Simulation Techniques for 5G Transmission

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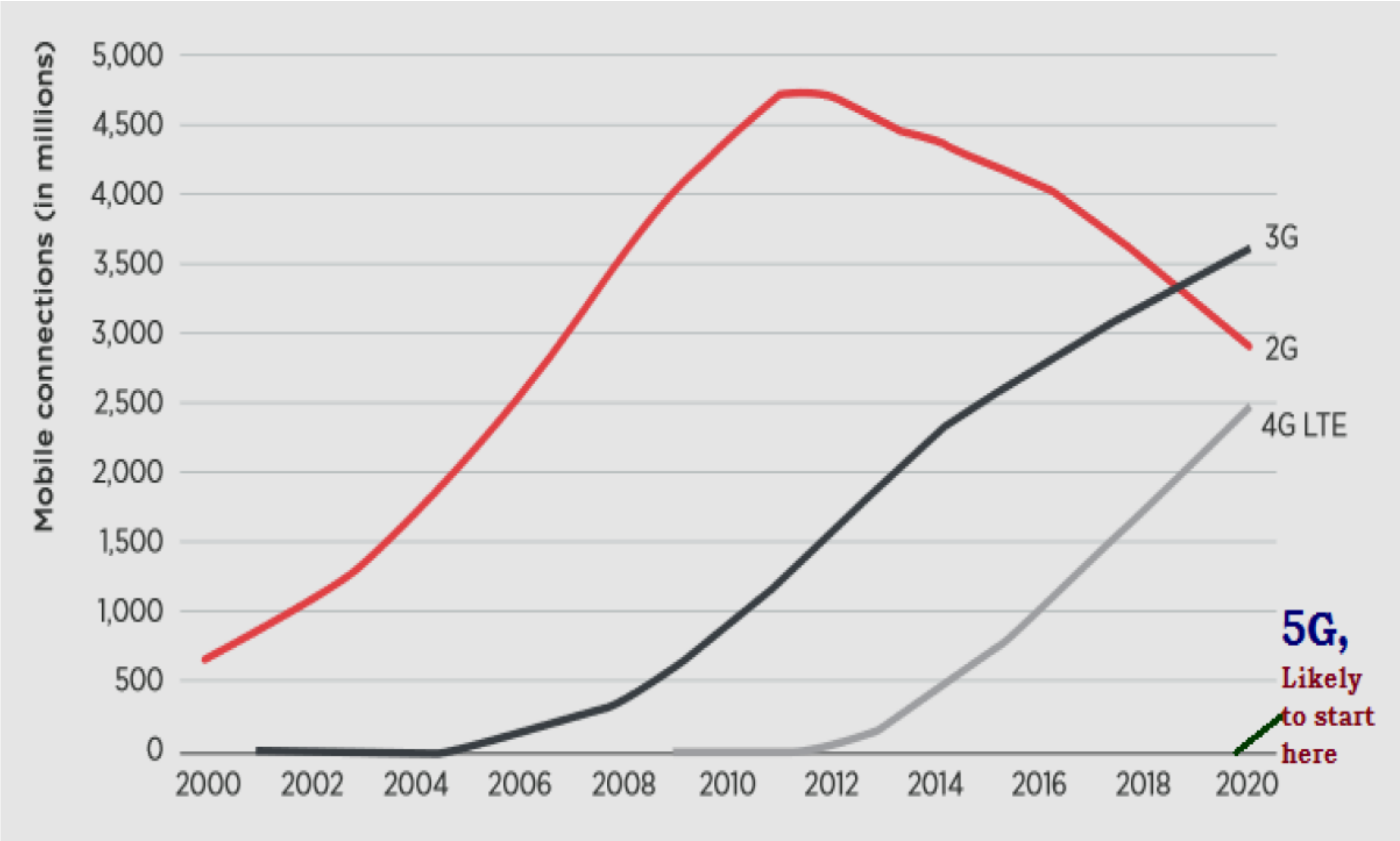
Content

1. Introduction to 5G

- Wireless communication has experienced a rapid growth and evolution since 1980s (1G, …4G, or now 5G).

- Evolution:



Mobile connections (in millions)
5,000
4,500
4,000
3,500
3,000
2,500
2,000
1,500
1,000
500
0
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
2020
3G
2G
4G LTE
5G,
Likely
to start
here

- What is 5G?
  - 5G is the fifth generation technology, and it has many advanced features potential enough to change our life dramatically.

- Targets:

https://5g-ppp.eu/#

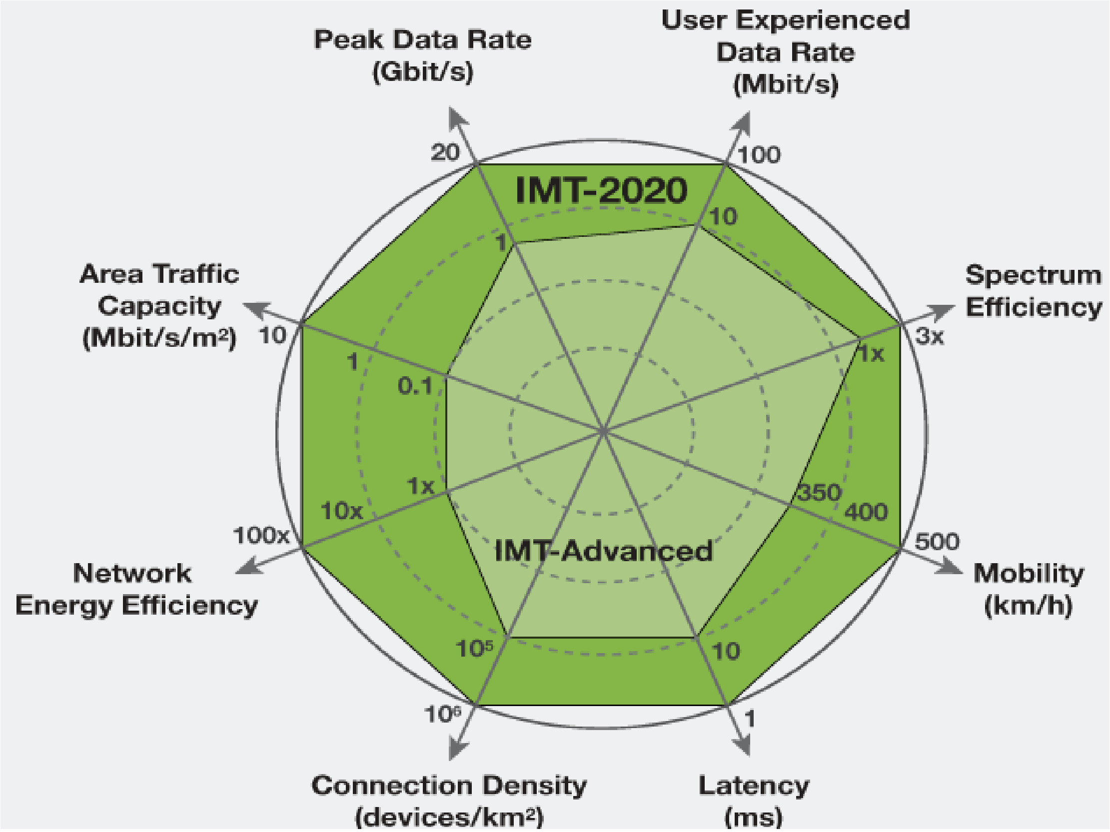
- Features:
  - High increased peak bit rate
  - Larger data volume per unit area
  - High capacity
  - Lower battery consumption
  - Better connectivity irrespective of the geographic region
  - Larger number of supporting devices
  - Lower cost of infrastructural development
  - Higher reliability of the communications

- How to increase the capacity by 1000 times?

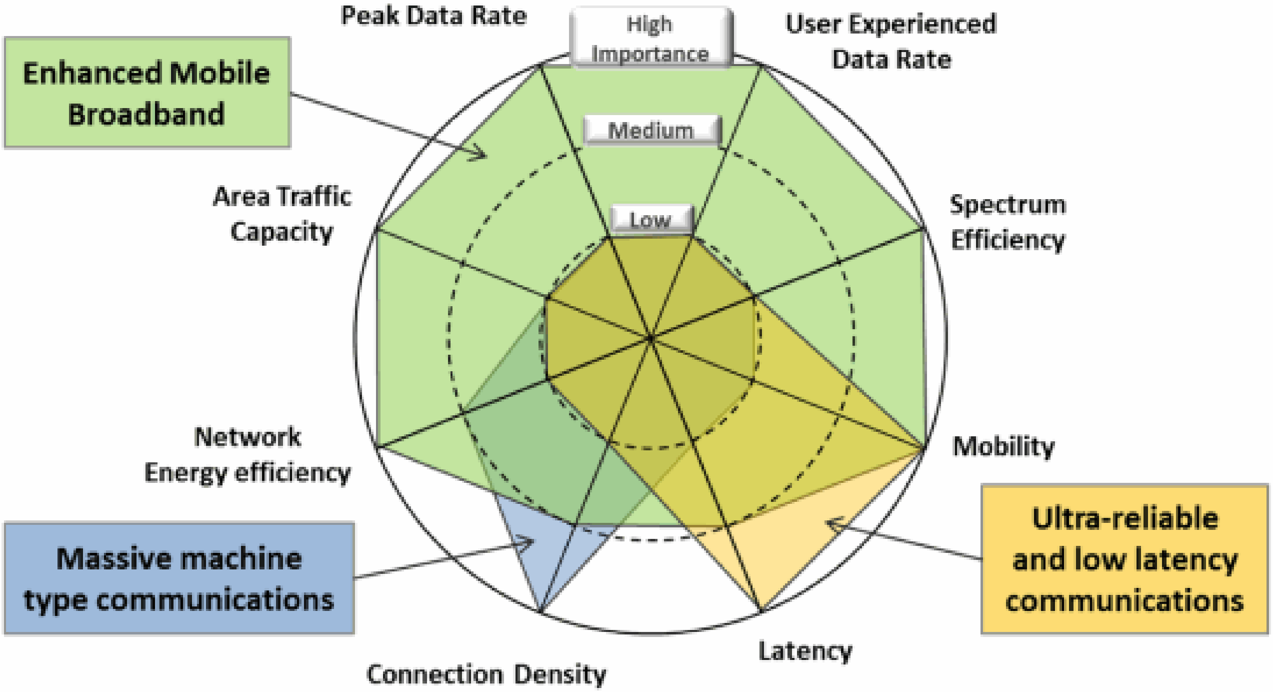
- Better spectral efficiency (4):
  - Spectral efficiency: 30 bps/Hz → ?
  - Key enabling technology, MIMO, …
- Larger spectrum (5):
  - Carrier bandwidth Increase: 100MHz →500MHz/1GHz
  - Spectrum availability?
    - Higher band: millimeter wave (mmWave)
    - Spectrum sharing
    - Unlicensed bands
- More cells, network densification (50):
  - Greatly reduce the coverage of a cell, i.e., dramatically increase the cell number
  - Key enabling technology, ultra-dense small cells

- Technology challenges:
  - Authorized shared access
  - Unlicensed bands
  - mmWave
  - Massive MIMO
  - Phase antenna array
  - Beam-forming and beam-tracking
  - Small cell
  - Interference management
  - Full duplex radios
  - SDN and NFV
  - …

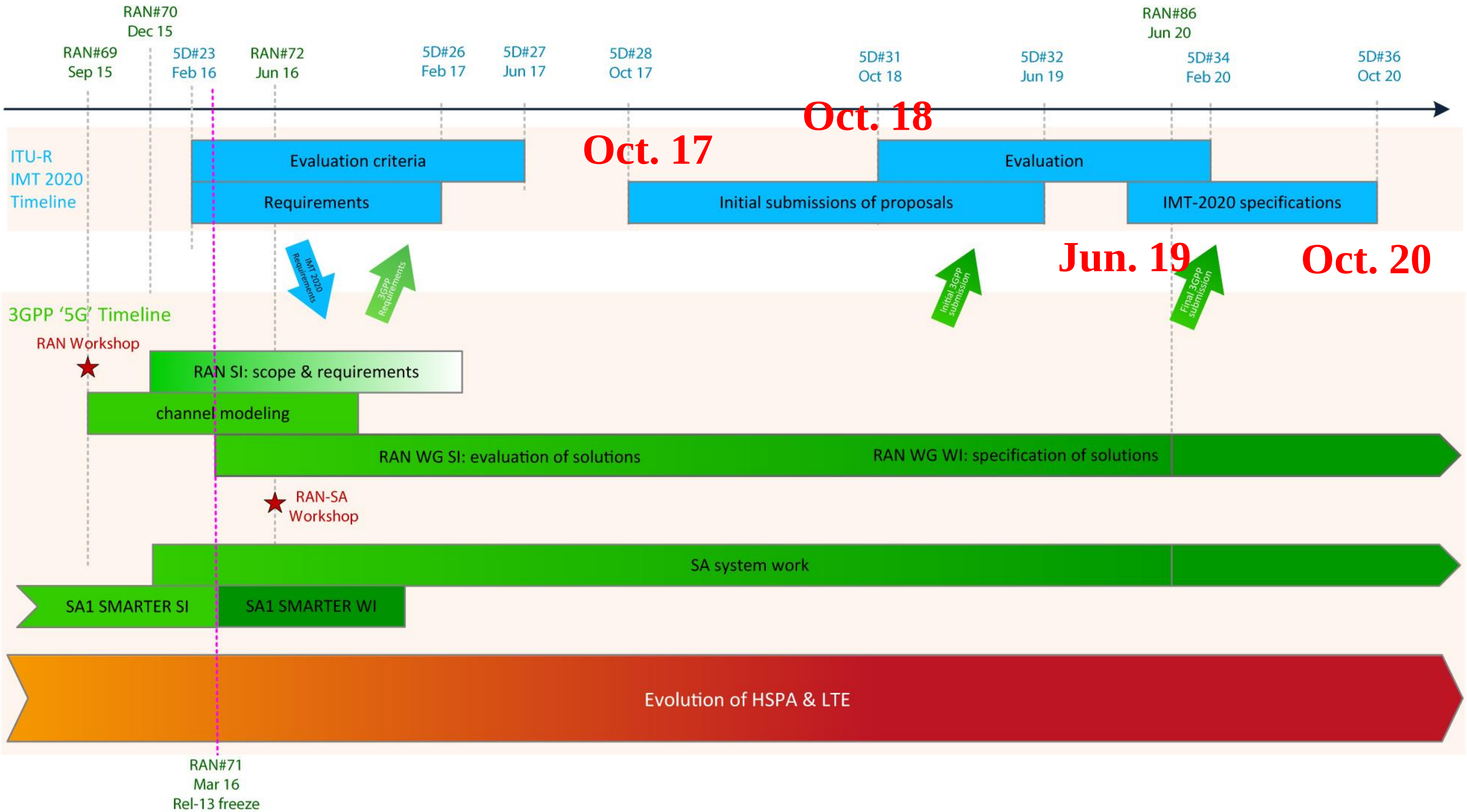
- ITU has defined three usage scenarios in 5G
  - Enhanced mobile broadband
  - Massive machine type communications
  - Ultra-reliable and low latency communications
- Key features:



- Three scenarios:

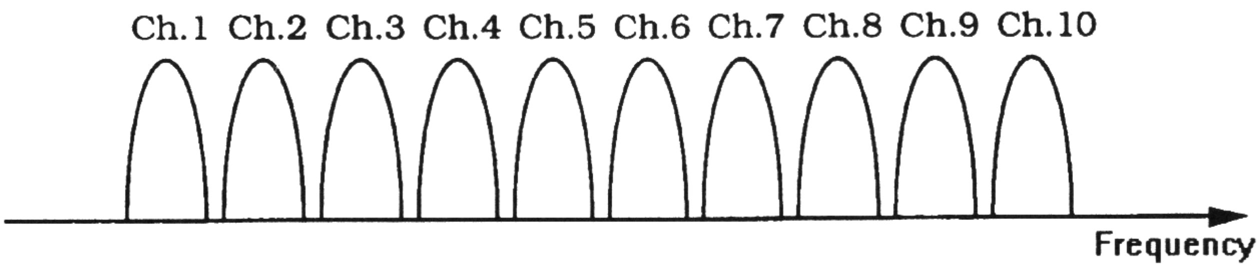


- 3GPP RAN workshop on 5G (9/18/2015):



2. Introduction to OFDM

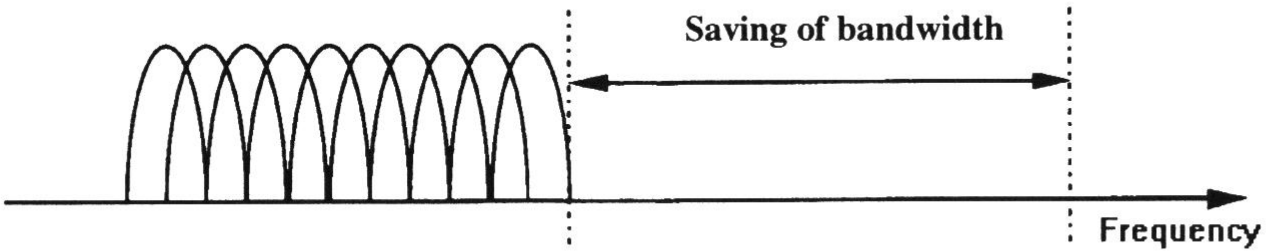
- OFDM can be defined with the framework of frequency-division-multiplexing (FDM).
- FDM : Split a high-rate data-stream into a number of lower rate streams transmitted simultaneously over a number of carriers.



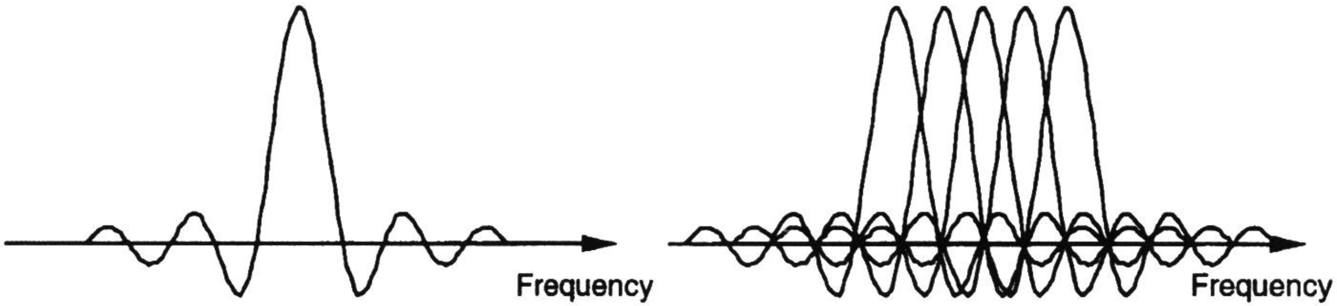
(a)

- Since the symbol duration increases for low rate carriers, the channel dispersion is decreased.
- Drawback: Guard bands make this approach inefficient.

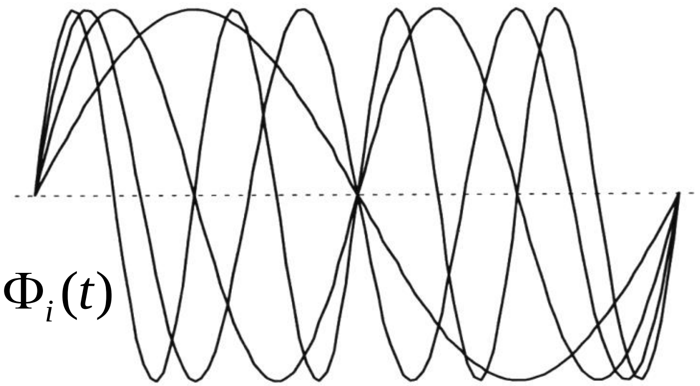
- Remedy: use overlapping sub-channels.



- It is possible to arrange carriers such that there is no interference between them. To do that the carriers must be mathematically orthogonal.



- A necessary and sufficient condition for this requirement is that all carriers must has a same period.
- Time domain waveform:



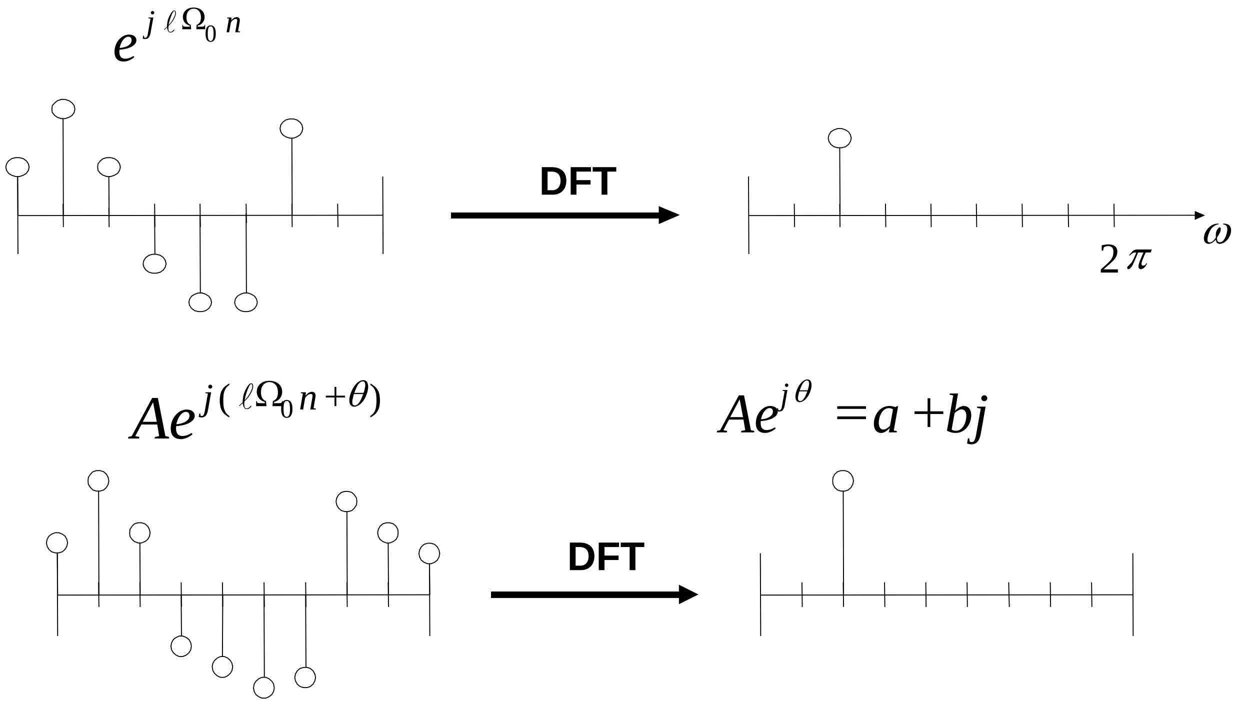
$$\int_{-\infty}^{\infty} \varphi_n(t)\varphi_l^*(t)dt = \begin{cases} 1, & n = l \\ 0, & n \neq l \end{cases}$$

- Consider the sampled Φ(t).

$$\Phi_k(n) = e^{jk\Omega_0 n}, \qquad \Omega_0 = \frac{2\pi}{N}, \quad k = 0,1,\cdots,N-1$$

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{j(k-m)\Omega_0 n} = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases}$$

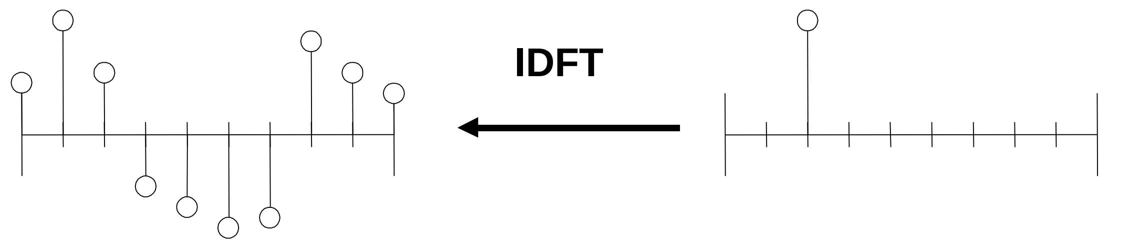
- The complex exponential exhibit an impulse in the DFT domain.



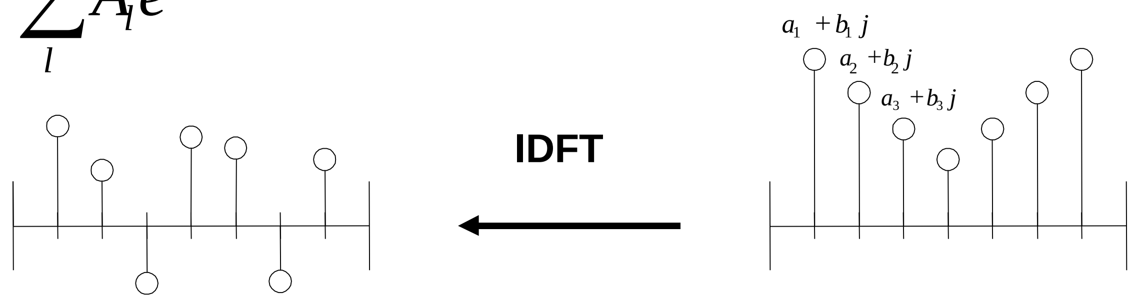
- Thus, we can conduct processing in the frequency domain using DFTs.

- The main idea is to use conduct modulation in the frequency domain.

$$Ae^{j(\ell\Omega_0 n+\theta)} \qquad\qquad Ae^{j\theta}=a+bj$$



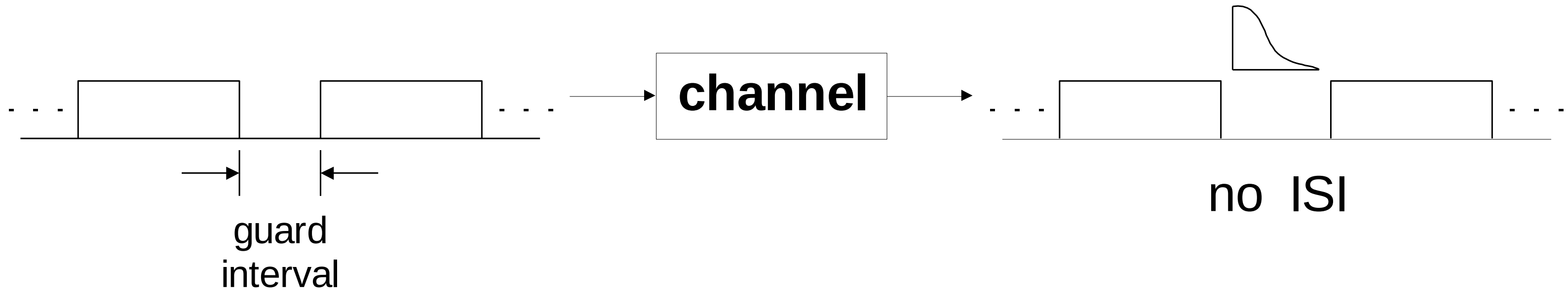
$$\sum_l A_l e^{j(l\Omega_0 n+\theta_l)}$$



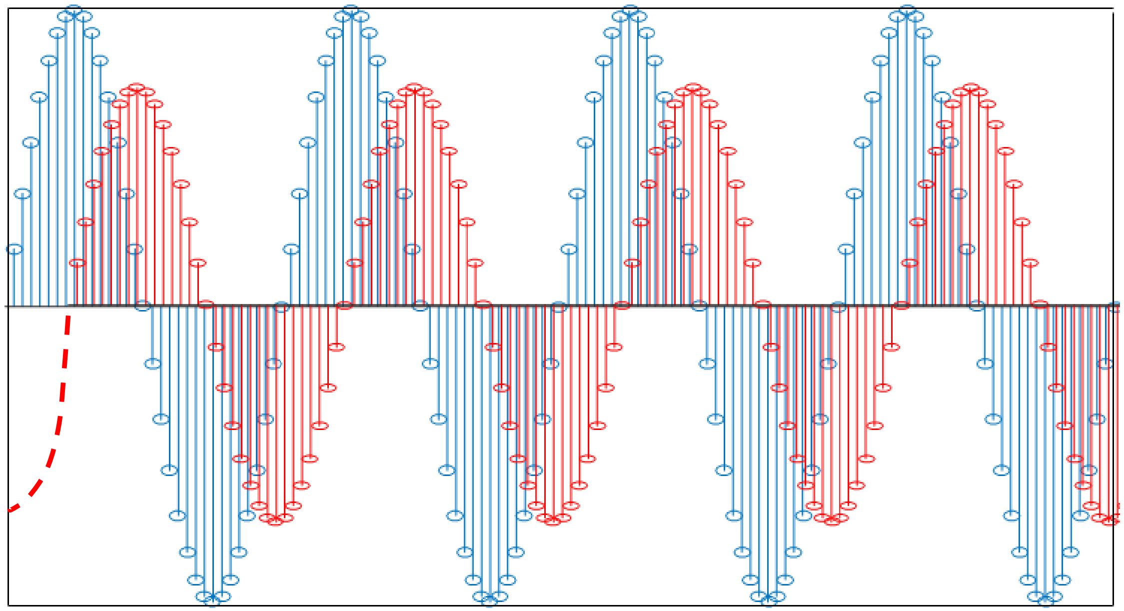
- In other words, we define QAM symbols in the frequency domain

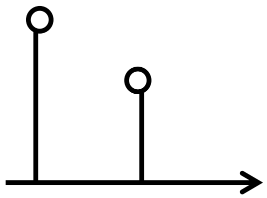
- To eliminate ISI completely, a guard time is introduced for each OFDM symbol. Since the guard time has no signal, the problem of intercarrier interference (ICI) arises.
- ISI and guard interval:

* Continuous transmission



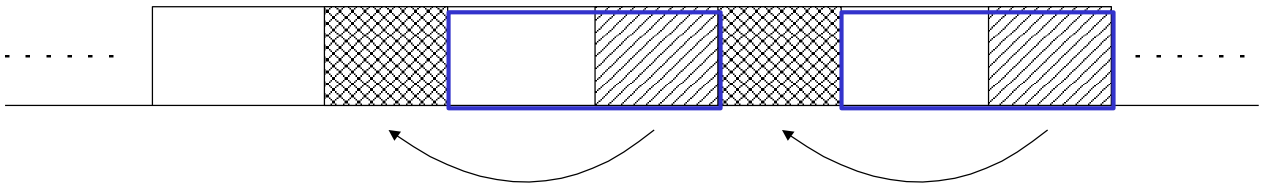
- ICI:





- The selected delayed version in the OFDM symbol is not a sinusoidal signal anymore.
- To solve the problem, cyclic prefix is added in the guard period.

- Cyclic prefix:



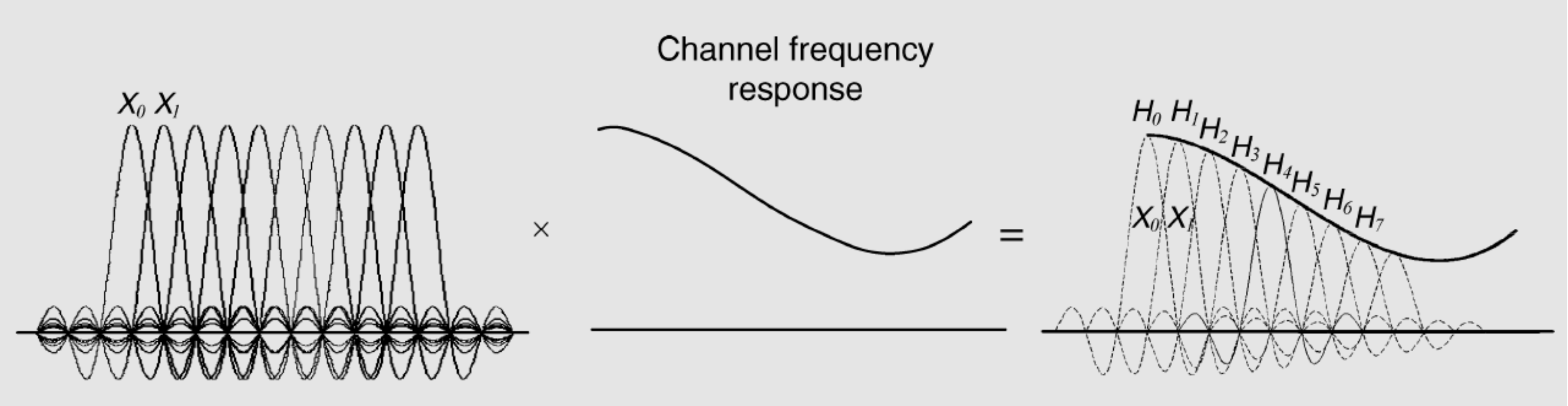
cyclic prefix

- Since the CP is added, the channel output will be a circular convolution of the channel response and the transmit signal. We then have

$$y^m(n) = x^m(n) \otimes h(n) \Rightarrow \tilde{x}^m(e^{j\omega_k}) = \frac{\tilde{y}^m(e^{j\omega_k})}{\tilde{h}(e^{j\omega_k})}$$ * In the DFT domain

- Thus, data in each channel can be recovered using a single-tap frequency domain equalizer.
- The system using this modulation technique is called orthogonal frequency division multiplexing (OFDM).

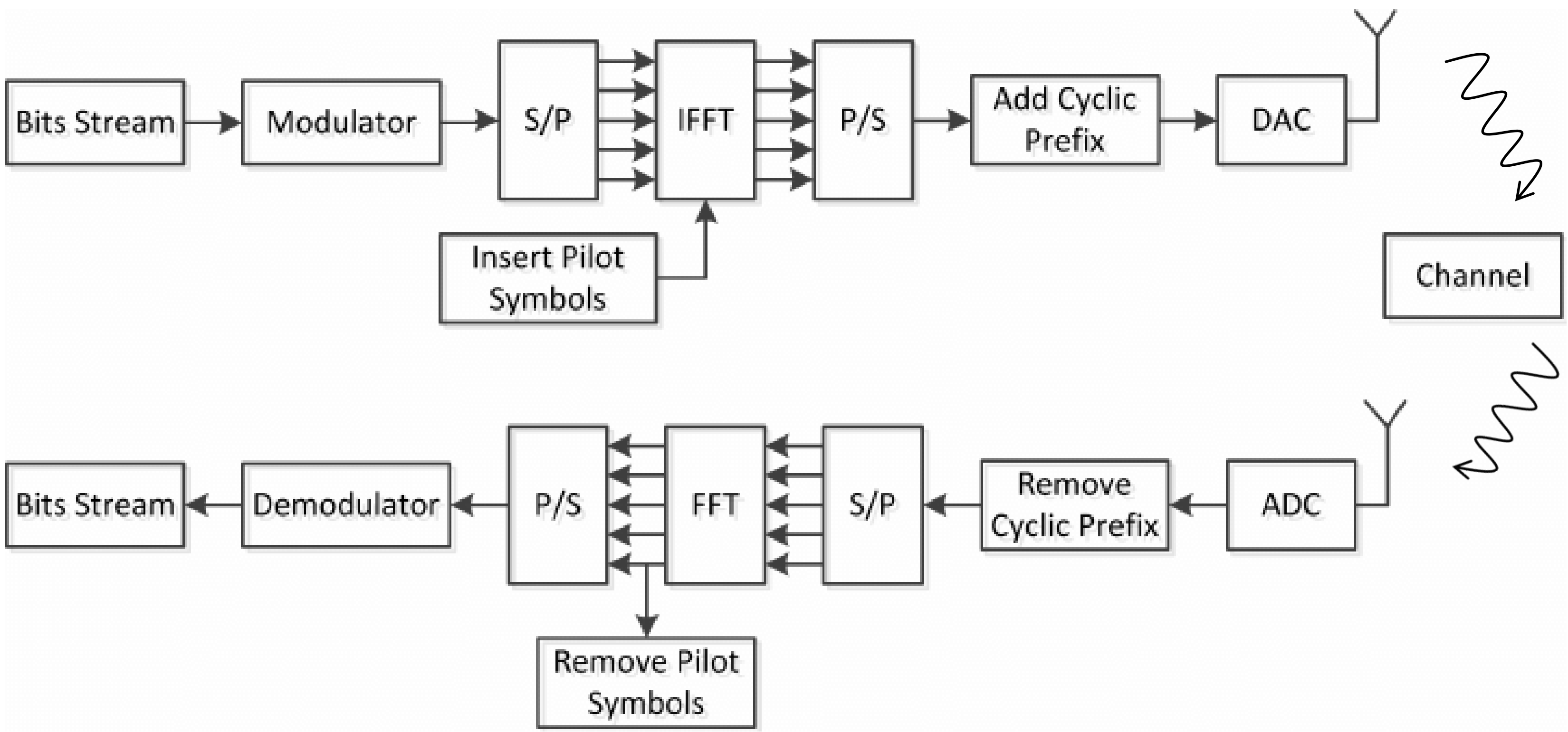
- Frequency domain response:



- Characteristics of OFDM systems:
  - High spectrum efficiency
  - Simple equalization
  - Simple multiple access
  - Sensitive to carrier frequency offset
  - High peak to average power ratio

- OFDM systems:

S/P: series to parallel S/P: series to parallel



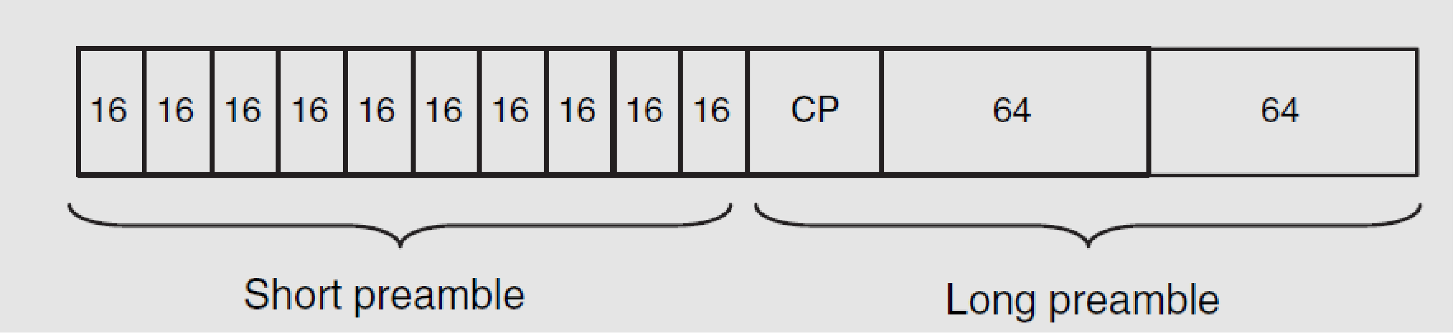
- Specifications of LTE:

Channel Bandwidth (MHz)	1.25	2.5	5	10	15	20
Frame Duration (ms)	10					
Subframe Duration (ms)	1					
Sub-carrier Spacing (kHz)	15					
Sampling Frequency (MHz)	1.92	3.84	7.68	15.36	23.04	30.72
FFT Size	128	256	512	1024	1536	2048
Occupied Sub-carriers (inc. DC sub-carrier)	76	151	301	601	901	1201
Guard Sub-carriers	52	105	211	423	635	847
Number of Resource Blocks	6	12	25	50	75	100
Occupied Channel Bandwidth (MHz)	1.140	2.265	4.515	9.015	13.515	18.015
DL Bandwidth Efficiency	77.1%	90%	90%	90%	90%	90%
OFDM Symbols/Subframe	7/6 (short/long CP)					
CP Length (Short CP) (μs)	5.2 (first symbol) / 4.69 (six following symbols)					
CP Length (Long CP) (μs)	16.67					

- Let the bandwidth for an OFDM system be f_s, the DFT size be N, and the CP size be μN where $0<\mu<1$.
- Let the sampling frequency of an OFDM be f_s. Then, the period will be $T=1/f_s$. Then, the subcarrier spacing (f_{ss}) will be $1/NT = f_b/N$.
- Let the number of subcarriers for data transmission be M. ($M \leq N$). The occupied bandwidth (f_b) is then Mf_s/N.
- The data rate (r) will be $QM/(1+\mu)NT = Qf_sM/(1+\mu)N$ where Q is the number of bits each QAM transmit.
- LTE system:
  - f_s=30.72MHz, N=2048, M=1200, μ=1/8.
  - f_{ss}=30.72MHz/2048=15KHz, f_b=1200x15KHz =18MHz (20MHz).
  - For QPSK, r=2x30.72MHzx1200/(1.125x2048)=32Mbps

- Packet format (IEEE802.11a/g):
  - Packet-based OFDM system
  - Client and access point (AP)
  - Data rate: 54Mbps
  - Bandwidth: 20MHz
  - CSMA/CA multiple access
  - FFT size: 64, CP: 16
- Packet format (IEEE802.11a/g):

Short preamble (10)	Long preamble (2)	Signal field	Payload



- Receiver:
  - Inner receiver: synchronization/channel estimation
  - Outer receiver: decoding
- Functions performed by preambles:
  1. Start-of-packet (SOP) detection – short preamble
  2. Automatic gain control (AGC) – short preamble
  3. Coarse frequency offset estimation- short preamble
  4. Coarse timing offset estimation – short/long preamble
  5. Fine timing offset estimation – long preamble
  6. Fine frequency offset estimation – long preamble
  7. Channel estimation – long preamble
- Function performed by pilots
  - Phase offset, residual CFO, and residual SFO.

- Packet detection:
  - The simplest method is (r_n: received signal, m_n: power estimate)

$$m_n = \sum_{k=0}^{L-1} r_{n-k} r_{n-k}^{*} = \sum_{k=0}^{L-1} | r_{n-k} |^2$$

  - The drawback of this approach is determination of threshold will be difficult (dependent on the channel).
- After packet detection, the symbol timing refines the estimate to *sample level* precision.

$$\hat{n}_s = \text{argmax} \left| \sum_{k=0}^{L-1} r_{n+k} t_k^{*} \right|^2$$

t_k : reference

- Packet detection – a better approach
  - Let $r_n = s_n + v_n$. Calculate autocorrelation of the received signal

$$c(n) = \sum_{k=0}^{L-1} r_{n+k} r^{*}_{n+k+D}$$

  - Normalize with the signal power:

L : length of the sliding window,
D : period of preamble

$$p(n) = \sum_{k=0}^{L-1} r_{n+k+D} r^{*}_{n+k+D} = \sum_{k=0}^{L-1} | r_{n+k+D} |^2$$

  - Decision statistic:

$$m(n) = \frac{|c(n)|^2}{p^2(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

  - If m(n) exceeds a threshold, a packet is claimed detected.

- Carrier frequency offset (CFO) estimation:
  - Assume that there are two consecutive repeated symbols (periodic).
  - The output signal from a channel is still periodic if its input signal is periodic.
- Transmit/receive signal:

$$y_n = s_n e^{j2\pi f_{tx} n T_s}$$

$$r_n = s_n e^{j2\pi f_{tx} n T_s} e^{-j2\pi f_{rx} n T_s} = s_n e^{j2\pi (f_{tx} - f_{rx}) n T_s} = s_n e^{j2\pi f_\Delta n T_s}$$

- CFO estimation:

$$z = \sum_{n=0}^{L-1} r_n r_{n+D}^* = \sum_{n=0}^{L-1} s_n s_{n+D}^* e^{j2\pi f_\Delta n T_s} e^{-j2\pi f_\Delta (n+D) T_s} = e^{-j2\pi f_\Delta D T_s} \sum_{n=0}^{L-1} | s_n |^2$$

$$\hat{f}_\Delta = -\frac{1}{2\pi D T_s} \angle(z)$$

* Unambiguous range: $\left[-\frac{1}{2DT_s}, \frac{1}{2DT_s} \right]$

- Symbol timing detection:
  - CFO has been compensated:

$$\hat{n}_s = \text{argmax} \left| \sum_{k=0}^{L-1} r_{n+k} t_k^* \right|^2$$

t_k : reference

- The reference signal can be part of the short preamble, combination of the short/long preamble, or part of the long preamble.
- Moreover, if the reference signal is composed of long preamble, we can set the long preamble as a complex Gaussian random sequence to see different result.
- This is essentially a matching operation. If periodic signal is used, periodic peaks will be observed.
- Partial matching can be conducted if CFO is not well compensated.

- Channel estimation:

$$\tilde{y}(k) = \tilde{h}(k) \times \tilde{x}(k)$$

* The received signal of the *k*th subcarrier

- Channel estimate:

$$\hat{h}_i(k) = \frac{\tilde{y}_i(k)}{\tilde{x}_i(k)}$$

* Estimated channel response of the *k*th subcarrier (*i*th preamble)

$$\hat{h}(k) = \frac{\hat{h}_1(k) + \hat{h}_2(k)}{2}$$

* Estimated channel response of the *k*th subcarrier

- To obtain better result, channel estimation methods discussed in the previous section can be used.

- Packet detection: a hypothesis testing problem:

$$m(n)=\frac{|c(n)|^2}{p^2(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \qquad \begin{aligned} H_0 &: r_n = v_n \\ H_1 &: r_n = s_n + v_n \end{aligned}$$

$* \, v_n \sim CN\left(0, 2\sigma_v^2\right)$

- For H_0:

$$c(n)=\sum_{k=0}^{L-1} v_{n+k} v_{n+k+D}^{*}, \quad p(n)=\sum_{k=0}^{L-1} | v_{n+k+D} |^2$$

$$f_c(x) \sim CN(0, 4L\sigma_v^4),\ f_p(x) \sim \chi_{2L}^2(2L\sigma_v^2, 4L\sigma_v^4)$$

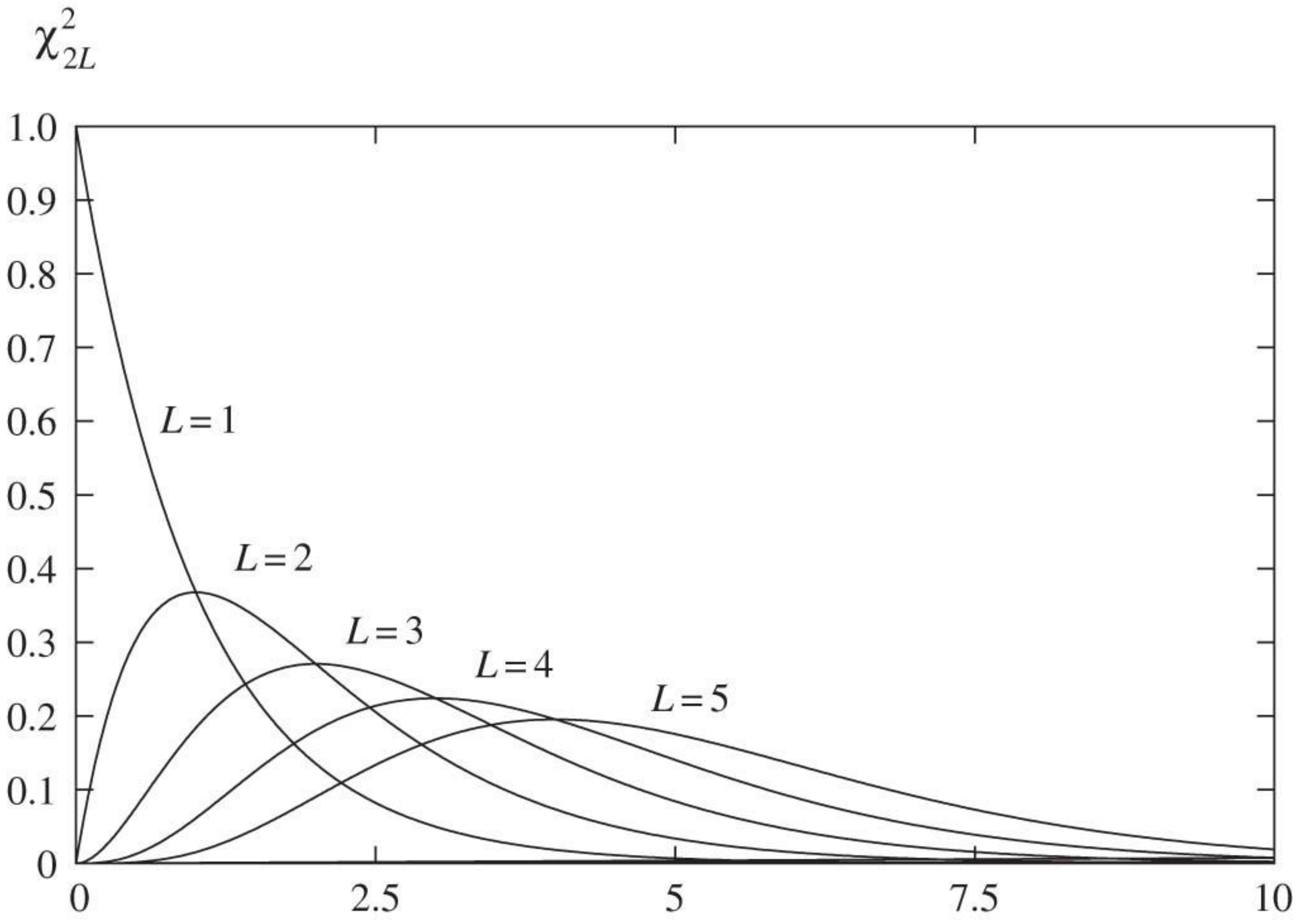
- Let $x_i \sim N(\mu_i, \sigma^2)$. Then, $y=x_1^2+x_2^2+\dots+x_N^2$ has a Chi-square distribution with degree of freedom N.

$\mu_i=0$, for all i's $\Rightarrow$ central χ^2: $E\{y\}=N\sigma^2,\ Var\{y\}=2N\sigma^4$

$\mu_i \neq 0$, for some i's $\Rightarrow$ noncentral χ^2: $E\{y\}=N\sigma^2+s^2,\ Var\{y\}=2N\sigma^4+4\sigma^2 s^2$

where $s^2=\sum_{i=1}^{N}\mu_i^2$

- Chi-square distribution:
  - It approaches Gaussian when degree-of-freedom is large.



- Then, $f_p(x)$ can also be approximated by a Gaussian.

$$* \; f_c(x) \sim CN(0, 2L\sigma_v^4)$$
$$f_p(x) \sim \chi^2_{2L}(2L\sigma_v^2, 4L\sigma_v^4)$$

$$f_{|c|^2}(x) \sim \chi_2^2(\mu_{c2}, \sigma_{c2}^2) = \chi_2^2(4L\sigma_v^4, 16L^2\sigma_v^8)$$
$$f_{|p|^2}(x) \sim \chi_2^2(\mu_{p2}, \sigma_{p2}^2) = \chi_2^2(4L\sigma_v^4 + (2L\sigma_v^2)^2, 2(4L\sigma_v^4)^2 + 4\times 4L\sigma_v^4(2L\sigma_v^2)^2)$$

- Let x and y denotes two random variables and y is zero mean. Let μ_y be a constant. If $\mu_y >> \sigma_y$, then

$$\frac{x}{\mu_y + y} = \frac{1}{\mu_y}\frac{x}{1+y/\mu_y} \approx \frac{1}{\mu_y}x\left(1-\frac{y}{\mu_y}\right) = \frac{x}{\mu_y} - \frac{xy}{\mu_y^2} \approx \frac{x}{\mu_y}$$

- So, $$m(n) = \frac{|c(n)|^2}{p^2(n)} \approx \frac{|c(n)|^2}{\mu_{p2}}, \qquad \mu_{p2} = 4L\sigma_v^4 + (2L\sigma_v^2)^2 \approx 4L^2\sigma_v^4$$

$$f_m(x) \sim \chi_1^2(L^{-1}, L^{-2}) \sim L\exp(-Lx)$$

- Recall the hypothesis testing problem:

$$m(n) = \frac{|c(n)|^2}{p^2(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \qquad \begin{aligned} H_0 &: r_n = v_n \\ H_1 &: r_n = s_n + v_n \end{aligned}$$

- Consider H$_1$. Define a new variable as: $q(n) = \sqrt{m(n)} = \dfrac{|c(n)|}{p(n)}$. When the preamble is perfected matched, we have following result:

$$c(n) = \sum_n |s(n)|^2 + \text{zero-mean r.v.}, \quad p(n) = \sum_n |s(n)|^2 + \sum_n |v(n)|^2 + \text{zero-mean r.v.}$$

Approximated by central Chi-square Central Chi-square

- When the mean of a Chi-square PDF is large, it can be approximated by a Gaussian.

$$q(n) = \sqrt{m(\mathrm{n})} = \frac{|c(n)|}{p(n)}, \quad c(n) \sim N(\mu_c, \sigma_c^2), \; p(n) \sim N(\mu_p, \sigma_p^2)$$

- Recall that $r_n = s_n + v_n$ and let

$$E\left\{\mathrm{Re}(s_n)^2\right\} = E\left\{\mathrm{Im}(s_n)^2\right\} = \sigma_s^2 \,, \quad E\left\{\mathrm{Re}(v_n)^2\right\} = E\left\{\mathrm{Im}(v_n)^2\right\} = \sigma_v^2$$

- In general, CFO exists. Then,

$$r_n = s_n e^{j2\pi\frac{\varepsilon}{N}n} + v_n, \quad r_{n+D} = s_{n+D} e^{j2\pi\frac{\varepsilon}{N}(n+D)} + v_{n+D}$$

$$r_n r_{n+D}^* = (s_n e^{j2\pi\frac{\varepsilon}{N}n} + v_n)(s_{n+D}^* e^{-j2\pi\frac{\varepsilon}{N}(n+D)} + v_{n+D}^*)$$

$$* \varphi = -2\pi\frac{\varepsilon}{N}D$$

$$= (s_n s_{n+D}^*)\mathrm{e}^{j\varphi} + (s_n v_{n+D}^*)\mathrm{e}^{j2\pi\frac{\varepsilon}{N}n} + (s_{n+D}^* v_n)\mathrm{e}^{-j2\pi\frac{\varepsilon}{N}(n+D)} + v_n^* v_{n+D}$$

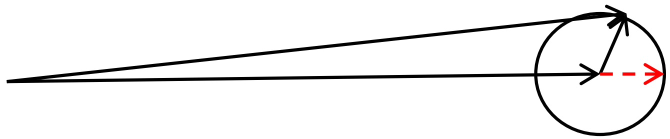
$$(r_n r_{n+D}^*)\mathrm{e}^{-j\varphi} = (s_n s_{n+D}^*) + (s_n v_{n+D}^*)\mathrm{e}^{j2\pi\frac{\varepsilon}{N}(n+D)} + (s_{n+D}^* v_n)\mathrm{e}^{-j2\pi\frac{\varepsilon}{N}n} + v_n^* v_{n+D}\,\mathrm{e}^{-j\varphi}$$

$$= \underbrace{s_n s_{n+D}^*}_{>0} + \mathrm{Re}\left\{\underbrace{(s_n v_{n+D}^*)\mathrm{e}^{j2\pi\frac{\varepsilon}{N}(n+D)}}_{(a)} + \underbrace{(s_{n+D}^* v_n)\mathrm{e}^{-j2\pi\frac{\varepsilon}{N}n}}_{(b)} + \underbrace{v_n^* v_{n+D}\,\mathrm{e}^{-j\varphi}}_{(c)}\right\}$$

$$+ j \times \mathrm{Im}\left\{(s_n v_{n+D}^*)\mathrm{e}^{j2\pi\frac{\varepsilon}{N}(n+D)} + (s_{n+D}^* v_n)\mathrm{e}^{-j2\pi\frac{\varepsilon}{N}n} + v_n^* v_{n+D}\,\mathrm{e}^{-j\varphi}\right\}$$

- Let $n = n_{opt}$ and $c(n)=\sum_{k=0}^{L-1} r_{n+k} r^{*}_{n+k+D}$

$$c(n)e^{-j\varphi} = \sum_{k=0}^{L-1} s_n s^{*}_{n+D} + \mathrm{Re}\{(a)+(b)+(c)\} + j \times \mathrm{Im}\{(a)+(b)+(c)\}$$



- If SNR is high or L is large enough, we can have

$$|c(n)e^{-j\varphi}| \approx \sum_{m=0}^{L-1} |s_n s^{*}_{n+D}| = \sum_{m=0}^{L-1} |s_n|^2, \quad E\{|c(n)|\} = L(2\sigma_s^2)$$

- Note that the variance of $|c(n)e^{-j\varphi}|$ is dominated by that of Re{(a)+(b)+(c)}.

$$(a) = (s_n v^{*}_{n+D})\,\mathrm{e}^{j2\pi\frac{\varepsilon}{N}(n+D)}, \ (b) = (\mathrm{s}^{*}_{n+D}\, v_n)\,\mathrm{e}^{-j2\pi\frac{\varepsilon}{N}n}, \ (c) = v^{*}_n v_{n+D}\,\mathrm{e}^{-j\varphi}$$

$$\mathrm{Re}\{(a)+(b)+(c)\} = \frac{\{(a)+(b)+(c)\} + \{(a)+(b)+(c)\}^{*}}{2}$$

- Note that (a), (b), and (c) are all zero mean.

$$(a) = (s_n v_{n+D}^*) \mathrm{e}^{j2\pi \frac{\varepsilon}{N}(n+D)}, \ (b) = (\mathrm{s}_{n+D}^* v_n) \mathrm{e}^{-j2\pi \frac{\varepsilon}{N} n}, \ (c) = v_n^* v_{n+D} \mathrm{e}^{-j\varphi}$$

$$E\left\{\mathrm{Re}\left\{(a)+(b)+(c)\right\}\right\}^2 = \frac{1}{4}\left\{\left\{(a)+(b)+(c)\right\} + \left\{(a)+(b)+(c)\right\}^*\right\}^2$$

- All cross terms will be zero and we have

$$E\left\{\left\{(a)+(b)+(c)\right\} + \left\{(a)+(b)+(c)\right\}^*\right\}^2$$

$$= E\left\{\left\{(a)+(b)+(c)\right\}^2 + 2\left\{(a)+(b)+(c)\right\}\left\{(a)+(b)+(c)\right\}^* + \left\{(a)+(b)+(c)\right\}^{*2}\right\}$$

$$= 2\left(4\sigma_s^2\sigma_v^2 + 4\sigma_s^2\sigma_v^2 + 4\sigma_v^4\right)$$

- Thus, $$|\,c(n)\,| = |\,c(n)e^{j\phi}\,| \sim N\left(L(2\sigma_s^2)\,,\, L(2\sigma_s^2+\sigma_v^2)(2\sigma_v^2)\right)$$

- For p(n):

$$p(n) = \sum_{k=0}^{L-1} | r_{n+k} |^2 = \sum_{k=0}^{L-1} | s_{n+k} + v_{n+k} |^2$$

* Expand and calculate

- Thus,

$$E \quad p(n) \quad = 2L(\sigma_s^2 + \sigma_v^2)$$

$$Var \quad p(n) \quad = 4L(2\sigma_s^2\sigma_v^2 + \sigma_v^4)$$

- Then,

$$q(n) = \frac{|c(n)|}{p(n)}, \quad \begin{cases} \mu_c = L(2\sigma_s^2) \\ \sigma_c^2 = L(2\sigma_s^2 + \sigma_v^2)(2\sigma_v^2) \\ \mu_p = 2L(\sigma_s^2 + \sigma_v^2) \\ \sigma_p^2 = 4L(2\sigma_s^2\sigma_v^2 + \sigma_v^4) \end{cases}$$

- Consider the following approximation:

$$\frac{\mu_x + x}{\mu_y + y} = \frac{\mu_x}{\mu_y}\frac{1 + x / \mu_x}{1 + y / \mu_y} \approx \frac{\mu_x}{\mu_y}\left(1 + \frac{x}{\mu_x}\right)\left(1 - \frac{y}{\mu_y}\right) \approx \frac{\mu_x}{\mu_y}\left(1 + \frac{x}{\mu_x} - \frac{y}{\mu_y}\right)$$

- Let

$$\tilde{c} = |c(n)| - \mu_c, \quad \tilde{p} = p(n) - \mu_p$$

- Then,

$$q(n) = \mu_q \left(1 + \frac{\tilde{c}}{\mu_c}\right)\left(1 - \frac{\tilde{p}}{\mu_p}\right) \approx \mu_q \left(1 + \frac{\tilde{c}}{\mu_c} - \frac{\tilde{p}}{\mu_p}\right), \quad \mu_q = \frac{\mu_c}{\mu_p}$$

$$E(q(n)) = \mu_q = \frac{\mu_c}{\mu_p}$$

$$Var(q(n)) = E\left\{(q(n) - \mu_q)^2\right\} = \mu_q{}^2 E\left\{\left(\frac{\tilde{c}}{\mu_c} - \frac{\tilde{p}}{\mu_p}\right)^2\right\}$$

$$= \mu_q{}^2 \left(E\left\{\left(\frac{\tilde{c}}{\mu_c}\right)^2\right\} + E\left\{\left(\frac{\tilde{p}}{\mu_p}\right)^2\right\} - 2E\left\{\left(\frac{\tilde{c}}{\mu_c}\right)\left(\frac{\tilde{p}}{\mu_p}\right)\right\}\right)$$

$$= \mu_q{}^2 \left(\frac{\sigma_c^2}{\mu_c^2} + \frac{\sigma_p^2}{\mu_p^2} - 2E\left\{\left(\frac{\tilde{c}}{\mu_c}\right)\left(\frac{\tilde{p}}{\mu_p}\right)\right\}\right)$$

- Recall that:

$$|c(n)| \approx \underbrace{\sum_{k=0}^{L-1}[s_{n+k}s^*_{n+k+D}]}_{\mu_c=L(2\sigma_s^2)} + \sum_{k=0}^{L-1}\mathrm{Re}\left\{s_{n+k}v^*_{n+k+D} + s^*_{n+k+D}v_{n+k} + v_{n+k}v^*_{n+k+D}\right\}$$

$$= \mu_c\left(1+\frac{1}{\mu_c}\underbrace{\sum_{k=0}^{L-1}\mathrm{Re}\{s_{n+k}v^*_{n+k+D} + s^*_{n+k+D}v_{n+k} + v_{n+k}v^*_{n+k+D}\}}_{A}\right)$$

$$p(n) = \underbrace{\sum_{k=0}^{L-1}|s_{n+k}|^2 + |v_{n+k}|^2}_{\mu_p=2L(\sigma_s^2+\sigma_v^2)} + \sum_{k=0}^{L-1}s_{n+k}v^*_{n+k} + s^*_{n+k}v_{n+k}$$

$$= \mu_p\left(1+\frac{1}{\mu_p}\underbrace{\sum_{k=0}^{L-1}s_{n+k}v^*_{n+k} + s^*_{n+k}v_{n+k}}_{B}\right)$$

- Then,

$$\frac{A}{\mu_c} \sim N\left(0, \frac{\sigma_c^2}{\mu_c^2}\right), \quad \frac{B}{\mu_p} \sim N\left(0, \frac{\sigma_p^2}{\mu_p^2}\right)$$

$$E\left\{\left(\frac{A}{\mu_c}\right)\left(\frac{B}{\mu_p}\right)\right\} = \frac{1}{2\mu_c\mu_p}\sum_{k=0}^{L-1} 2\left(2\sigma_s^2 2\sigma_v^2\right) = \frac{4L(\sigma_s^2\sigma_v^2)}{\mu_c\mu_p}$$

$*$ Expand and note $s_{n+k} = s_{n+k+D}$.

- Finally, we have

$$Var(q(n)) = \mu_q{}^2\left(\frac{\sigma_c^2}{\mu_c^2} + \frac{\sigma_p^2}{\mu_p^2} - 2Cov\left(\frac{A}{\mu_c}, \frac{B}{\mu_p}\right)\right) = \mu_q{}^2\left(\frac{\sigma_c^2}{\mu_c^2} + \frac{\sigma_p^2}{\mu_p^2} - \frac{8L(\sigma_s^2\sigma_v^2)}{\mu_c\mu_p}\right)$$

- Then, q(n) can also be approximated by a Gaussian PDF.

$$E\{q(n)\} = \frac{\mu_c}{\mu_p} = \frac{L(2\sigma_s^2)}{L(2\sigma_s^2 + 2\sigma_v^2)} = \mu_q$$

* n=n $_{opt}$: perfectly matched

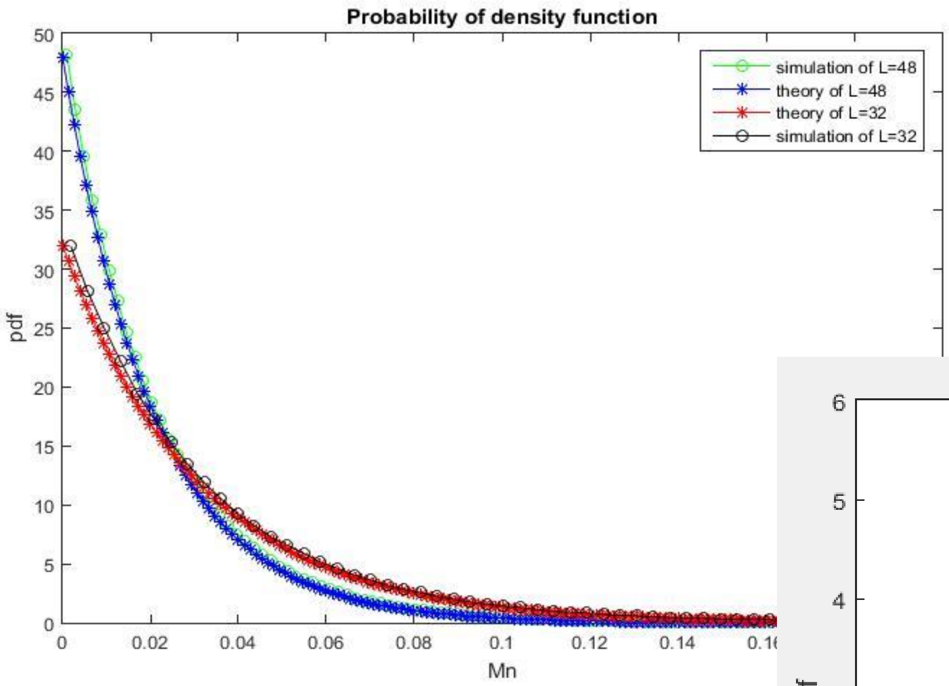
$$Var\{q(n)\} = \mu_q{}^2 \left(\frac{\sigma_c^2}{\mu_c^2} + \frac{\sigma_p^2}{\mu_p^2} - \frac{8L(\sigma_s^2\sigma_v^2)}{\mu_c\mu_p} \right) = \mu_q^2 \left(\frac{4\sigma_s^2\sigma_v^2 + 6\sigma_v^4 + 8\frac{\sigma_v^6}{\sigma_s^2} + 2\frac{\sigma_v^8}{\sigma_s^4}}{L(2\sigma_s^2 + 2\sigma_v^2)^2} \right) = \sigma_q^2$$

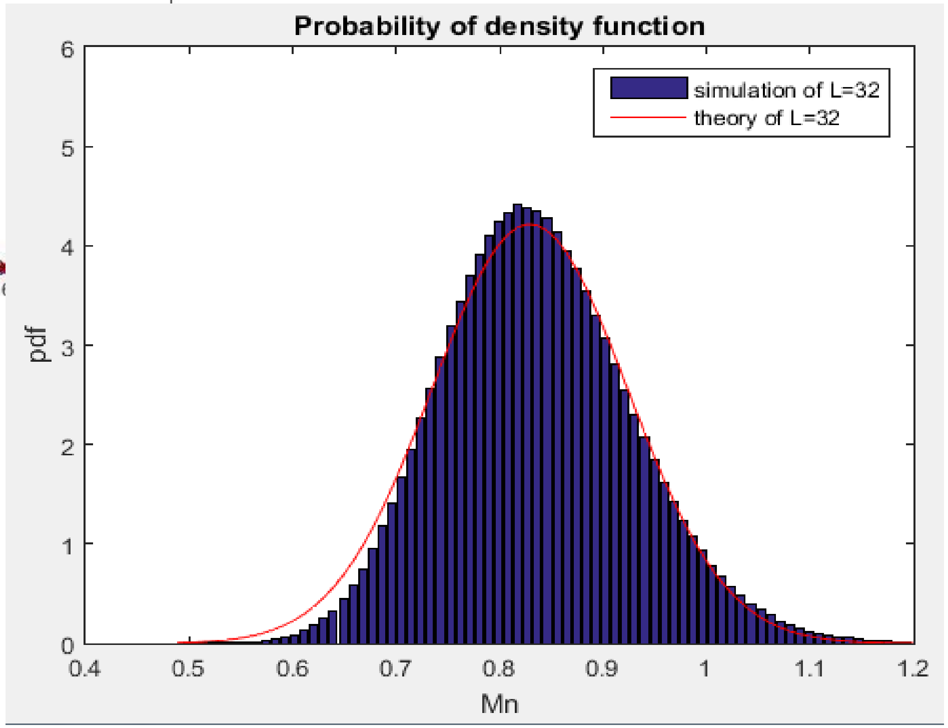
- Note that m(n) =q²(n), and we can approximate m(n) with a non-central Chi-square distribution.
- Finally, we have

$$E\{m(n)\} = \sigma_q^2 + \mu_q^2 \approx \mu_q^2 = \mu_m$$

$$Var\{m(n)\} = 2\sigma_q^4 + 4\sigma_q^2\mu_q^2 \approx 4\sigma_q^2\mu_q^2 = \sigma_m^2$$

- Examples (PDF of $m(n_{opt})$ for H_0 and H_1):





- For H_0, the PDF of m(n) is

$$p_m(x) = L\mathrm{e}^{-Lx},\ x \geq 0$$

- Thus, we can calculate the probability of false alarm as:

$$p\left(m(n) \geq \eta \mid H_0\right) = \int_{\eta}^{\infty} L e^{-Lx} dx = e^{-L\eta}$$

- For H_1, the PDF of m(n) is

$$p_m(x) = \frac{1}{\sqrt{2\pi}\sigma_m} e^{-\frac{(x-\mu_m)^2}{2\sigma_m{}^2}}$$

- We can calculate the probability of detection as:

$$p\left(m(n) \geq \eta \mid H_1\right) = \int_{\eta}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-\frac{(x-\mu_m)^2}{2\sigma_m{}^2}} dx = Q\left(\frac{\eta - \mu_m}{\sigma_m}\right)$$

- Define SNR as $$\mathrm{SNR} = \frac{\sigma_s^2}{\sigma_v^2}.$$
- We can rewrite μ_m and σ_m^2 as:

$$\mu_m = \frac{\mathrm{SNR}^2}{(1+\mathrm{SNR})^2} + \frac{(1+\mathrm{SNR})^3 + \mathrm{SNR}^2(2+\mathrm{SNR})}{2\times L(1+\mathrm{SNR})^4} \approx \frac{\mathrm{SNR}^2}{(1+\mathrm{SNR})^2}$$

$$\sigma_m{}^2 = \frac{2\times\mathrm{SNR}^2\left[(1+\mathrm{SNR})^3 + \mathrm{SNR}(1+\mathrm{SNR}^2)\right]}{L(1+\mathrm{SNR})^6}$$

- Carrier frequency offset (CFO) estimation:
  - Assume that there are two consecutive repeated symbols (periodic).
  - The output signal from a channel is still periodic if its input signal is periodic.
- Transmit/receive signal:

$$y_n = s_n e^{j2\pi f_{tx} nT_s}$$

$$r_n = s_n e^{j2\pi f_{tx} nT_s} e^{-j2\pi f_{rx} nT_s} = s_n e^{j2\pi (f_{tx} - f_{rx}) nT_s} = s_n e^{j2\pi f_\Delta nT_s}$$

- CFO estimation:

$$z = \sum_{n=0}^{L-1} r_n r_{n+D}^* = \sum_{n=0}^{L-1} s_n s_{n+D}^* e^{j2\pi f_\Delta nT_s} e^{-j2\pi f_\Delta (n+D)T_s} = e^{-j2\pi f_\Delta DT_s} \sum_{n=0}^{L-1} | s_n |^2$$

$$\hat{f}_\Delta = -\frac{1}{2\pi DT_s} \angle(z)$$

- In OFDM (N: DFT size):

$$\angle z = 2\pi f_\Delta D T_s = 2\pi \frac{f_\Delta}{f_s} D = 2\pi \frac{K+\varepsilon}{D} D, \quad K: \text{intger}, \; 0 < \varepsilon < 1$$

* K/D is called integer CFO while ε/D is fractional

- Note that the definition of integer CFO varies with D. If using CP for CFO estimation, D=N and integer CFO corresponds to K/N.
- Fractional CFO is usually estimated first.

$$z = \sum_{n=0}^{L-1} r_n r_{n+D}^* = e^{-j2\pi D \frac{\varepsilon}{N}} \sum_{n=0}^{L-1} |s_n|^2 \Rightarrow \frac{\hat{\varepsilon}}{N} = -\frac{1}{2\pi D} \angle(z)$$

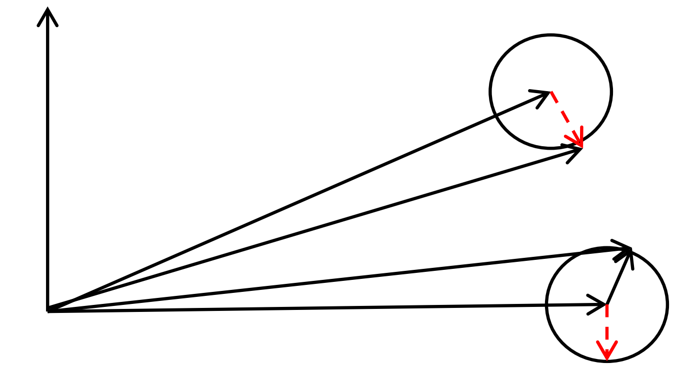
- Once fractional CFO is estimated and compensated, the integer CFO can be estimated by matching known pilots in frequency domain.

- From the previous result, we have

$$r_n r^*_{n+D} = (s_n e^{j2\pi\frac{\varepsilon}{N}n} + v_n)(s^*_{n+D} e^{-j2\pi\frac{\varepsilon}{N}(n+D)} + v^*_{n+D})$$

$$= (s_n s^*_{n+D})e^{j\phi} + (s_n v^*_{n+D})\,\mathrm{e}^{j2\pi\frac{\varepsilon}{N}n} + (\mathrm{s}^*_{n+D}\, v_n)\,\mathrm{e}^{-j2\pi\frac{\varepsilon}{N}(n+D)} + v^*_n v_{n+D}$$

$$= |\, s_n\,|^2\, e^{j\phi} + \mathrm{Re}\{(a)+(b)+(c)\} + j\times \mathrm{Im}\{(a)+(b)+(c)\}$$



$$*\ \phi = -2\pi\frac{\varepsilon}{N}D$$

- The phase estimate is perturbed mainly by the Im{(a)+(b)+(c)} term. If ϕ=0 and SNR is high, then

$$\hat{\phi} = \tan^{-1}\left(\frac{\mathrm{Im}(r_n r^*{}_{n+D})}{\mathrm{Re}(r_n r^*{}_{n+D})}\right) \approx \phi + \frac{\mathrm{Im}\ \ (a)+(b)+(c)}{|\ s_n\ |^2}$$

- Thus, we have $\quad * \; c(n) = \sum_{k=0}^{L-1} r_{n+k} r^{*}_{n+k+D}$

$$|c(n_{opt})| \sim N\left(L(2\sigma_s^2),\, L(2\sigma_s^2+\sigma_v^2)(2\sigma_v^2)\right),$$

$$* \; \mathrm{Var}\left[\mathrm{Im}\;\; (a)+(b)+(c)\;\;\right] = \mathrm{Var}\left[\mathrm{Re}\;\; (a)+(b)+(c)\;\;\right]$$

$$E\{\hat{\phi}\} = \phi$$

$$Var\{\hat{\phi}\} = \frac{L(2\sigma_s^2+\sigma_v^2)(2\sigma_v^2)}{(L(2\sigma_s^2))^2} = \frac{\sigma_v^2(2\sigma_s^2+\sigma_v^2)}{2L\sigma_s^4} \approx \frac{\sigma_v^2}{L\sigma_s^2} = \frac{1}{L\times \mathrm{SNR}}$$

  – Note that the result also holds for $\phi \neq 0$.

- For two long preambles: $\quad r_n = s_n + v_n,$

$$\tilde{s}_{1,k} = \sum_{n=0}^{N-1} s_n e^{-j\frac{2\pi k}{N}n}, \quad \tilde{s}_{2,k} = \sum_{n=N}^{2N-1} s_n e^{-j\frac{2\pi k}{N}n} = \sum_{n=0}^{N-1} s_{n+N} e^{-j\frac{2\pi k}{N}(n+N)} = \sum_{n=0}^{N-1} s_n e^{-j\frac{2\pi k}{N}n}$$

* Does not consider CP

* Does not have CFO

- If CFO presents, then

$$s_{n+N} = s_n e^{j2\pi\varepsilon} \Rightarrow \tilde{s}_{2,k} = \tilde{s}_{1,k} e^{-j2\pi\varepsilon}$$

- This is to say that CFO can be estimated in frequency domain.

$$y_{1,k} = \tilde{r}_{1,k} + \tilde{v}_{1,k}$$

$$y_{2,k} = \tilde{r}_{1,k} e^{j2\pi\varepsilon} + \tilde{v}_{2,k}$$

- With maximum likelihood estimate, CFO can be estimated by:

$$\hat{\varepsilon} = \frac{1}{2\pi} \tan^{-1}\left(\frac{\sum_{k=0}^{K-1} \mathrm{Im}[y_{2,k} y_{1,k}^{*}]}{\sum_{k=0}^{K-1} \mathrm{Re}[y_{2k} y_{1k}^{*}]} \right)$$

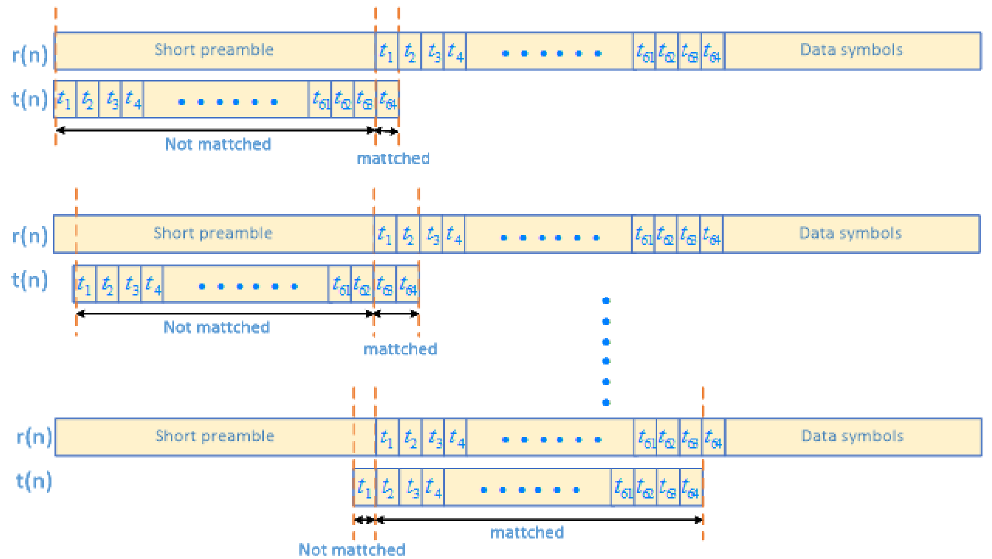
- It has been shown in [3] that

$$Var\{\hat{\phi}\} \leq \frac{1}{K \times \mathrm{SNR}}$$

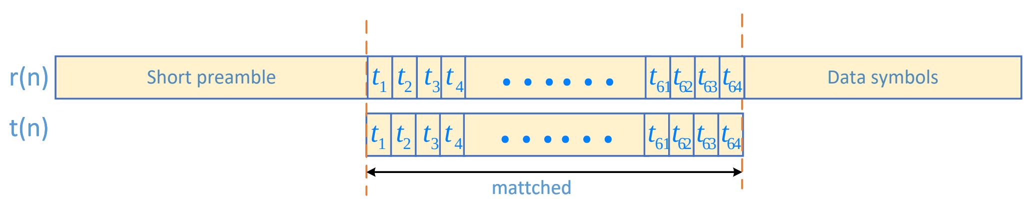
- Symbol timing (CFO has been compensated):

$$\hat{n}_s = \text{argmax}\left|\sum_{k=0}^{L-1} r_{n+k} t_k^*\right|^2 \qquad t_k : \text{reference}$$

- Similar to that in packet detection, we can formulate two hypotheses.
- H_0 (partially matched):



- H_1(completely matched):



- First, we let the reference signal be the long preamble which is deterministic.
- Let

$$z(n)=\sum_{k=0}^{L-1} r\left(n-k\right)t^{*}(L-k)$$

- For H_0, r(.) and t(.) are partially matched and z(n) can be written as

$$z(n)=\underbrace{\left(\sum_{k=m}^{L-1} s(n-k)t^{*}\left(L-k\right)+\sum_{k=0}^{m-1} s\left(n-k\right)t^{*}\left(L-k\right)\right)}_{Signal}+\underbrace{\sum_{k=0}^{L-1} v(n-k)t^{*}\left(L-k\right)}_{Noise}$$

  - L is the length of the long training sequence.
  - The first half of the signal part is s(n-k)=p(L-k+m) where p(.) is the short training sequence with length L.
  - The second half of the signal is s(n-k)=t(m-k)
  - m is the length of the input signal that matches the referene signal.

- Mean and variance of z(n):

$$* \, \mathrm{var}(p_n) = \sigma_s^2$$
$$\mathrm{var}(t_n) = \sigma_t^2$$

$$E\{z(n)\} = \sum_{k=m}^{L-1} p(L-k+m)t^*(L-k) + \sum_{k=0}^{m-1} t(m-k)t^*(L-k) = \sum_{k=0}^{m-1} t(m-k)t^*(L-k) = \mu_z(n)$$

$$Var(z_o) = E\left\{|z_o - \mu_z|^2\right\} = E\left\{\left|\sum_{k=0}^{L-1} v(n-k)t^*(L-k)\right|^2\right\} = \sum_{k=1}^{L} 2\sigma_v{}^2 |t(L-k)|^2 = 2L\sigma_v{}^2\sigma_t{}^2 = \sigma_z{}^2$$

- By the CLT, $z(n) \sim CN(\mu_z, \sigma_z{}^2)$.
- Then, $|z(n)|^2$ can be approximated by a non-central Chi-square distribution as:

$$\left|\sum_{k=0}^{L-1} r(n-k)t^*(L-k)\right|^2 \sim \chi_2{}^2(\mu_{z2}, \sigma_{z2}{}^2)$$

$$\mu_{z2} = 2 \times (\frac{1}{2}\sigma_z{}^2) + |\mu_z(n)|^2$$

$$\sigma_{z2}{}^2 = 2 \times 2 \times (\frac{1}{2}\sigma_z{}^2)^2 + 4 \times |\mu_z(n)|^2 (\frac{1}{2}\sigma_z{}^2)$$

- For H_1, r(.) and t(.) are completely matched and z(n) can be written as:

$$z(n) = \sum_{k=0}^{L-1} r(L-k)t^*(L-k) = \sum_{k=0}^{L-1} \big(t(L-k) + v(L-k)\big) \times t^*(L-k)$$

$$= \sum_{k=0}^{L-1} t(L-k)t^*(L-k) + v(L-k)t^*(L-k)$$

- Mean and variance of z(n) can be derived as:

$$E\{z(n)\} = E\left\{\sum_{k=0}^{L-1}\Big(t(L-k)t^*(L-k) + v(L-k)t^*(L-k)\Big)\right\} = \sum_{k=0}^{L-1}|t(L-k)|^2 = L\sigma_t^2 = \mu_z$$

$$Var\{z(n)\} = E\left\{\left|\sum_{k=0}^{L-1}\Big(t(L-k)t^*(L-k) + v(L-k)t^*(L-k)\Big) - \mu_z\right|^2\right\}$$

$$= \sum_{k=0}^{L-1} E\left\{\left|v(L-k)t^*(L-k)\right|^2\right\} = \sum_{k=0}^{L-1} 2\sigma_v{}^2\,|t(L-k)|^2 = 2L\sigma_t{}^2\sigma_v{}^2 = \sigma_z{}^2$$

- By the CLT, the distribution of z(n) can be approximated as a complex Gaussian random variable:

$$z(n) \sim CN(\mu_z, {\sigma_z}^2).$$

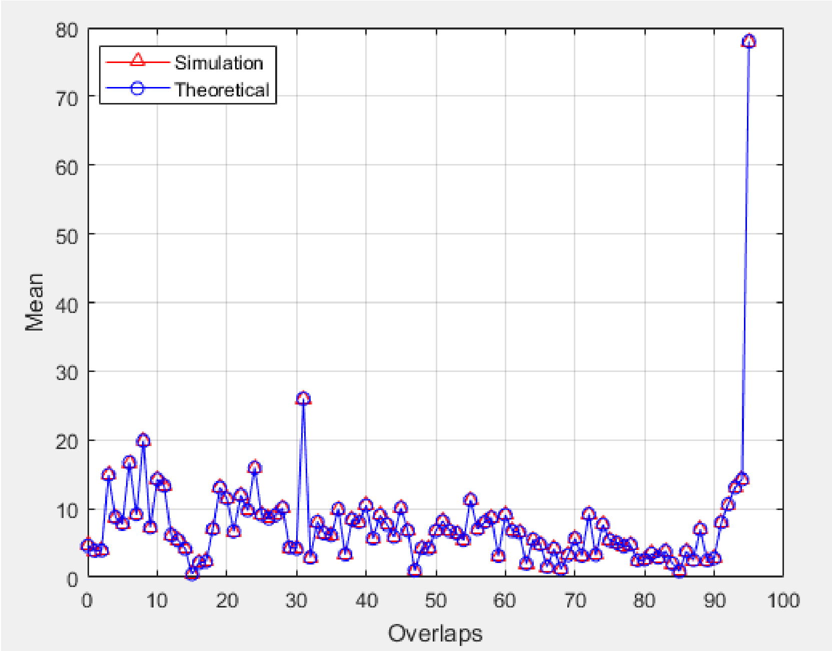
- Again, $|z(n)|^2$ can be approximated by a non-central Chi-square distribution as:

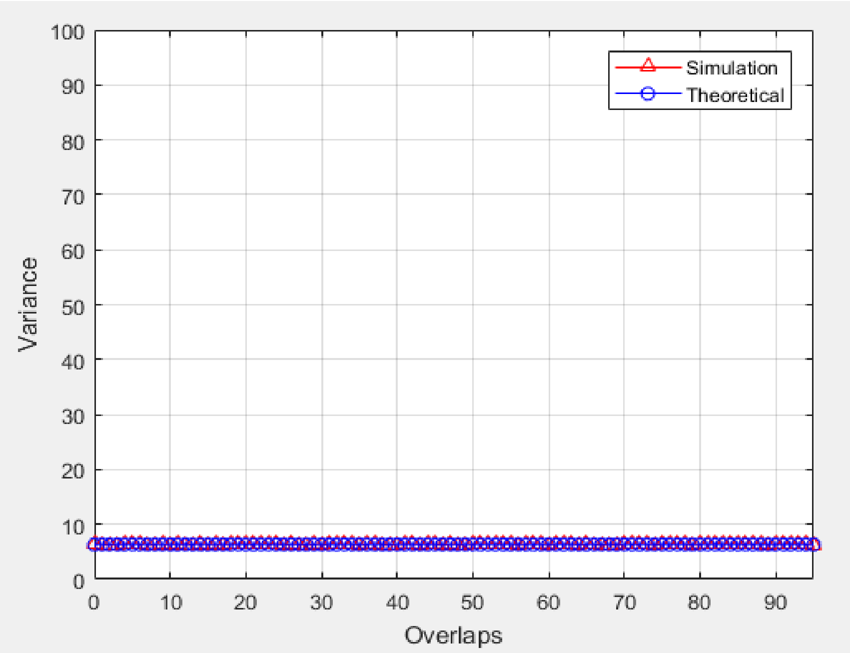
$$|z(n)|^2 = \left|\sum_{k=0}^{L-1} r\left(n-k\right) t^*\left(L-k\right)\right|^2 \sim {\chi_2}^2(\mu_{z2}, {\sigma_{z2}}^2)$$

$$\mu_{z2} = 2 \times (\frac{1}{2}{\sigma_z}^2) + {\mu_z}^2,$$

$$ {\sigma_{z2}}^2 = 2 \times 2 \times (\frac{1}{2}{\sigma_z}^2)^2 + 4 \times {\mu_z}^2 (\frac{1}{2}{\sigma_z}^2)$$

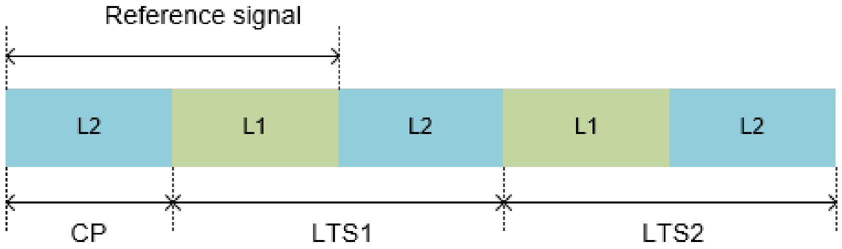
- Simulation result (SNR=10dB):





  - Before complete matching, the mean fluctuates.
  - When complete matching, the mean spikes.
  - The variance remains the same for all cases.

- The reference signal can be treated as a complex Gaussian random process also ($t(n)$~$CN(0,\sigma_t^2)$).



- For H_0:

$*t \sim CN\left(0,\sigma_t^2\right)$

$\mathrm{var}(p_n) = \sigma_s^2 = \sigma_t^2$

L : length of reference signal

$$E\{z(n)\} = \sum_{k=m}^{L-1} p(L-k+m)t^*(L-k) + \sum_{k=0}^{m-1} t(m-k)t^*(L-k) = 0$$

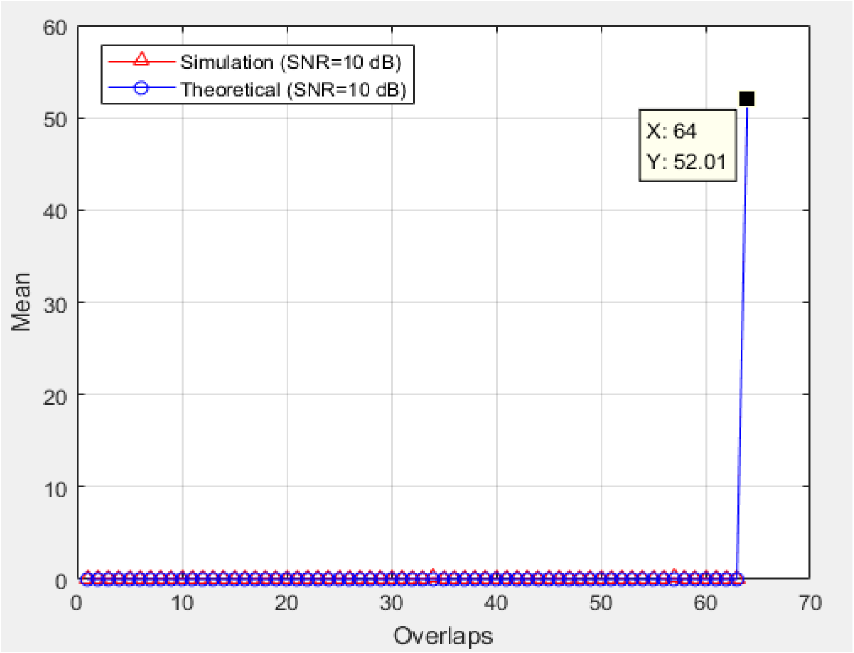
$$Var(z) = Var\left\{\sum_{k=m}^{L-1} p(L-k+m)t^*(L-k) + \sum_{k=0}^{m-1} t(m-k)t^*(L-k) + \sum_{k=0}^{L-1} v(n-k)t^*(L-k)\right\}$$
$$= (L-m)\sigma_s^2\sigma_t^2 + m\sigma_t^2 + L\sigma_v^2\sigma_t^2$$

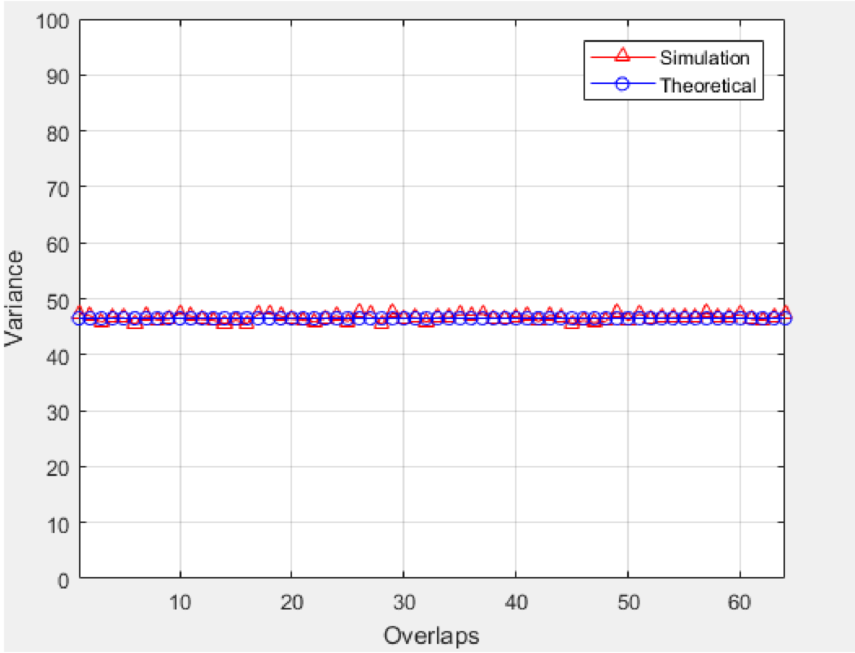
- For H_1:

$$E\{z(n)\} = \sum_{k=0}^{L-1} t(L-k)t^*(L-k) = L\sigma_t^2$$

$$Var(z) = Var\left\{\sum_{k=0}^{L-1} t(L-k)t^*(L-k) + \sum_{k=0}^{L-1} v(n-k)t^*(L-k)\right\} = L\sigma_t^2 + L\sigma_v^2\sigma_t^2$$

- Simulation result (SNR=10dB):





  - Before complete matching, the mean is zero.
  - When complete matching, the mean spikes.
  - The variance is the same for all cases.

- By the CLT, the distribution of z can be approximated as a complex Gaussian random variable.

$$z \sim CN(\mu_z, \sigma_z{}^2)$$

- Then, |z(n)| can be approximated as non-central Chi-square distribution with DoF two.

$$\left|z(n)\right|^2 = \left|\sum_{k=0}^{L-1} r\left(n-k\right) t^*\left(L-k\right)\right|^2 \sim \chi_2{}^2(\mu_{z2}, \sigma_{z2}{}^2)$$

$$\mu_{z2} = 2\times(\frac{1}{2}\sigma_z{}^2) + \mu_z{}^2,\ \sigma_{z2}{}^2 = 2\times2\times(\frac{1}{2}\sigma_z{}^2)^2 + 4\times\mu_z{}^2(\frac{1}{2}\sigma_z{}^2)$$

- Channel estimation:

$$\tilde{y}(k) = \tilde{h}(k) \times \tilde{x}(k)$$

* The received signal of the *k*th subcarrier

- Channel estimate:

$$\hat{h}_i(k) = \frac{\tilde{y}_i(k)}{\tilde{x}_i(k)}$$

* Estimated channel response of the *k*th subcarrier (*i*th preamble)

$$\hat{h}(k) = \frac{\hat{h}_1(k) + \hat{h}_2(k)}{2}$$

* Estimated channel response of the *k*th subcarrier

- In many OFDM systems, channel response needed to be interpolated.
- In 802.11a/g, no interpolation is needed, giving the simple method reasonably good performance.

- Using the vector representation (OFDM symbol), we have

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

$$\mathbf{h} = \underbrace{[h(1), h(2), ..., h(K), 0, ..., 0]}_{FFT\ size}, \quad \mathbf{H}\text{: circult matrix by } \mathbf{h}$$

- Let

$$\tilde{\mathbf{y}} = \mathbf{F}\mathbf{y}, \quad \tilde{\mathbf{x}} = \mathbf{F}\mathbf{x}, \quad \mathbf{v} = \mathbf{F}\mathbf{v}, \quad \tilde{\mathbf{h}} = \mathbf{F}\mathbf{h}, \quad \mathbf{F}\text{: DFT matrix}$$

- Then, $$\mathbf{F}\mathbf{y} = \mathbf{F}\mathbf{H}\mathbf{F}^{*}\mathbf{F}\mathbf{x} + \mathbf{F}\mathbf{v} \Rightarrow \tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{v}}$$

$$\tilde{\mathbf{H}}\text{: diag}\,(\tilde{\mathbf{h}}), \quad \tilde{\mathbf{h}} = \mathbf{F}\mathbf{h}$$

- And (for a long preamble),

* $\bar{h}_k$: estimate of $\tilde{h}_k$

$$\tilde{y}_k = \tilde{h}_k \tilde{x}_k + \tilde{v}_k \Rightarrow \bar{h}_k = \frac{\tilde{y}_k}{\tilde{x}_k} = \tilde{h}_k + \frac{\tilde{v}_k}{\tilde{x}_k} = \tilde{h}_k + \tilde{\delta}_k$$

- For data detection (data symbol): $* \ \bar{x}_k$: estimate of $\tilde{x}_k$

$$\tilde{y}_k = \tilde{h}_k \tilde{x}_k + \tilde{v}_k \Rightarrow \bar{x}_k = \frac{\tilde{y}_k}{\bar{h}_k} = \frac{\tilde{h}_k \tilde{x}_k}{\bar{h}_k} + \frac{\tilde{v}_k}{\bar{h}_k} = \frac{\tilde{h}_k \tilde{x}_k}{\tilde{h}_k + \tilde{\delta}_k} + \frac{\tilde{v}_k}{\tilde{h}_k + \tilde{\delta}_k}$$

$$\bar{x}_k = \frac{1}{\tilde{h}_k}\left(\frac{\tilde{h}_k \tilde{x}_k}{1 + \tilde{\delta}_k / \tilde{h}_k} + \frac{\tilde{v}_k}{1 + \tilde{\delta}_k / \tilde{h}_k}\right)$$

- Assuming that the error is small, then we can have

$$\bar{x}_k = \frac{1}{\tilde{h}_k}\left(\frac{\tilde{h}_k \tilde{x}_k}{1 + \tilde{\delta}_k / \tilde{h}_k} + \frac{\tilde{v}_k}{1 + \tilde{\delta}_k / \tilde{h}_k}\right) \approx \frac{1}{\tilde{h}_k}\left(\left[1 - \frac{\tilde{\delta}_k}{\tilde{h}_k}\right]\tilde{h}_k \tilde{x}_k + \left[1 - \frac{\tilde{\delta}_k}{\tilde{h}_k}\right]\tilde{v}_k\right)$$

$$= \tilde{x}_k + \frac{\tilde{v}_k}{\tilde{h}_k} \underbrace{- \frac{\tilde{\delta}_k}{\tilde{h}_k}\tilde{x}_k - \frac{\tilde{\delta}_k}{\tilde{h}_k^2}\tilde{v}_k}_{Extra\ noise}$$

- Let the preamble be QPSK modulated (and normalized) and the channel gain be one.

- In addition, the channel is estimated with two long preambles. Then,

$$\sigma_{ex}^2 = \sigma_{\tilde{\delta}}{}^2 \sigma_{\tilde{x}}{}^2 + \sigma_{\tilde{\delta}}{}^2 \sigma_{\tilde{v}}{}^2 = \frac{1}{2}\sigma_{\tilde{v}}{}^2 + \frac{1}{2}\sigma_{\tilde{v}}{}^2 \sigma_{\tilde{v}}{}^2$$

- The MSE in data detection becomes $\sigma_v^2 + \sigma_{ex}^2$. And, the SNR becomes

$$-10\log_{10}\left(\sigma_v^2 + \sigma_{ex}^2\right) = -10\log_{10}\sigma_v^2\left(1 + \frac{\sigma_{ex}^2}{\sigma_v^2}\right) = -10\log_{10}\sigma_v^2 - 10\log_{10}\left(1 + \frac{\sigma_{ex}^2}{\sigma_v^2}\right)$$

- Due to the channel estimation, the SNR is then degraded by

$$10\log_{10}\left(1 + \frac{\sigma_{ex}^2}{\sigma_v^2}\right).$$

- Note that noise in channel estimate and data detection is different (preamble and data symbol).

- Data detection:

$$\hat{x}(k) = \frac{\tilde{y}(k)}{\overline{h}(k)}$$

* Estimated data in the *k*th subcarrier

- The final channel estimate is the average of the two estimates with two long preambles.
- With the degraded SNR, we can then calculate the probability of QAM symbol error.
- Note that the effect of residual CFO and SFO will be accumulated. Without proper tracking, the error rate will be increased for later OFDM symbols.

- Performance evaluation of synchronization:
  - Set a window size (W_p) for packet detection.
  - If the packet is detected within W_p, the detection is successful (right timing plus/minus $W_p/2$).
  - Probabilities of detection, missing, and false alarm can then be calculated.
  - If packet detection is successful, CFO estimation, symbol timing, and channel estimation can then be conducted (searching window for symbol timing: W_s).
  - MSEs can then be calculated to evaluate the performance of CFO estimation/symbol timing.
- The size of W_s is a tradeoff between the performance and the computational complexity.
- All the derived theoretical values can be compared with the simulated, verifying the correctness of simulations.

- Practice 1:
  - Build an OFDM system with LTE specifications including
  - Transmitter
  - AWGN channel, and
  - Receiver (perfect synchronization).
  - Plot SER vs. SNR.
  - Assume QPSK symbols
  - FFT size=2048 (number of used subcarriers=1200)

* Note: SNR in time domain is different from that seen in each subcarrier

- Practice 2:
  - Redo the practice with a multipath channel.

- Result: