathbf{H}\mathbf{A}\mathbf{D}\mathbf{D}^H\mathbf{A}^H\mathbf{H}^H\right|\right).$$

- The corresponding sum-rate optimization problem:

$$\begin{aligned}(\mathbf{A}^{\mathrm{opt}}, \mathbf{D}^{\mathrm{opt}}) &= \arg\max_{\mathbf{A},\mathbf{D}} R(\mathbf{A},\mathbf{D}),\\ \text{s.t} \quad & \mathbf{A} \in \mathcal{F},\\ & \|\mathbf{AD}\|_F^2 = N_s,\end{aligned}$$

Single-User Hybrid Precoding

where F : the set containing all feasible analog beamformers

the set of $N_t \times N_t^{RF}$ matrices with constant-magnitude entries

- It is known that there are no general solution due to the nonconvex amplitude constraint $\mathbf{A} \in \mathrm{F}$
  - To find practical solutions using an approximation scheme, e.q., transforming the sum rate into "distance" between **AD** and optimal unconstrained precoder $\mathbf{P}_{\text{opt}}$.
- First , using the SVD of $\mathbf{H} \doteq \mathbf{U\Sigma V}^H$, the sum rate:

$$R(\mathbf{A},\mathbf{D}) = \log_2\left(\left|\mathbf{I} + \frac{\rho}{N_s\sigma_n^2}\boldsymbol{\Sigma}^2\mathbf{V}^H\mathbf{A}\mathbf{D}\mathbf{D}^H\mathbf{A}^H\mathbf{V}\right|\right)$$

where $\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_2 \end{bmatrix}$, $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_2]$, $\Sigma_1 : N_s \times N_s$

$V_1 : N_t \times N_s$

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Single-User Hybrid Precoding

$\Rightarrow$ the optimal unconstrained precoder $\mathbf{P}_{opt} = \mathbf{V}_1$,

it cannot be expressed by AD with $\mathbf{A} \in \mathrm{F}$

- Second, the practical hybrid precoder AD is designed to close the optimal unconstrained precoder $\mathbf{V_1}$. (Get near-optimal performance)

$\Rightarrow$ max $tr(\mathbf{V}_1^H \mathbf{AD})$

$\Rightarrow$ equivalent to min $\left\| \mathbf{P}_{opt} - \mathbf{AD} \right\|_F$

- Third , the near-optimal sum-rate approximated by

$$(\mathbf{A}^{opt}, \mathbf{D}^{opt}) = \underset{\mathrm{AD}}{\arg\min} \left\| \mathbf{P}_{opt} - \mathbf{AD} \right\|_F$$

$$\text{s.t. } \mathbf{A}(:,\mathrm{i}) \in \{\mathbf{a}_t(\phi_\ell^t, \theta_\ell^t), \forall \ell\}$$

$$\left\| \mathbf{AD} \right\|_F^2 = N_s$$

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Single-User Hybrid Precoding

- Fourth, finding the best low-dimensional representation of $\mathrm{P}_{\mathrm{opt}}$ using the basis vector $\mathbf{a}_t(\phi_\ell^t, \theta_\ell^t)$.
  - Selecting the "best" N_t^{RF} array response vectors and finding their optimal baseband combination, the optimal objective function:

$$\widetilde{\mathbf{D}}^{\mathrm{opt}} = \arg\min_{\widetilde{\mathbf{D}}} \left\| \mathbf{P}_{\mathrm{opt}} - \mathbf{A}_{\mathrm{t}} \widetilde{\mathbf{D}} \right\|_F,$$

$$\text{s.t} \quad \left\| \mathrm{diag}\left(\widetilde{\mathbf{D}} \widetilde{\mathbf{D}}^H \right) \right\|_0 = N_{\mathrm{t}}^{\mathrm{RF}},$$

$$\left\| \mathbf{A}_{\mathrm{t}} \widetilde{\mathbf{D}} \right\|_F^2 = N_{\mathrm{s}},$$

where

$$\mathbf{A}_t = \left[\mathbf{a}_t(\phi_1^t, \theta_1^t), \cdots, \mathbf{a}_t(\phi_L^t, \theta_L^t) \right] \Rightarrow L \le N_t^{RF}$$

$$\tilde{\mathbf{D}} : L \times N_s \ \textit{matrix}, \left\| \textit{diag}(\tilde{\mathbf{D}}\tilde{\mathbf{D}}^H) \right\|_0 = N_t^{RF}, L \le N_t^{RF} \ (\textit{select } N_t^{RF} \textit{ rows of } \tilde{\mathbf{D}})$$

Single-User Hybrid Precoding

- It is equivalent to the typical problem of sparse signal recovery. The above problem can be solved by the well-known concept of orthogonal matching pursuit (OMP).

ALGORITHM 1 SPATIALLY SPARSE PRECODING

Input: $\mathbf{P}_{\text{opt}}$

$\mathbf{A} = \text{Empty Matrix}$

$\mathbf{P}_{\text{res}} = \mathbf{P}_{\text{opt}}$

for $i \leq N_{\text{t}}^{\text{RF}}$ **do**

$\quad \boldsymbol{\Psi} = \mathbf{A}_{\text{t}}^{H} \mathbf{P}_{\text{res}}$

$\quad k = \arg\max_{l=1,\ldots,L} \left(\boldsymbol{\Psi}\boldsymbol{\Psi}^{H} \right)_{l,l}$

$\quad \mathbf{A} = \left[\mathbf{A} \middle| \mathbf{A}_{\text{t}}^{(k)} \right]$

$\quad \mathbf{D} = \left(\mathbf{A}^{H}\mathbf{A} \right)^{-1} \mathbf{A}^{H} \mathbf{P}_{\text{opt}}$

$\quad \mathbf{P}_{\text{res}} = \dfrac{\mathbf{P}_{\text{opt}} - \mathbf{A}\mathbf{D}}{\left\| \mathbf{P}_{\text{opt}} - \mathbf{A}\mathbf{D} \right\|_{F}}$

end for

$\mathbf{D} = \sqrt{N_{\text{s}}} \dfrac{\mathbf{D}}{\left\| \mathbf{A}\mathbf{D} \right\|_{F}}$

return A,D

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Single-User Hybrid Precoding

- In Algorithm1:

(1) Step1&2: Initialization

(2) Step 5: Finding the vector $\mathbf{a}_t(\phi_\ell^t, \theta_\ell^t)$,which the optimal precoder has the maximum projection.

(3) Step 6: Appending the selected $\mathbf{a}_t(\phi_\ell^t, \theta_\ell^t)$ to the analog beamformer **A**.

(4) Step 7: LS solution D is calculated by the dominant **A**

(5) Step 8: Removing the dominant **A** contribution, the residual precoding matrix $\mathbf{P}_{\mathrm{res}}$ to find the next $\mathbf{a}_t$ and get largest projection.

(6) Step 9: Until all N_t^{RF} precoding vectors have been selected, the A and D are determined to $\min\left\|\mathbf{P}_{opt} - \mathbf{AD}\right\|_F$

Single-User Hybrid Precoding

(7) Step 10: Ensuring to satisfy the TX power constraint $\|\mathbf{AD}\|_F^2 = N_s$

Single-User Hybrid Precoding

<Performance Evaluation>

- Simulation parameters:
  - Three methods comparision:
    - (1). Spatially sparse precoding
    - (2). Optimal unconstrained precoding ($\mathbf{P}_{opt} = \mathbf{V}_1$, fully digital precoding)
    - (3). Beam steering precoding (fully analog beamforming)
  - L=3 propagation paths with uniformly distributed AoAs/AoDs
  - BS sector angle: AZ=60^o, EL=20^o wide

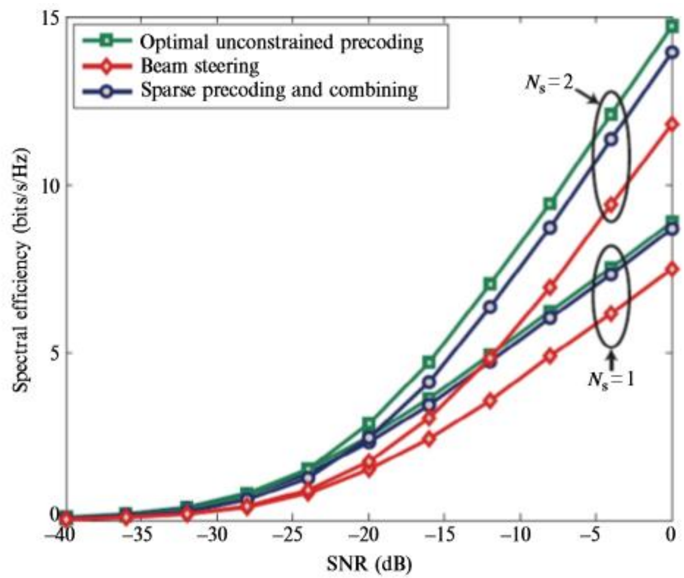
    MS: smaller antenna arrays of omnidirectional elements (steer any direction)
  - Element spacing $d = \lambda/2, \quad SNR = \rho/\sigma_n^2$

Single-User Hybrid Precoding

- 64x16 mmWave massive MIMO system with planar arrays at BS and MS
- BS uses $N_t^{RF} = 4$ RF chains to TX $N_s = 1$ or 2 streams

Simulation results:

- Sum-rate comparison in 64x16 mmWave massive MIMO system with $N_t^{RF} = 4$



Sum-rate comparison in a 64 × 16 mmWave massive MIMO system with $N_t^{RF} = 4$.

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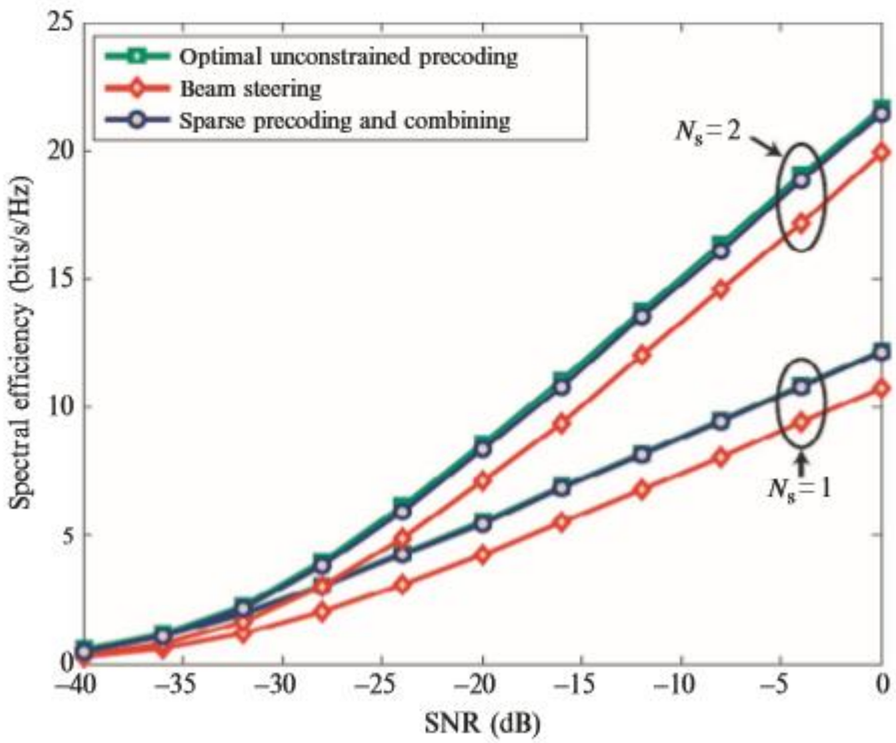
Single-User Hybrid Precoding

- Sum-rate comparison in 256x64 mmWave massive MIMO system with $N_t^{RF} = 6$



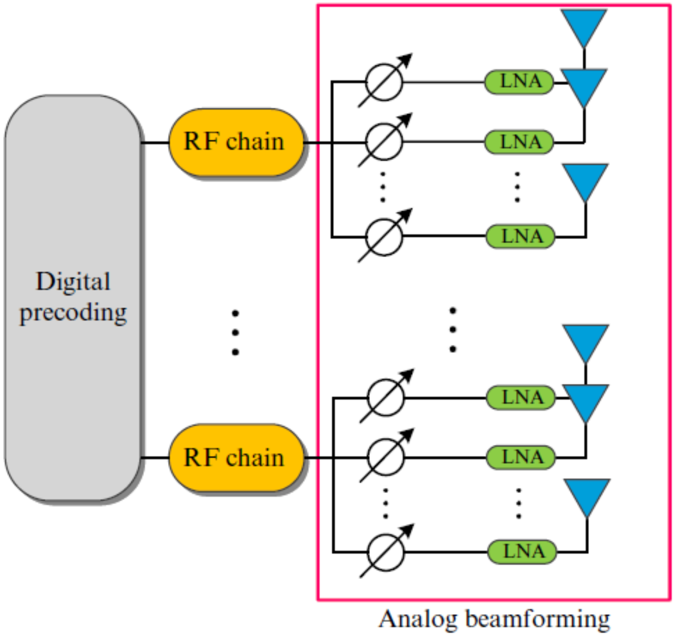
Sum-rate comparison in a 256 × 64 mmWave massive MIMO system with $N_t^{RF} = 6$.

$N_t^{RF} \uparrow$

Sparse precoding close to optimal precoding

Single-User Hybrid Precoding
SIC-Based Hybrid Precoding (Subconnected)

- For the subconnected architecture, each RF chain is connected to only a subset of BS antennas.



Subconnected architecture of hybrid precoding for single-user mmWave massive MIMO system.

- Reducing the number of required PSs from $N_t \times N_t^{RF}$ to N_t

→low computation complexity

SIC-Based Hybrid Precoding (Subconnected)

- Consider the single-user mmWave massive MIMO system:
  - BS with N_t antennas and N_t^{RF} RF chains
  - Each RF chain connected to one subantenna array with M antennas, i.e., $N_t = N_t^{RF} M$
  - BS TX $N_s = N_t^{RF}$ streams (fully spatial multiplexing gain)
  - MS user with N_r receiver antennas
  - The digital precoder $\mathbf{D}_{N_t^{RF} \times N_s = N_s \times N_s}$ being specialized to a diagonal matrix $\mathbf{D} = diag[d_1\ d_2 \cdots d_{N_s}]$ (**D**: power allocation)

SIC-Based Hybrid Precoding (Subconnected)

- The analog beamformer $\mathbf{A}_{N_t \times N_t^{RF} = N_t \times N_s}$ is a special block diagonal structure

$$\mathbf{A} = \begin{bmatrix} \bar{\mathbf{a}}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{a}}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \bar{\mathbf{a}}_{N_s} \end{bmatrix}_{N_t \times N_s}, \qquad \bar{\mathbf{a}}_n \in \mathbb{C}^{M \times 1}$$

where the elements $\bar{\mathbf{a}}_n$ have the same amplitude $1/\sqrt{M}$ but different phases.

SIC-Based Hybrid Precoding (Subconnected)

<Basic Idea>

- Maximizing the total achievable rate R(**P**) to design the hybrid precoder **P**=**AD**

$$R(\mathbf{P}) = \log_2\left(\left|I + \frac{\rho}{N_s\sigma_n^2}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\right|\right)$$

where

$$\mathbf{P} = \mathbf{A}\mathbf{D} = diag\left\{\bar{\mathbf{a}}_1,\ldots,\bar{\mathbf{a}}_{N_s}\right\}\cdot diag\left\{d_1,\cdots d_{N_s}\right\}$$

SIC-Based Hybrid Precoding (Subconnected)

- Satisfying the three constraints:

(1) $\mathbf{P} = diag\left\{\bar{\mathbf{p}}_1, \ldots, \bar{\mathbf{p}}_{N_s}\right\}, \bar{\mathbf{p}}_n = d_n \bar{\mathbf{a}}_n$ is the $M \times 1$ vector of $\mathbf{p}_n$ of $\mathbf{P}$

$$\mathbf{p}_n = [\mathbf{0}_{1\times M(n-1)}, \bar{\mathbf{p}}_n^T, \mathbf{0}_{1\times M(N_s - n)}]^T$$

(2) The amplitude of the elements of **A** is fixed to $\frac{1}{\sqrt{M}}$

The elements of each column of **P** have the same amplitude due to the diagonal D.

(3) $\left\|\mathbf{P}\right\|_F \le N_s$ meeting the total TX power constraint

→(1) & (2) nonconvex constraints

SIC-Based Hybrid Precoding (Subconnected)

- Based on the special block diagonal hybrid precoding **P**, the precoding on different subarrays is independent.
  - Decompose the total sum rate into a series of subrate optimation problems, each subrate only considers one subarray.
- Diving **P** as $\mathbf{P} = \left[\mathbf{P}_{N_s-1}\ \mathbf{p}_{N_s}\right]$, $\mathbf{p}_{N_s}$ is the $N_s th$ columns of **P**, $\mathbf{P}_{N_s-1}$ is the first (N_s-1) columns of **P**. $(N_s M \times (N_s-1))$

The total achievable rate R(**P**):

$$R(\mathbf{P}) \overset{(a)}{=} \log_2(\left|\mathbf{T}_{N_s-1}\right|) + \log_2\left(\left|\mathbf{I} + \frac{\rho}{N_s\sigma_n^2}\mathbf{T}_{N_s-1}^{-1}\mathbf{H}\mathbf{p}_{N_s}\mathbf{p}_{N_s}^H\mathbf{H}^H\right|\right)$$

$$\overset{(b)}{=} \log_2(\left|\mathbf{T}_{N_s-1}\right|) + \log_2\left(\left|\mathbf{I} + \frac{\rho}{N_s\sigma_n^2}\mathbf{p}_{N_s}^H\mathbf{H}^H\mathbf{T}_{N_s-1}^{-1}\mathbf{H}\mathbf{p}_{N_s}\right|\right)$$

SIC-Based Hybrid Precoding (Subconnected)

where

$$(a): \mathbf{T}_{N_s-1} = \mathbf{I} + \frac{\rho}{N_s\sigma_n^2}\mathbf{H}\mathbf{P}_{N_s-1}\mathbf{P}_{N_s-1}^H\mathbf{H}^H$$

$$(b): \left|\mathbf{I}+\mathbf{XY}\right| = \left|\mathbf{I}+\mathbf{YX}\right| \ , \mathbf{X} = \mathbf{T}_{N_s-1}^{-1}\mathbf{H}\mathbf{p}_{N_s} \ , \mathbf{Y} = \mathbf{p}_{N_s}^H\mathbf{H}^H$$

The right side of (b) is the achievable subrate of the $N_s th$ subarray.

→ Next, further decomposing $\log_2\left(\left|\mathbf{T}_{N_s-1}\right|\right)$:

$$\log_2\left(\left|\mathbf{T}_{N_s-2}\right|\right) + \log_2(\mathbf{I} + \frac{\rho}{N_s\sigma_n^2}\mathbf{p}_{N_{s-1}}^H\mathbf{H}^H\mathbf{T}_{N_s-2}^{-1}\mathbf{H}\mathbf{p}_{N_{s-1}})$$

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SIC-Based Hybrid Precoding (Subconnected)

- After N decompositions, the total achievable rate R:

$$R = \sum_{n=1}^{N_s} \log_2 \left(I + \frac{\rho}{N_s \sigma_n^2} \mathbf{p}_n^H \mathbf{H}^H \mathbf{T}_{n-1}^{-1} \mathbf{H} \mathbf{p}_n \right)$$

$$\mathbf{T}_n = I + \frac{\rho}{N_s \sigma_n^2} \mathbf{H} \mathbf{P}_n \mathbf{P}_n^H \mathbf{H}^H \quad , \mathbf{T}_0 = \mathbf{I}_{N_s}$$

→Optimal the subrate of 1st subantenna and update the $\mathbf{T}_1$ matrix , until the last subarray , i.e. , SIC-based hybrid precoding

SIC-Based Hybrid Precoding (Subconnected)

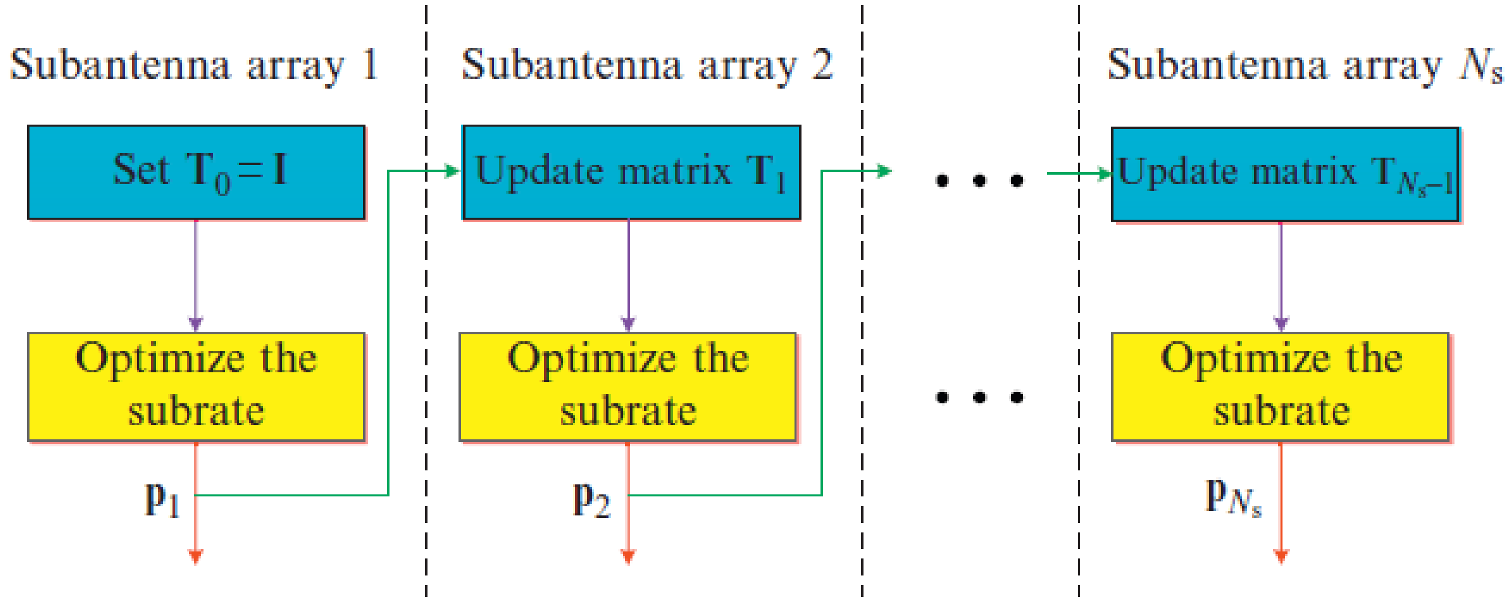

Diagram of the SIC-based hybrid precoding.

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SIC-Based Hybrid Precoding (Subconnected)

- Focusing on subrate optimization problem of the *n*th subarray:

$$\mathbf{p}_n^{\mathrm{opt}} = \underset{\mathbf{p}_n \in \mathcal{F}}{\arg\max} \log_2\left(1 + \frac{\rho}{N_s \sigma_n^2} \mathbf{p}_n^H \mathbf{G}_{n-1} \mathbf{p}_n\right)$$

where

$$\mathbf{G}_{n-1} = \mathbf{H}^H \mathbf{T}_{n-1}^{-1} \mathbf{H}$$

F is the set of the feasible vectors satisfying three constraints.

- $\mathbf{p}_n$ only has M nonzero elements $\bar{\mathbf{p}}_n$, the subrate optimization problem:

$$\bar{\mathbf{p}}_n^{\mathrm{opt}} = \underset{\bar{\mathbf{p}}_n \in \bar{\mathcal{F}}}{\arg\max} \log_2\left(1 + \frac{\rho}{N_s \sigma_n^2} \bar{\mathbf{p}}_n^H \bar{\mathbf{G}}_{n-1} \bar{\mathbf{p}}_n\right)$$

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SIC-Based Hybrid Precoding (Subconnected)

- where

$$\bar{\mathbf{G}}_{n-1}(\text{Submatrix of } \mathbf{G}_n) = \mathbf{R}\mathbf{G}_{n-1}\mathbf{R}^H = \mathbf{R}\mathbf{H}^H\mathbf{T}_{n-1}^{-1}\mathbf{H}\mathbf{R}^H$$

$$\mathbf{R} = \left[\mathbf{0}_{M\times M(n-1)} \ \mathbf{I}_M \ \mathbf{0}_{M\times M(N_s-n)}\right]$$

$\bar{\mathbf{F}}$ is the $M\times 1$ feasible vectors satisfying constraints 2 & 3

→ $\bar{\mathbf{G}}_{n-1} = \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^H$, the optimal unconstrained precoding vector:

$\bar{\mathbf{p}}_n^{opt} = \mathbf{v}_1$ (the first column of $\mathbf{V}$)

(Not obey the constraint 2 , same amplitude)

SIC-Based Hybrid Precoding (Subconnected)

- Convert to the distance optimization problem : (equivalent)

$$\overline{\mathbf{p}}_n^{\text{opt}} = \arg\min_{\overline{\mathbf{p}}_n \in \overline{\mathcal{F}}} \|\mathbf{v}_1 - \overline{\mathbf{p}}_n\|_2^2.$$

  - The distance can be expressed by : (Using $\overline{\mathbf{p}}_n = d_n \overline{\mathbf{a}}_n$ constraints)

$$\left\|\mathbf{v}_1 - \overline{\mathbf{p}_n}\right\|_2^2 = 1 + d_n^2 - 2d_n \operatorname{Re}(\mathbf{v}_1^H \overline{\mathbf{a}}_n)$$

$$= (d_n - \operatorname{Re}(\mathbf{v}_1^H \overline{\mathbf{a}}_n))^2 + (1 - [\operatorname{Re}(\mathbf{v}_1^H \overline{\mathbf{a}}_n)]^2)$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

$$1st \ \ \text{min.} \qquad\qquad\qquad 2nd \ \text{min.}$$

SIC-Based Hybrid Precoding (Subconnected)

where $\mathbf{v}_1^H \mathbf{v}_1 = 1$, $\bar{\mathbf{a}}_n^H \bar{\mathbf{a}}_n = 1$, each elements $\bar{\mathbf{a}}_n$ has the same amplitude $\frac{1}{\sqrt{M}}$

→1st min. : $d_n^{\text{opt}} = \mathrm{Re}\left(\mathbf{v}_1^H \bar{\mathbf{a}}_n\right)$

$$2^{\text{nd}}\,\text{min.} = \max\left|\mathrm{Re}\left(\mathbf{v}_1^H \bar{\mathbf{a}}_n\right)\right| \quad (\because \mathbf{v}_1^H \mathbf{v}_1 = 1, \bar{\mathbf{a}}_n^H \bar{\mathbf{a}}_n = 1)$$

$$\therefore \bar{\mathbf{a}}_n^{opt} = \frac{1}{\sqrt{M}} e^{j angle(\mathbf{v}_1)} \quad \textit{(elements of } \bar{\mathbf{a}}_n \textit{ with same amplitude)}$$

$$\therefore get\ d_n^{opt}:$$

$$d_n^{opt} = \mathrm{Re}(\mathbf{v}_1^H \bar{\mathbf{a}}_n) = \frac{1}{\sqrt{M}} \cdot \mathrm{Re}(\mathbf{v}_1^H e^{j angle(\mathbf{v}_1)}) = \frac{\|\mathbf{v}_1\|_1}{\sqrt{M}}$$

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SIC-Based Hybrid Precoding (Subconnected)

- Get the optimal $\overline{\mathbf{p}}_n^{\mathrm{opt}}$

$$\overline{\mathbf{p}}_n^{\mathrm{opt}} = d_n^{\mathrm{opt}} \overline{\mathbf{a}}_n^{\mathrm{opt}} = \frac{1}{M} \|\mathbf{v}_1\|_1 e^{j\mathrm{angle}(\mathbf{v}_1)}$$

where each element v_i of $\mathbf{v}_1$ has amplitude <u>less than one</u>,

i.e. , $\left\|\overline{\mathbf{p}}_n^{\mathrm{opt}}\right\|_2^2 \leq 1$

$$\therefore \|\mathbf{P}^{\mathrm{opt}}\|_F^2 = \left\|\mathrm{diag}\left\{\overline{\mathbf{p}}_1^{\mathrm{opt}}, \ldots, \overline{\mathbf{p}}_N^{\mathrm{opt}}\right\}\right\|_F^2 \leq N_s$$ (satisfying constraint 3)

TX total PWR

- After acquiring $\overline{\mathbf{p}}_n^{\mathrm{opt}}$, $\mathbf{T}_n$ and $\overline{\mathbf{G}}_n$ can be updated to find next $\overline{\mathbf{p}}_{n+1}^{opt}$

SIC-Based Hybrid Precoding (Subconnected)

- Summary the following three steps:

Step 1: Execute the SVD of $\overline{\mathbf{G}}_{n-1}$ to obtain $\mathbf{v}_1$.

Step 2: Let $\overline{\mathbf{p}}_n^{\mathrm{opt}} = \frac{1}{M}\|\mathbf{v}_1\|_1 e^{j\mathrm{angle}(\mathbf{v}_1)}$ be the optimal solution to the current nth subantenna array.

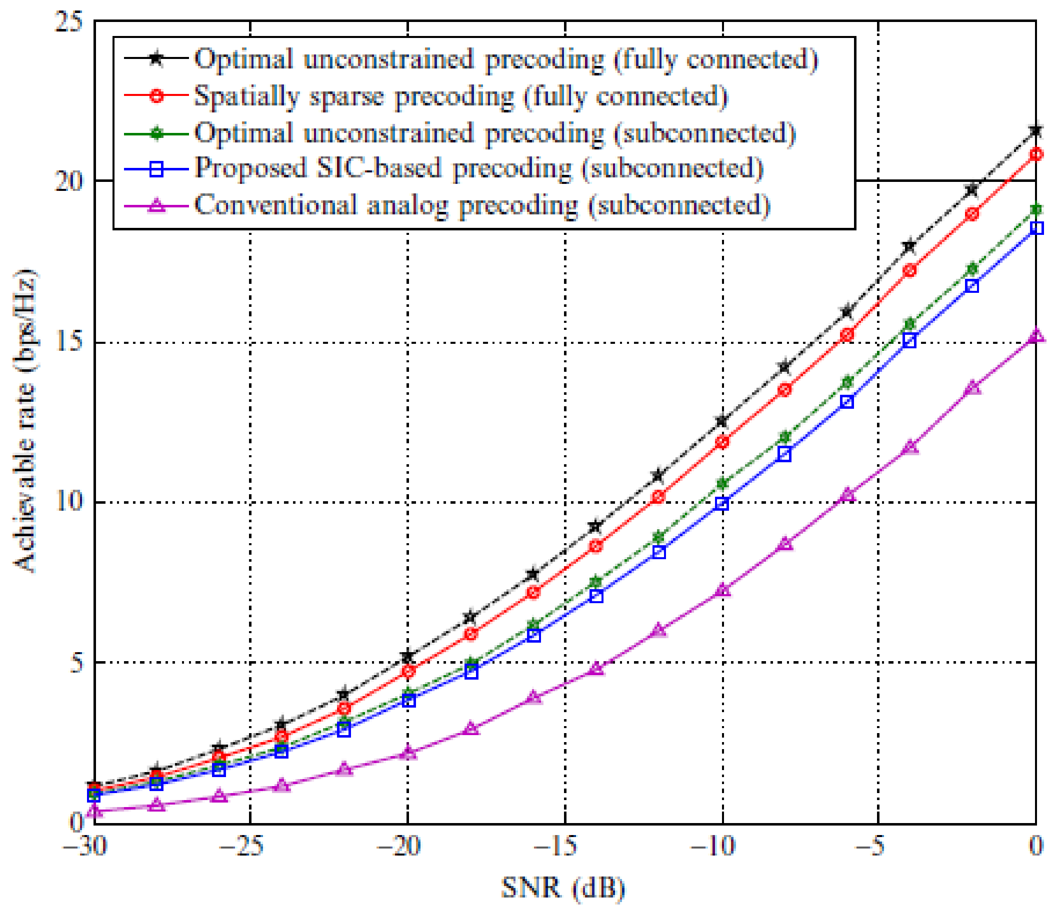
Step 3: Update matrices $\mathbf{T}_n = \mathbf{I} + \frac{\rho}{N_s \sigma_n^2}\mathbf{H}\mathbf{P}_n\mathbf{P}_n^H\mathbf{H}^H$ and $\overline{\mathbf{G}}_n = \mathbf{R}\mathbf{H}^H\mathbf{T}_n^{-1}\mathbf{H}\mathbf{R}^H$ for the next $(n+1)$th subantenna array.

SIC-Based Hybrid Precoding (Subconnected)

<Performance Evaluation>

- Simulation parameters:
  - .. L=3 (channel paths)
  - .. carrier frequency =28GHz
  - .. $d = \lambda/2$
  - .. BS (AoDs = $[-\pi/6, \pi/6]$) , MS (AoAs =$[-\pi, \pi]$)
  - .. $N_t \times N_r = 64 \times 16$ ($N_t^{RF} = N_s = 8$)

SIC-Based Hybrid Precoding (Subconnected)



Achievable rate comparison for an $N_t \times N_r = 64 \times 16$ ($N_t^{RF} = N_s = 8$) mmWave massive MIMO system.

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SIC-Based Hybrid Precoding (Subconnected)

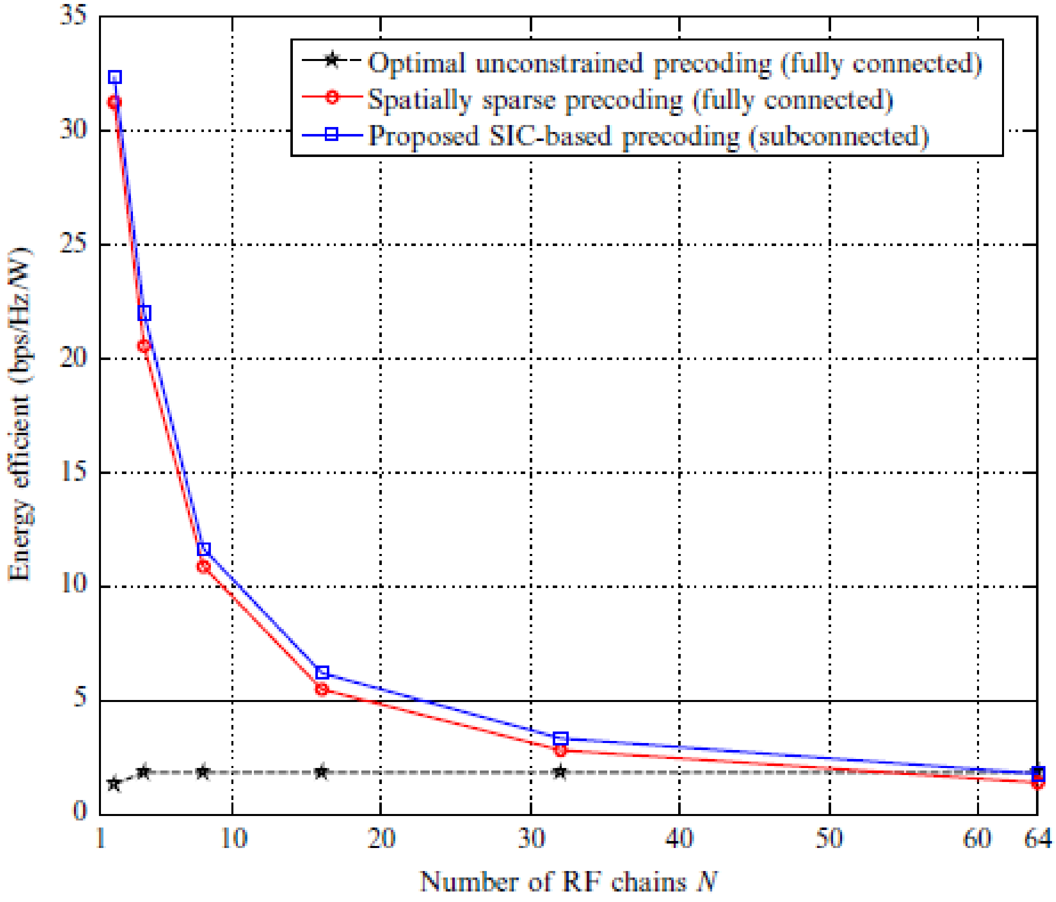
- Energy efficiency η:

$$\eta = \frac{R}{P_{\text{total}}} = \frac{R}{P_{\text{t}} + N_{\text{RF}} P_{\text{RF}} + N_{\text{PS}} P_{\text{PS}}} \ (\text{bps/Hz/W})$$

($P_{\text{RF}} = 250\,\text{mW}$, $P_{\text{PS}} = 1\,\text{mW}$, $P_t = 1W(30dBm)$, $N_{\text{t}} \times N_{\text{r}} = 64 \times 64$)

($N_{\text{t}}^{\text{RF}} = 1, 2, 4, \ldots, 64$)

SIC-Based Hybrid Precoding (Subconnected)

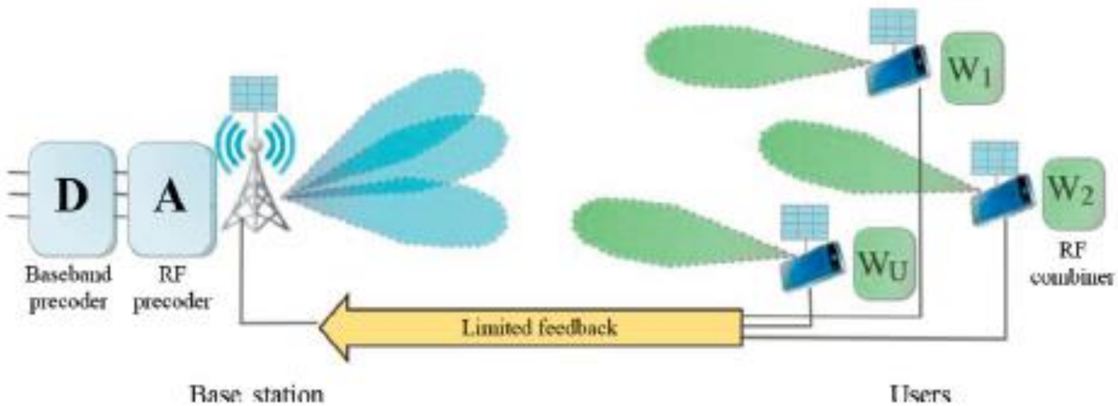


Energy efficiency comparison against the number of RF chains N_t^{RF}, where $N_t \times N_r = 64 \times 64$, SNR $= 0$ dB.

5.2 Multiuser Hybrid Precoding

<System Model>

- Hardware architecture of hybrid precoding for multiuser mmWave massive MIMO Systems



Hardware architecture of hybrid precoding for multiuser mmWave massive MIMO systems.

- BS : N_{BS} antennas, N_{RF} RF chains $(N_{\mathrm{RF}} \le N_{\mathrm{BS}})$,communicating with U MSs
- Each MS: N_{MS} antennas, only one RF chain

Multiuser Hybrid Precoding

- BS communicates with every MS via only one stream
- Total data streams , $N_S = U \le N_{RF}$
- Fully connected architecture

- For the downlink , BS employs a $U \times U$ digital precoder $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_U]$ and an $N_{BS} \times U$ analog precoder $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_U]$. , TX signal:

$$\mathbf{x} = \mathbf{A}\mathbf{D}\mathbf{s}$$

where $\underline{s}_{U\times 1}$ original signal vector, $\mathbb{E}(\mathbf{s}\mathbf{s}^H) = (\rho/U)\mathbf{I}_U$,

ρ : average TX power each user with equal power allocation

Multiuser Hybrid Precoding

all the elements of A with the same constant amplitude N_{BS}^{-1}
the total TX power constraint with normalizing **D** to satisfy
$\|\mathbf{AD}\|_F^2 = U$.

Multiuser Hybrid Precoding

- Consider the narrowband block-fading channel model. The RX signal vector $\underline{r}_u$ of the uth MS:

$$\mathbf{r}_u = \mathbf{H}_u \sum_{n=1}^{U} \mathbf{A}\mathbf{d}_n s_n + \mathbf{n}_u$$

where $\mathbf{H}_u$: $N_{\text{MS}} \times N_{\text{BS}}$ channel matrix between BS and the uth MS

$\mathbf{n}_u$: $\mathcal{CN}\left(0, \sigma_n^2\right)$

- Analog combiner $\mathbf{w}_u$ of the uth MS is used to combine the RX signal $\mathbf{r}_u$:

$$y_u = \mathbf{w}_u^H \mathbf{r}_u = \mathbf{w}_u^H \mathbf{H}_u \sum_{n=1}^{U} \mathbf{A}\mathbf{d}_n s_n + \mathbf{w}_u^H \mathbf{n}_u$$

Multiuser Hybrid Precoding

where $\mathbf{w}_u$ is the same constraint as the analog precoder $\mathbf{A}$, i.e. , all the elements of $\mathbf{w}_u$ having the same amplitude N_{MS}^{-1} but different phases. (MS only analog BF, lower cost)

Multiuser Hybrid Precoding

<Two-Stage Hybrid Precoding>

- Goal : Designing BS analog precoder **A** and digital precoder **D**, MS analog combiner $\{\mathbf{w}_u\}_{u=1}^{U}$ to max sum rate:

$$R = \sum_{u=1}^{U} R_u$$

$$R_u = \log_2\left(1 + \frac{\frac{P}{U}\left|\mathbf{w}_u^H \mathbf{H}_u \mathbf{A} \mathbf{d}_u\right|^2}{\frac{P}{U}\sum_{n \neq u}\left|\mathbf{w}_u^H \mathbf{H}_u \mathbf{A} \mathbf{d}_n\right|^2 + \sigma_n^2}\right)$$

Multiuser Hybrid Precoding

- Precoding design problem to find $\mathbf{A}^{\text{opt}}, \mathbf{D}^{\text{opt}}, \{\mathbf{w}_u^{\text{opt}}\}_{u=1}^{U}$ that solve

$$\left\{\mathbf{A}^{\text{opt}}, \mathbf{D}^{\text{opt}}, \{\mathbf{w}_u^{\text{opt}}\}_{u=1}^{U}\right\} = \arg\max \sum_{u=1}^{U} \log_2\left(1 + \frac{\frac{P}{U}\left|\mathbf{w}_u^H \mathbf{H}_u \mathbf{A} \mathbf{d}_u\right|^2}{\frac{P}{U}\sum_{n \neq u}\left|\mathbf{w}_u^H \mathbf{H}_u \mathbf{A} \mathbf{d}_n\right|^2 + \sigma_n^2}\right)$$

$$\begin{aligned} \text{s.t.} \quad & \mathbf{a}_u \in \mathcal{F}, \quad u = 1,2,\ldots,U, \\ & \mathbf{w}_u \in \mathcal{W}, \quad u = 1,2,\ldots,U, \\ & \|\mathbf{A}\mathbf{D}\|_F^2 = U. \end{aligned}$$

- Step 1: Search over the entire $\mathcal{F}^U \times \mathcal{W}^U$ of all possible $\{\mathbf{a}_u\}_{u=1}^{U}$ and $\{\mathbf{w}_u\}_{u=1}^{U}$ to max each user power.
- Step 2: Using iterative method to find the digital precoder **D** and suppress interferences.

Multiuser Hybrid Precoding

ALGORITHM 2 TWO-STAGE HYBRID PRECODING

Inputs: $\mathcal{F}$BS analog precoding codebook

$\mathcal{W}$MS analog combining codebook

First stage: Single-user analog precoding/combining design

For each MS u, $u = 1, 2, \ldots, U$

The BS and MS u select $\mathbf{v}_u^{\text{opt}}$ and $\mathbf{g}_u^{\text{opt}}$ respectively that solve

$$\left\{\mathbf{g}_u^{\text{opt}}, \mathbf{v}_u^{\text{opt}}\right\} = \underset{\substack{\forall \mathbf{g}_u \in \mathcal{W} \\ \forall \mathbf{v}_u \in \mathcal{F}}}{\arg\max} \left\| \mathbf{g}_u^H \mathbf{H}_u \mathbf{v}_u \right\|$$

MS u sets $\mathbf{w}_u = \mathbf{g}_u^{\text{opt}}$

BS sets $\mathbf{A} = \left[\mathbf{v}_1^{\text{opt}}, \mathbf{v}_2^{\text{opt}}, \ldots, \mathbf{v}_U^{\text{opt}}\right]$

Second stage: Multiuser digital precoding design

For each MS u, $u = 1, 2, \ldots, U$

MS u estimates its effective channel $\overline{\mathbf{h}}_u = \mathbf{w}_u^H \mathbf{H}_u \mathbf{A}$

MS u feeds back $\overline{\mathbf{h}}_u$ to the BS

BS employs ZF digital precoder $\mathbf{D} = \overline{\mathbf{H}}^H \left(\overline{\mathbf{H}}\,\overline{\mathbf{H}}^H\right)^{-1}$, where $\overline{\mathbf{H}} = \left[\overline{\mathbf{h}}_1^T, \overline{\mathbf{h}}_2^T, \ldots, \overline{\mathbf{h}}_U^T\right]^T$

$$\mathbf{d}_u = \frac{\mathbf{d}_u}{\|\mathbf{A}\mathbf{d}_u\|_F}, \; u = 1, 2, \ldots, U$$

Multiuser Hybrid Precoding

- In first stage , using the beam training algorithms to design the analog precoding and combining vectors (without channel estimation)
- In second stage , each MS u estimates the effective channel $\bar{\mathbf{h}}_u = \mathbf{w}_u^H \mathbf{H}_u \mathbf{A}$. Then, $\bar{\mathbf{h}}_u$ is fed back to BS. ($\bar{\mathbf{h}}_{U\times 1} \square \ \mathbf{H}_u$)
- Finally , BS uses ZF digital precoder via $\bar{\mathbf{H}} = [\bar{\mathbf{h}}_1, \bar{\mathbf{h}}_2, \ldots, \bar{\mathbf{h}}_U]$ to find $\mathbf{D}$. (with normalization)

Multiuser Hybrid Precoding

<Performance Evaluation>

- Simulation parameters:
  - BS: 8×8 UPA with 4 MSs , AZ AoAs/AoDs with uniformly $[0, 2\pi]$
  - MS: each having 4×4 UPA , EL AoAs/AoDs with uniformly $[-\pi/2, \pi/2]$
  - Channel of each user has " one path "

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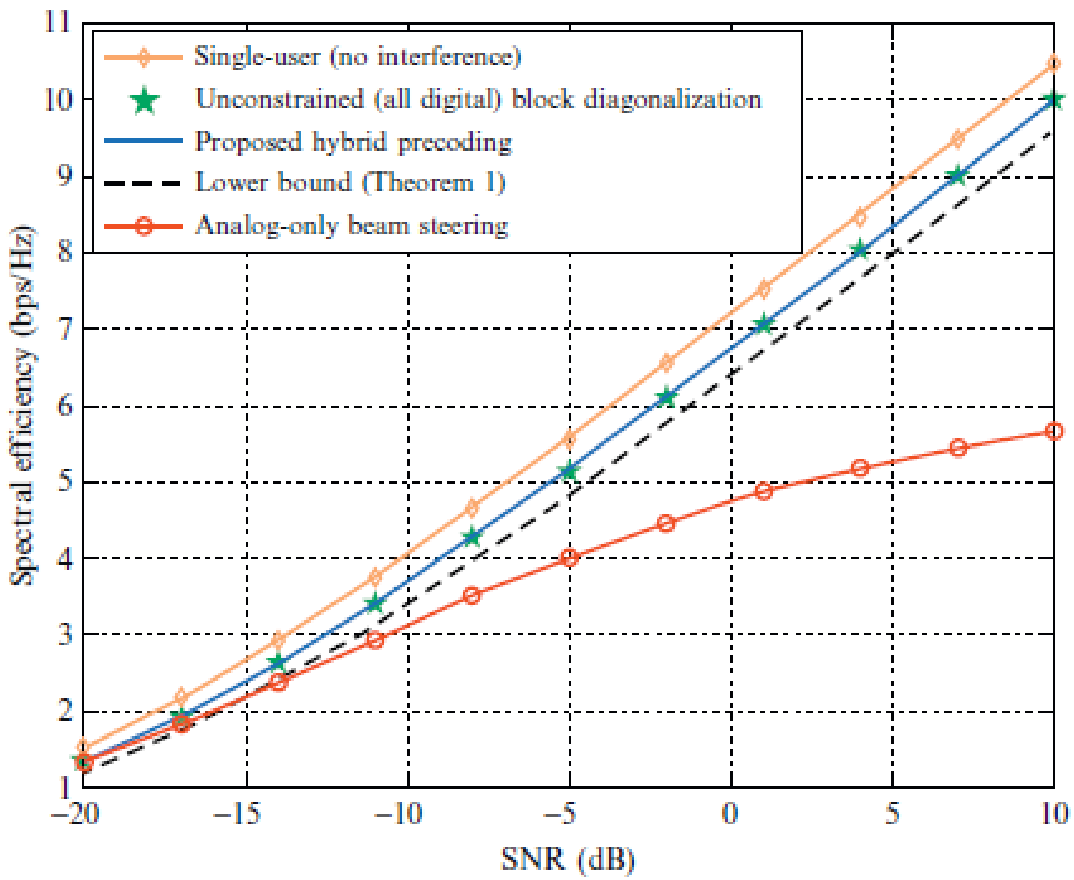
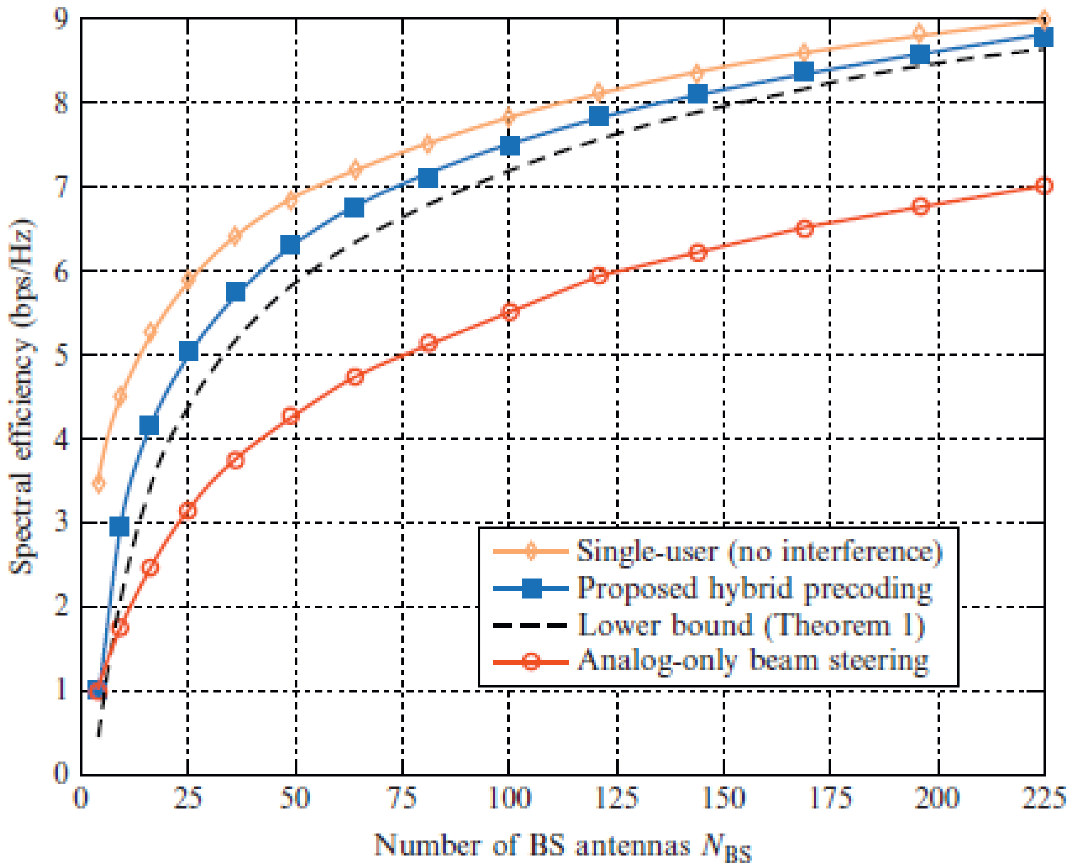
Multiuser Hybrid Precoding



Achievable rate comparison for an $N_{BS} = 64$, $N_{MS} = 16$, and $U = 4$ multiuser mmWave massive MIMO system.

Multiuser Hybrid Precoding



Achievable rate comparison against the number of BS antennas N_{BS}.

6. Conclusions

- Digital precoding aims to cancel data streams interference.
- Analog beamforming designs to improve the antenna array gain.
- Hybrid precoding combines the advantages of digital precoding and analog beamforming.
- Hybrid precoding seems more appropriate for future mmWave massive MIMO systems.
- Only a small-size digital precoder via a smaller number of RF chains will be required.

Thanks for your attention!

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